

Investigating phonological abstraction through feature induction

*Features in Phonology, Morphology, Syntax: What are they?
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Overview

- Introduction
 - should grammars always refer to features?
 - approach from perspective of machine learning

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 - approach from perspective of machine learning
- Computational simulation: how does a learner abstract over domains of application?
 - model, data, method
 - results: grammars with features in some constraints only

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 - approach from perspective of machine learning
- Computational simulation: how does a learner abstract over domains of application?
 - model, data, method
 - results: grammars with features in some constraints only
- Discussion: implications of grammars referring to features as well as other units

Introduction: background

- Features help generalize over domains of application of rules or constraints
- Phonology: features generalize over segment/phoneme categories

E.g., /-z/ → [-s] / [p,t,k,f,θ,s,ʃ,tʃ]_ ⇒
/-z/ → [-s] / [-voice]_

Introduction: background

- Question:

Is it always advantageous (both for the analyst and the speaker) to state every rule or constraint in the grammar in terms of features?

- In other words: is it unreasonable for grammar to refer to sound event through levels of abstraction other than features?

(Not counting prosodic units, suprasegmentals)

Introduction: background

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 - establishes preference for phonetically natural rules
- (see Chomsky & Halle 1968, Postal 1968, Kenstowicz & Kisseberth 1979 for more)

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 - adapting categorical versions of phonetic features is most economical hypothesis of representation
 - establishes preference for phonetically natural rules

(see Chomsky & Halle 1968, Postal 1968, Kenstowicz & Kisseberth 1979 for more)
- Models with richer representations lead to longer grammars, therefore are disfavored

Introduction: empirical issue

- Phonological patterns may apply to groups of segments, or to single segments.

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- English (Jensen 1993, Mielke 2007):
 - sibilants [s,z,ʃ,ʒ,tʃ,dʒ] may not precede [s,z] word-finally: *[bʌs-s, bʌz-z, pæʃ-s, peɪdʒ-z]

p	t	k	Red: disallowed before [s,z] word-finally
b	d	g	
f	θ	s ʃ tʃ	
v	ð	z ʒ dʒ	
m	n	ŋ	
w	r l	j	

Introduction: empirical issue

- Phonological patterns may apply to groups of segments, or to single segments.
- English (Jensen 1993, Mielke 2007):
 - only [s] may start a three-consonant word-initial cluster: [strit], *[ftrit, ntrit, tʃtrit]

p	t	k	Red: disallowed before [s,z] word-finally	
b	d	g	Purple: allowed as C1 in word-initial CCC	
f	θ	s	ʃ	tʃ
v	ð	z	ʒ	dʒ
m	n	ŋ		
w	ɹ	l	j	

Introduction: empirical issue

- Phonological patterns may apply to groups of segments, or to single segments.
- P-base cross-linguistic database of phonological classes (Mielke 2007):
 - 13 patterns encoded as applying to one segment
 - 11 additional cases (apply to all segments but one) found by manual search of languages starting with A alone

Introduction: empirical issue

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p	t	k	Red: [+anterior]
b	d	g	
f	θ	s	ʃ
v	ð	z	ʒ
m	n	ŋ	
w	ɹ	l	j

Introduction: empirical issue

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v	ð	z	ʒ dʒ
m	n		ŋ
w	ɹ l	j	

Red: [+anterior]
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Introduction: empirical issue

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m	n	ŋ		
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Red: [+anterior]

Blue: [-voice]

Green: [+strident]

Introduction: always features?

- Featural representation of one-segment class will always be longer and more complex
- Is it desirable (for analyst/speaker) to represent one-segment classes in this way?

Introduction: always features?

- Featural representation of one-segment class will always be longer and more complex
- Is it desirable (for analyst/speaker) to represent one-segment classes in this way?
 - If features are *a priori* specified as building blocks of grammars: yes
 - Is this still the case when this *a priori* assumption is taken away?

Introduction: machine learning

- I will approach this question in terms of machine learning
- Given a choice between representing a pattern in terms of segments and in terms of features:
 - How will data containing both one-segment and multi-segment patterns be learned?
 - Learning algorithm not explicitly instructed to aim for a certain level of abstraction

Introduction: machine learning

- Possible outcomes:
 1. The grammars have constraints referring only to segments
 2. The grammars have constraints referring only to features
 3. The grammars have constraints referring to both features and segments

Introduction: assumptions

- Essential assumptions for this simulation:
 1. Atomic segment units are available to the language user:
 - active in on-line processing of speech
(Jesse et al. 2007, Nielsen 2011)
 - active in phonological processes, e.g.,
consonant OCP
(Coetzee & Pater 2008 and references therein)

Introduction: assumptions

- Essential assumptions for this simulation:
 2. Phonological features are learned:
 - assuming universal features, the same feature is realized differently across languages
(Cho & Ladefoged 1999)
 - therefore, phonetic information cannot be sufficient for mapping perception/articulation to features

Introduction: assumptions

- Essential assumptions for this simulation:
 2. Phonological features are learned:
 - contextual information must be used
 - grammar contains contextual information
 - use contextual information from grammar (rather than contextual information outside of grammar)
(see Mielke (2004) on learning features from phonological patterns)

Introduction: assumptions

- Consequences of these assumptions:
 1. Segment-to-feature mapping must be learned simultaneously with grammar
 2. Constraints/rules referring to features gradually become available during grammar learning process

Introduction: assumptions

- Non-essential working assumptions:
 - Features are induced only from contextual information: no phonetic content
(Substance-free phonology: Morén 2006, 2007 (and many others))
 - All phonological constraints are induced instead of innate
(see Hayes & Wilson 2008 on constraint induction)

Introduction: summary

- Question: Is it always advantageous (both for the analyst and the speaker) to state every constraint in the grammar in terms of features?
- Crucial empirical phenomenon: one-segment patterns
- Learning one-segment and multi-segment patterns:
all-feature grammars as outcome?
- *Preview*: segment/feature grammars obtained

Simulation: overview

- Machine learning simulation based on paradigm established by Hayes & Wilson (2008):
 - phonotactic constraint-based grammar is built up from positive data
 - violable constraints selected and weighted to optimally predict the attested data

Simulation: overview

- Departure from Hayes & Wilson's learner:
 - features are not built into the model, but induced at intermediate stages of grammar learning

- Questions:
 - will features be learned at all?
 - will all constraints in grammars learned by this procedure always use features?

Simulation: model

- Maximum Entropy model
(Della Pietra et al. 1997, Hayes & Wilson 2008)
 - probability distribution over possible representations based on weighted violable constraints (*à la* OT/Harmonic Grammar)
 - constraints weighted to make this distribution maximally similar to what is observed

(see Appendix for more)

Simulation: model

- Regularization:
 - Optimization of constraint weights constrained by L2 prior (Hastie et al. 2009):
 - keeps sum of constraint weights as small as possible
 - encourages more general constraints: one general constraint with larger weight yields smaller sum of weights than several specific constraints with smaller weights

Simulation: model

- Information gain:
 - Value which estimates how much a constraint will improve the current grammar
(bring it closer to predicting the observed data)
 - Information gain of a constraint correlates with how accurately it captures a (sub)pattern in the data

(see Appendix for more)

Simulation: model

- Constraints:
 - phonotactic constraints against two- and three-element sequences of word-boundaries, segments or features
 - examples: *#m, *km, *u[labial]u

Simulation: model

- Constraints:
 - selected probabilistically based on information gain:
 - start with random seed constraint
(subject to information gain threshold)
e.g. $*\#pi$
 - seed constraint repeatedly manipulated until this does not lead to increase in information gain
e.g. $*\#pi \rightarrow *\#mi \rightarrow *\#m$

Simulation: model

- Features found by clustering information gain of closely related constraints
 - Intuition:
a feature denotes a class of segments that participates in the same phonological pattern

Simulation: model

- Features found by clustering information gain of closely related constraints
 - Implementation:
 - a feature denotes a class of segments which yields high-valued constraints when inserted in the same context

	i	a	u	p	t	k	b	d	g	m	n	ŋ
*#_	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.015	0.015	0.015

Simulation: model

- Features found by clustering information gain of closely related constraints
 - Cluster analysis (Mixture of Gaussians, Everitt 2011) divides same-context constraints into high and low information gain value clusters (whenever appropriate)

	i	a	u	p	t	k	b	d	g	m	n	ŋ
*#_	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.015	0.015	0.015

Simulation: model

- Features found by clustering information gain of closely related constraints
 - Focus segments extracted from cluster of high information-value constraints
 - Feature label assigned to these segments
(phonetics not taken into account - labels are arbitrary)

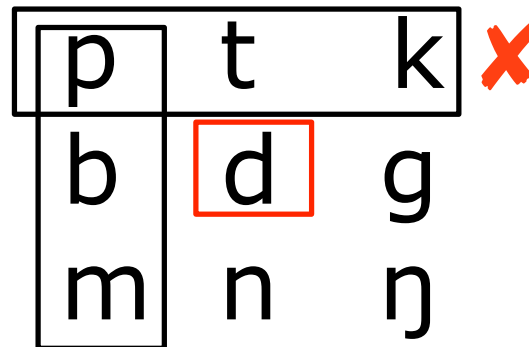
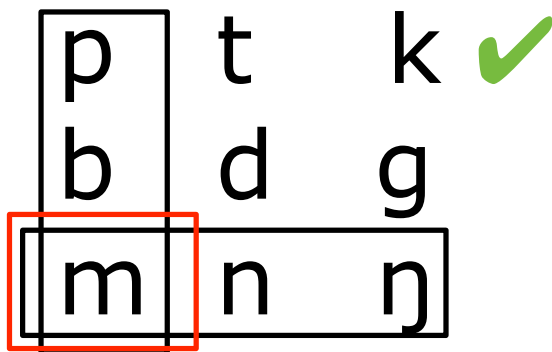
[nasal]

	i	a	u	p	t	k	b	d	g	m	n	ŋ
*#_	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.015	0.015	0.015



Simulation: data

- Nature of data to consider:
 - both one-segment and multi-segment patterns must be present
 - single segment in one-segment pattern must be representable as intersection of segment classes appealed to in multi-segment patterns



Simulation: data

- Example: English (Jensen 1993, Mielke 2007)

p	t	k	Red: disallowed before [s,z] word-finally	
b	d	g		
f	θ	s	ʃ	tʃ
v	ð	z	ʒ	dʒ
m	n	ŋ		
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Simulation: data

- Example: English (Jensen 1993, Mielke 2007)

p	t	k	Red: disallowed before [s,z] word-finally	
b	d	g	Blue: allowed as C3 in word-final CCC	
f	θ	s	ʃ	tʃ
v	ð	z	ʒ	dʒ
m	n	ŋ		
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Simulation: data

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p	t	k	Red: disallowed before [s,z] word-finally
b	d	g	Blue: allowed as C3 in word-final CCC
f	θ	ʃ	Purple: allowed as C1 in word-initial CCC
v	ð	z	
m	n	ŋ	
w	ɹ	l	j

Simulation: data

- Example: English (Jensen 1993, Mielke 2007)

p	t	k	Red: disallowed before [s,z] word-finally
b	d	g	Blue: allowed as C3 in word-final CCC
f	θ	ʃ	Purple: allowed as C1 in word-initial C
v	ð	z	
m	n	ŋ	
w	ɹ	l	j

- Other examples like this found in, e.g., Yoruba (Pulleyblank 1988)

Simulation: data

- The actual data used for the simulations was a toy language which shared the crucial properties of these examples:

p t k

b d g

m n ŋ

Red: no nasals word-initially

Simulation: data

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p	t	k
b	d	g
m	n	ŋ

Red: no nasals word-initially

Blue: no labials between high vowels [i,u]

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p	t	k	Red: no nasals word-initially
b	d	g	Blue: no labials between high vowels [i,u]
m	n	ŋ	Purple: no [m] word-finally

- All possible CVCVC forms obeying these restrictions present in input to the learner

Simulation: procedure

- Initial state: no constraints, features unavailable
- All potential representations (given in segments) equally probable

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- Initial state: no constraints, features unavailable
- All potential representations (given in segments) equally probable
- All CVCVC sequences over toy language inventory are potential representations
- Observed forms have no initial nasals, no labials between high Vs, no final [m]

possible: ... padam padan ... nitun ditun dibun
 observed: ... padan ... ditun

Simulation: method

- Step 1: Find a group of constraints which forms a local peak in gain value

e.g., $\{*\#m, *\#n, *\#\eta\}$

These have higher information gain than,
e.g., $*\#p, *\#am, *\#n$:

$*\#p, *\#am, *\#n$ ban (more) observed forms in the data and bring the empty grammar less close to predicting the observed data

Simulation: method

- Step 2: Find all possible contexts that can be made from these constraints.

The constraints $\{*\#m, *\#n, *\#\eta\}$ can be factored into the following contexts

*#_
*_m
*_n
*_η

Simulation: method

- Step 3: for every context, find if there is a cluster of segments which yields a high information gain value when inserted in that context; assign feature labels to those clusters

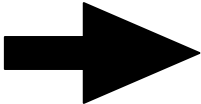
	i	a	u	p	t	k	b	d	g	m	n	ŋ
*#_	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.015	0.015	0.015

$[m, n, \eta] \Rightarrow [\text{nasal}]$

Simulation: method

- Step 4: add the selected constraints to the grammar, and optimize their weights

Grammar:

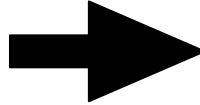
*#m: 0		*#m: 6
*#n: 0		*#n: 6
*#η: 0		*#η: 6

Simulation: method

- Steps 1-4 repeated until final goal is reached (observed data have at least 95% total likelihood)
- Features induced at step 3 available for use in constraints at next occurrence of step 1
 - Once $*\#m$, $*\#n$, $*\#n$ are in the grammar, and the feature label $[\text{nasal}] = [m, n, n]$ is induced,
 - the constraint $*\#[\text{nasal}]$ becomes available

Simulation: method

- E.g., $*#[\text{nasal}]$ has high information gain value (*not in current grammar, tightly fits data pattern*)
- If selected and weighted, $*#[\text{nasal}]$ takes away all the weight of $*\#m$, $*\#n$, $*\#\eta$
- zero weight equivalent to absence from grammar

		$*#[\text{nasal}]: 8$
$*\#m: 6$		$*\#m: 0$
$*\#n: 6$		$*\#n: 0$
$*\#\eta: 6$		$*\#\eta: 0$

Simulation: method

- Reset to 0 because of regularization prior:
 - higher weight on one constraint is better than lower weights on three constraints combined
- This effect occurs when the candidates punished by a new constraint are a strict superset of those punished by individual existing constraints:
 - *#[nasal] *versus* *#m, *#n, *#ŋ
 - *[hi][labial][hi] *versus* *ibi, *ibu, *umi ...

Simulation: method

- Reset to 0 does not happen when feature-based constraint and segment-based constraint are homonymous:
 - $*[\text{labial, nasal}]\# = *m\#$

Simulation: method

- Reset to 0 does not happen when feature-based constraint and segment-based constraint are homonymous:
 - $*[\text{labial}, \text{nasal}]\# = *m\#$
- Homonymous feature-based constraint has lower information gain (*repeats existing constraint*)
 - $*[\text{lab}, \text{nas}]\#$ less likely to be selected
- Even when it is selected, no reset to 0
 - $*m\#$ retains some weight next to $*[\text{lab}, \text{nas}]\#$

Simulation: results

- 31 out of 32 runs yielded grammars referring to both segments and features
- Most frequent grammar:
*#[nasal], *[high][labial][high], *m#
- One all-feature grammar:
*#[nasal], *[hi][labial][hi], *[labial,nasal]#
- All other grammars were variations of the most frequently observed grammar (see Appendix)

Simulation: results

- The learner strongly prefers a segmental representation for the one-segment pattern, and a featural representation for the multi-segment patterns.
- By extrapolation, languages with at least one one-segment pattern are expected not to represent that one-segment pattern (entirely) in terms of features.

Discussion

- Machine learning simulation shows:
 - when *a priori* assumption of all-feature grammars is lifted:
 - despite bias in favor of generalization,
 - one-segment patterns not represented in terms of features
- This is because features are more efficient only for multi-segment patterns

Discussion

- These results show that:
 - features can be learned in a bottom-up fashion from phonological patterns
 - grammars that represent one-segment patterns without features emerge despite bias towards generalization (from regularization)

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- These results show that:
 - features can be learned in a bottom-up fashion from phonological patterns (see also Archangeli et al. 2012)
 - grammars that represent one-segment patterns without features emerge despite bias towards generalization (from regularization)

(Procedure relies only on structural factors: these methods may also be applied to other domains of language, e.g., syntax)

Discussion: implications

- Implication for (phonological) analysis:
 - when a (phonological) pattern is analyzed, it is not trivial that it is stated in terms of features
 - rather, question of appropriate level of abstraction asked for every pattern

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- Implication for (phonological) analysis:
 - when a (phonological) pattern is analyzed, it is not trivial that it is stated in terms of features
 - rather, question of appropriate level of abstraction asked for every pattern

- Why would level of abstraction matter?

Discussion: implications

- There are psycholinguistic techniques to probe into levels of abstraction:
 - Bach testing (Halle 1978)
 - Priming (Jesse et al. 2007)
 - Talker adaptation (McQueen et al. 2006, Nielsen 2011)
- Ergo: level of abstraction in hypothesized rules/constraints matters empirically
- Important direction for future research

Discussion: implications

- Another consequence of grammars with both featural and lower-order descriptions:
 - same sound event may be described at different levels of abstraction
e.g., [m] or [labial,nasal]
 - this means: multiple autonomous levels of representation for sounds

Discussion: implications

- This property is reminiscent of models such as
 - Turbidity (Goldrick 2001)
 - Abstract Declarative Phonology (Bye 2006)
 - Colored Containment (Van Oostendorp 2004, 2008)
 - Bidirectional Phonology (Boersma 2007)
- Grammars with multiple levels of abstraction need little extension to have the extra power of such models (Nazarov 2012, 2013)
- Another direction for further investigation

Conclusion

- Are features always better for representing phonological patterns?
- Investigation through machine learning of features:
 - no: one-segment patterns favor representation by segment units
- Grammars which refer both to features and lower-order units (segments) are worthy of consideration by speakers and analysts

Thank you!

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Appendix: Maximum Entropy model

- Observed distribution p

$$p(\mathbf{x}) = \text{count}(\mathbf{x}) / \sum_{y \in \Omega} \text{count}(y)$$

- Predicted distribution q : based on harmony scores H for every candidate

$$H(\mathbf{x}) = \sum (w_i \times C_i(\mathbf{x}))$$

$$q(\mathbf{x}) = e^{H(\mathbf{x})} / \sum_{y \in \Omega} e^{H(y)}$$

Ω stands for the set of possible representations

Appendix: Maximum Entropy model

- Objective of the model: manipulate weights to minimize K-L divergence of observed distribution from predicted distribution

$$D_{\text{KL}}(t \parallel w) = \sum [t(x) * \ln(t(x) / w(x))]$$

$$\text{Obj} = \min_W [D_{\text{KL}}(p \parallel q) + \sum_{w \in W} [(w - \mu)^2 / 2\sigma]]$$

regularization term;

$\mu = 0$ and $\sigma = 10,000$

Appendix: Information gain

- Let C^* be a proposed new constraint, and w^* its weight
- Let q' be the distribution predicted by the current grammar augmented with C^* with weight w^*
- Information gain: maximum descent in K-L divergence of observed from predicted when C^* is added to the grammar

(L2 regularization with $\mu = 0$ and $\sigma = 10,000$ added to this maximization also)

$$G(w^*, C^*) = \max_{w^*} [D_{KL}(p \parallel q) - D_{KL}(p \parallel q')]$$

Appendix: Results

- Word-initial pattern:
 - 26 grammars: represented by *#[nasal]
 - 3 grammars: *#[nasal], *#[nasal]V
 - 3 grammars:

(42) *the three runs at which the word-initial restriction was represented by non-overlapping constraints*

Run 11			Run 16			Run 17		
Constraint	Traditional notation	Weight	Constraint	Traditional notation	Weight	Constraint	Traditional notation	Weight
*#m	*#m	2.68	*#{nŋ}	*#[nasal, -labial]	2.78	*#{nŋ}	*#[nasal, -labial]	3.37
*#{nŋ}	*#[nasal, -labial]	1.12	*#{mŋ}	*#[nasal, -coronal]	2.78	*#m	*#m	2.68
*#{nŋ} {aiu}	*#[nasal, -labial]V	1.12						
*#{nŋ}	*#[nasal, -labial]	1.12						

Appendix: Results

- Word-medial pattern:
 - Combination of one or more of the following constraints:

(43) a survey of all 18 constraints attested in the final grammars which represented (part of) the word-medial pattern

*{iu}{pbm}{iu}	*{iu}{pbm}u	*{iu}{pbm}	*mi	*{aiu}m
*{iu}{pm}{iu}	*{iu}{pbm}i	*{pbm}{iu}	*mu	*m{aiu}
*{iu}{pb}{iu}	*u{pbm}{iu}	*{iu}{pb}		
*{iu}b{iu}	*u{pm}{iu}	*{iu}m		
*{iu}m{iu}	*{iu}{pb}u			

- E.g.: *{iu}{pb}{iu}, *{iu}m{iu}

Appendix: Results

- Word-final pattern:
 - 28 grammars: only *m#
 - 1 grammar: only *[nasal,labial]#
 - 3 grammars:

(44) *the three runs (not counting run 23) at which the word-final restriction was not solely represented with the constraint *m#*

Run 12			Run 16			Run 17		
Constraint	Traditional notation	Weight	Constraint	Traditional notation	Weight	Constraint	Traditional notation	Weight
*m#	*m#	2.29	*m#	*m#	2.27	*m#	*m#	2.15
*{aiu}m#	*Vm#	0.05 ¹⁵	*{m}#	*[nasal,labial]#	0.16	*{aiu}m#	*Vm#	0.25