PHILLIP BRICKER

REDUCING POSSIBLE WORLDS TO LANGUAGE

(Received 12 September, 1986)

Can possible worlds be reduced to language?[1] That is, can talk about possible worlds be reconstrued as talk about respectable linguistic entities, thus allaying ontological worries as to what strange manner of entity a possible world might be? I will argue in the final section of this paper that no such reduction can succeed. But if my argument is to be conclusive, it will have to be directed against the strongest possible case for reduction, not against the proposals most commonly heard. The commonly heard proposals succumb to a simple cardinality argument: on quite modest assumptions, it can be shown that there are more possible worlds than there are linguistic entities provided by the proposal; it follows straightway that the linguistic entities cannot *be* the possible worlds. One might be tempted to think that some version of the cardinality argument could be used quite generally to show that any attempt at reducing possible worlds to language must fail. This, however, is not the case. In this paper I will show how the standard proposals for reduction can be generalized in a natural way so as to make better use of the resources available to them, and thereby circumvent the cardinality argument. Once we see just what the limitations are on these more general proposals for reduction, we will be able to see more clearly where the real difficulty lies with any attempt to reduce possible worlds to language. Roughly, the difficulty is this: no actual language could have the descriptive resources needed to represent all the ways things might have been. I will conclude by arguing that this same difficulty spells doom for any nominalist or conceptualist proposal for reducing possible worlds.

1. INTRODUCTION

In order to set the stage for what will follow, let me briefly recapitulate

Philosophical Studies **52** (1987) 331–355.

some of the ontological positions that might be held with respect to possible worlds. The *realist* with respect to possible worlds takes possible worlds to be primitive to his ontology. He means his talk about possible worlds to be taken literally, not as disguised talk about some other kind of entity. Non-realists can take either of two stances towards possible worlds: they can simply reject talk of possible worlds outright, or they can try to reinterpret talk of possible worlds in a way they find ontologically acceptable. The first sort of non-realist, if consistent, must also reject the philosophical analyses that have been based upon the notion of a possible world, or, at any rate, take them to have at most heuristic value and to lack any serious philosophical import. On the other hand, the *reductionist* with respect to possible worlds often finds such analyses to be fruitful, illuminating, and philosophically important. The problem is not that he finds talk of possible worlds meaningless or incomprehensible, but that his philosophical conscience will not permit him to take such talk at face value. Possible worlds are nowhere to be found in his ontology.

In this paper, I will be concerned only with *ontological* reduction: an ontological reduction of possible worlds leaves talk of possible worlds syntactically unchanged, but reinterprets the values of the variables by identifying possible worlds with entities that the reductionist accepts.[2] Moreover, I will assume that the reductionist is an actualist and a nominalist (of sorts): his ontology consists only of actual, concrete entities together with what can be constructed out of such entities by means of set theory.[3] Some philosophical consciences, it is true, have even balked at an ontology of sets; but, as will become clear, any attempt at reduction would be crippled at the outset if the reductionist did not at least have set theory at his disposal. Now, what the reductionist must do if he is to succeed in appeasing his philosophical conscience is to show that all talk about possible worlds that he wants to preserve can be interpreted as disguised talk about some kind of entity that he does accept.

It is natural at this point for the reductionist to turn to language. For among the entities that he does accept are *linguistic entities*: finite sequences of types of concrete marks or sounds (where sequences and types are given their standard interpretations in terms of sets), and set-theoretic constructions out of such sequences. Moreover, we do commonly gain access to the notion of a possible world through language, through what purport to be descriptions of possible worlds. So the reductionist might hope that talk about possible worlds could be made respectable by identifying possible worlds with their purported linguistic descriptions, or, at any rate, constructions out of such descriptions.[4]

To what language shall we assume that these descriptions belong? If we allow the reductionist to take the notion of a language too broadly, the resulting reduction is likely to be circular. For example, if he can merely *stipulate* that the language contains a name for every possible world, then the existence of the language (with this interpretation) is at least as dubious as the existence of the possible worlds themselves. The surest safeguard against such circularity is to require that possible worlds be reduced to a *natural* language, perhaps enriched by the languages of science, mathematics, and formal logic. Unfortunately, many sentences of a natural language are unsuitable for use in constructing the possible worlds. I will thus assume that the reductionist has extracted from the enriched natural language a sublanguage satisfying all of the following conditions:

(1) All sentences are declarative sentences.
(2) Truth values of sentences are independent of contexts of inscription or utterance.
(3) All sentences are unambiguous.
(4) There is no vagueness in the truth conditions for sentences, let alone indeterminacy of a more radical sort.
(5) Sentences can be uniquely parsed so as to exhibit their truth-functional and quantificational form.

Any sublanguage of a natural language satisfying these conditions will be called a *reasonable* language — reasonable in the sense that it is appropriate for the project of reducing the possible worlds.[5] One characteristic of the original natural language will be shared by all of its sublanguages: the expressions of the language are *finite* sequences over a *finite* alphabet, and thus the language has at most a *countably* infinite number of distinct expressions. One of the chief contentions of this paper, however, is that this limitation is not as severe as might first appear.

2. THREE NECESSARY CONDITIONS FOR REDUCTION

What conditions must be met by a successful reduction of possible worlds to language? Let us suppose that the reductionist has fixed upon a particular theory of possible worlds, that is, the set of those sentences about possible worlds whose truth he wishes to preserve. Let us call this the *possible worlds theory*. Just which statements about possible worlds will be included in this theory is largely up to the reductionist. But I assume that at the very least he will want the theory to be strong enough to support the standard analysis of the alethic modalities as quantifiers over possible worlds. Of course, the reductionist cannot hope to preserve the truth of *everything* that a realist would assert about possible worlds. For example, a realist would assert that possible worlds are not linguistic entities, whereas this cannot be preserved by any reduction of possible worlds to language. Similarly, other statements directly or indirectly about the ontological status of possible worlds must remain a source of disagreement between the realist and the reductionist. What the reductionist must do is to fix upon a theory that is strong enough to support the possible worlds analyses that he wants to accept, but not so strong that the very possibility of reduction is excluded by the theory itself. Now, a minimal condition that the reduction must satisfy is that it provide a translation of sentences about possible worlds into sentences about linguistic entities that maps truths of the possible worlds theory into truths, and falsehoods into falsehoods.

But the sort of reductionist here considered not only wants the translation to preserve the truth of the possible worlds theory, he wants it to preserve the structure of the theory as well. For example, sentences that make existential claims about possible worlds are to be translated into sentences that make existential claims about linguistic entities. The reduction is not intended to provide a restructuring of logical form, only a switch of the underlying ontology. If a reduction provides a truth- and structure-preserving translation of the possible worlds theory, then I will call the translation *faithful*, and say that the reduction satisfies the *faithfulness condition*.

There is a routine method for providing such a faithful translation, at least if we assume that the possible worlds theory is couched within an extensional language. To each possible world one assigns a linguistic entity to serve as that world's representative — in effect, to *be* the possible world. Distinct possible worlds are assigned distinct linguistic entities if they are discernible[6], but in general the assignment need not be one-to-one. Any such assignment uniquely determines a faithful translation of the possible worlds theory into a theory of the linguistic entities. For example, given any unary predicate expressing some property of possible worlds, a corresponding predicate expressing some property of the linguistic entities is introduced as follows: the new predicate is true of just those linguistic entities that were assigned to some possible world of which the old predicate was true. More generally, the entire possible worlds ideology (to use Quine's term), as represented by the predicates and function symbols of the language of the possible worlds theory, is transferred by way of the assignment into a corresponding ideology of the linguistic entities, as represented by corresponding new predicates and function symbols. Every sentence about possible worlds can then be translated into a sentence about linguistic entities simply by replacing old predicates and function symbols by corresponding new ones. The translation clearly preserves logical structure. Moreover, by wending one's way up Tarski's inductive definition of truth, it can easily be shown that the translation is truth-preserving as well. So, any such assignment of linguistic entities to possible worlds can be used to ensure that the faithfulness condition be satisfied.

But this whole procedure has an air of circularity about it. The reductionist wishes to show that he need not admit a primitive notion of possible world to his ontology. According to the above procedure, he does this by showing that, for each possible world, a corresponding linguistic entity can be found. But if he thus invokes the possible worlds in selecting the linguistic entities, how can he claim to have eliminated the possible worlds? I do not think that he can unless the reduction meets the following *non-circularity condition*, which then shows that the circular way of describing the reduction can be avoided: the possible worlds, and all the possible worlds ideology, must be *constructed* out of the actualist ontology and ideology that is already available to be reductionist. In other words, all the primitive predicates and function symbols of the possible worlds theory must be translated by predicates and function symbols that are *definable* within the reductionist's theory of the language in question. Thus, the reduction will be circular if the possible worlds are eliminated by stepping up the ideology of the reductionist's theory of language. As we shall see, the non-circularity condition is difficult to apply in practice if it is unclear just what the reductionist should be allowed to include within his ideology. My general policy will be to allow the reductionist whatever property of linguistic entities he thinks he can make sense of; he will need all the help he can get.[7]

There is a further condition that, I believe, must be satisfied by any successful reduction of possible worlds to lingusitic entities. As yet I have said nothing to require that the linguistic entities correspond in a natural way with the possible worlds that they are to replace. It might be that the entire syntactic structure of the possible worlds theory was duplicated, sentence for sentence, by the syntactic structure of some wholly unrelated actualist theory of linguistic entities. But the existence of such a duplication of structure cannot by itself provide grounds for concluding that the linguistic entities are fit to play the role of the possible worlds. We need to require that the linguistic entities be naturally linked to the possible worlds that they are to replace. Now, the obvious place to look for such a natural correlation is to the semantics of the language: the linguistic entities represent possible worlds in virtue of what they mean. More exactly, I will say that the reduction satisfies the *naturalness condition* as long as each possible world is replaced by a linguistic construction that can serve as a complete description of that possible world, where a linguistic construction *completely describes* a possible world if it is true at that world, and perhaps at worlds indiscernible from that world, but at no others.

Again I have taken the realist's perspective and spoken as if there were possible worlds existing over and above the linguistic entities. Only the realist can speak of a correlation between possible worlds and linguistic entities as being more or less natural. But I think that the naturalness condition can be restated in a way that the reductionist can accept. For although it makes no sense to the reductionist to speak of a linguistic construction as providing a complete description of a possible world, it does make sense to speak of a linguistic construction as *purporting* to provide such a description, even if he doesn't believe that anything exists for the description to describe. So the reductionist too can recognize which linguistic entities are naturally suited to play the role of the possible worlds, although, as we shall see, modal notions may be needed for this purpose.

3. FIRST PROPOSAL

With these three conditions in hand — faithfulness, non-circularity, and naturalness — let us turn to the evaluation of specific proposals for reducing possible worlds to language. Suppose that the reductionist has fixed upon some particular language L satisfying the five requirements listed in the introduction to this paper. If two worlds are such that there is a sentence of L true at one of the worlds but not the other, then I will say that the worlds are *linguistically discernible with respect to L*, or, *L-discernible*, for short.[8] The first proposal to be considered is based upon the following idea: a class of possible worlds (in particular, the class of all possible worlds) is reducible to L as long as any two discernible worlds from that class are also *L-discernible*. For if discernible worlds are always *L-discernible*, then the reductionist can succeed in assigning distinct linguistic entities to discernible worlds by assigning to each possible world the set of those sentences of L that are true at the world. Note that the reductionist is not required to assign distinct linguistic entities to indiscernible possible worlds (if any there be); I assume that the principle of the identity of indiscernible worlds is compatible with the possible worlds theory.

Which sets of sentences of L will be identified with possible worlds under this proposal? Such a set of sentences must be *consistent*, that is, there must be a possible world at which all of the sentences of the set are true. Moreover, let us assume that it follows from the possible worlds theory that possible worlds are fully determinate, and thus that for any sentence of L and for any possible world, either that sentence or its negation is true at that world. Then the set of sentences true at a world will be a *maximal* consistent set, containing for any sentence of L either that sentence or its negation. Thus the proposal for reduction that we are considering can be formulated as follows:

PROPOSAL 1. *Possible worlds are maximal consistent sets of sentences of L.*

Note that the notion of consistency used in Proposal 1 is a modal notion; it cannot be taken to be narrowly logical consistency, where this is defined, for example, as truth under some reinterpretation of the non-logical vocabulary, lest there turn out to be possible worlds in which bachelors are married.

Proposal 1 is the proposal made by Richard Jeffrey [8]: 196—197, who called such maximal consistent sets of sentences *novels*. It has been defended by Frank Jackson [7]: 24, and, more extensively, by Andrew Roper [15].[9] It is related to Carnap's well-known proposal to identify possible worlds with *state descriptions*, where a state description is defined as a set of sentences (of some given language) which contains for every atomic sentence either that sentence or its negation, but not both, and no other sentences.[10] One of the ways in which Carnap's proposal differs from Proposal 1 is that it places additional restrictions upon the language in question.[11] Thus, Carnap must assume that the language contains names (or there would be no atomic *sentences* at all) and that distinct names denote distinct individuals. More significantly, Carnap must assume that the primitive predicates of the language have been chosen (and so *can* be chosen) in such a way as to guarantee that all the state descriptions are consistent (in the modal sense). This second assumption, which is quite strong, can be avoided by changing the proposal to read that possible worlds are to be identified, not with state descriptions, but with maximal consistent sets of *basic sentences*, where a basic sentence is either an atomic sentence or the negation of an atomic sentence. Then the proposal can be compared with Proposal 1 by assuming that the language in question is a reasonable language, and that it contains a name for every individual that can be uniquely picked out by some formula of the language. But this revised Carnapian proposal is still not equivalent to Proposal 1. In the 'possible worlds' it provides, only individuals that can be given names exist. Thus the proposal cannot account for the fact that other individuals *might* have existed, and it thereby misrepresents the facts of modality. In what follows I will say no more about Carnap's proposal, and focus upon the more plausible Proposal 1 and its generalizations.[12]

4. THE CIRCULARITY OBJECTION

It might seem that Proposal 1 violates the non-circularity condition because it makes use of the notion of a consistent set of sentences, and consistency has simply been characterized in terms of truth at a possible world. The reductionist, if he is to succeed, must be able to define consistency in a way that makes reference only to entities and notions that he already accepts. Here, I think, the reductionist must turn to empirical linguistics. Recall that L is assumed to be a sublanguage of an actual language used by actual people. For the reductionist, whatever facts there are about the consistency of sets of sentences of L will be grounded in actual usage. Now, empirical data of actual usage is available to the reductionist in constructing the possible worlds. Contained within the data will be direct or indirect reports of modal intuitions that the reductionist can cash out in terms of the consistency of sets of sentences. Indeed, whenever the realist uses his modal intuitions to decide the truth or falsity of some sentence about possible worlds, the reductionist can use reports of those modal intuitions in constructing a notion of possible world that preserves the truth value of the sentence. Presumably, there will not be enough data to answer all questions about the consistency of sets of sentences. But all that matters for the reduction is that there be enough data from which a notion of possible world can be constructed that provides a faithful reinterpretation of the possible worlds theory.

Of course, where modal intuitions give out and are unable to decide the truth value of some sentence about possible worlds, the realist and the reductionist disagree as to what to say. The realist maintains that there is a fact of the matter, but that the fact is unknown and perhaps unknowable. The reductionist maintains that there is no fact, and that the truth value of the sentence is to be conventionally decided one way or the other. But this difference need not hinder the reductionist in his attempt to preserve the possible worlds theory. For although the realist thinks that the reductionist is bound to get many of the unknown facts of modality wrong, he cannot object in this way without merely begging the question whether or not there are any such facts.

These brief remarks suggest how I think Proposal 1 could be vindicated with respect to the charge of circularity. Much more would have to be said as to what methods the reductionist should be permitted, and, in particular, as to what can count as admissible data. But I want to bracket the problem of circularity for the rest of this discussion and turn instead to the problem that results from considerations of the cardinality of the class of possible worlds. This will provide a decisive refutation of Proposal 1.

5. THE CARDINALITY OBJECTION

We have numerous beliefs about what is possible, and about which possibilities exclude which other possibilities — that is, about *com*possibility, or what I have called consistency. If we accept the thesis that whatever is possible is true at some (fully determinate) possible world, then our beliefs about possibility and compossibility will lead us to beliefs about the number of possible worlds: we should believe that there are *at least* as many possible worlds as are needed to support our beliefs about possibility and compossibility. In this section, I shall use this method to set a lower bound on the cardinality of the class of possible worlds.[13]

Here are some modal beliefs that I take to be fairly uncontroversial. I believe that the world might have consisted of nothing but (a single kind of) uniformly dense matter distributed throughout a Euclidean space and time. Moreover, the world might have contained nothing but a single solid cube of such matter, persisting without change throughout all eternity. Let us fix our attention upon one such world containing one such cube. I believe that that world might have had less matter than it had. For example, the cube might have been missing one of its corners, or it might have had holes through it like a swiss cheese. Indeed, all of the cube's matter might have been missing except for that of a single point. In general, for any collection of points of matter of the original cube, the world might have had the matter of just those points, spatially arranged in just that way.

If these beliefs about what is possible are correct, how many possible worlds must there be? Let the worlds that result from the elimination of some of the matter of the original cube be called the *cube worlds.* Different metaphysical positions with respect to transworld identity will lead one to count the cube worlds in different ways, although all methods of counting ultimately give the same result. Let us first consider how a haecceitist with respect to matter would count the cube worlds.[14] A *haecceitist* is someone who believes that there are primitive facts as to whether individuals inhabiting different worlds are numerically identical or not, and that therefore two worlds might be just alike with respect to all their qualitative properties, but nevertheless differ with respect to which individuals inhabit them. A haecceitist *with respect to matter*, then, believes that there are primitive facts as to whether individuals inhabiting different worlds are composed of the numerically same matter. Now, the haecceitist with respect to matter calculates the number of cube worlds as follows. Sets of points of the original cube are in one-to-one correspondence with cube worlds, each set of points corresponding with the unique cube world in which the cube has retained just the matter occupying the points of that set, in just that spatial configuration. Since space is assumed to be Euclidean, there are continuum many, or $\beth_1$, points of the original cube, and thus there are power-set-of-the-continuum many, or $\beth_2$, sets of such points. It follows that there are $\beth_2$ cube worlds. Since every cube world is a possible world, there are then *at least* $\beth_2$ possible worlds. So calculates the haecceitist.

According to the *anti-haecceitist*, however, distinct worlds must differ in some qualitative feature. In particular, distinct cube worlds must differ in the size or shape of their respective aggregates of matter, for there are no other qualitative properties that could distinguish them. (Note that qualitative does not exclude quantitative on this usage.) But then the anti-haecceitist will find the above calculation guilty of double counting: where the haecceitist sees many worlds, the anti-haecceitist often sees only one. For example, imagine the original cube to be divided into two equal halves. Call the cube world in which just the matter in one half is retained w_1, and the cube world in which just the matter in the other half is retained w_2. The haecceitist claims that w_1 and w_2 are distinct worlds because they differ as to which half of the original cube, and thus as to which matter, they contain. But the anti-haecceitist claims that w_1 and w_2 are one and the same world, namely, that cube world that can be completely described in qualitative terms as the Euclidean world consisting of nothing but a solid, rectangular block of a certain kind of matter, of a certain shape and size, persisting unchanged throughout all time. So according to the anti-haecceitist, the calculation done above counted the same world twice.[15]

More generally, wherever the haecceitist sees distinct cube worlds whose respective aggregates of matter have the same shape and size, the anti-haecceitist sees but a single world. This can be made precise as follows. Two aggregates of matter contained within the original cube, *a* and *b*, have the *same shape and size* just in case one can be superimposed upon the other by some combination of translations, rotations, and reflections; that is, just in case one can be superimposed upon the other by a *Euclidean transformation.* (Note that this definition also covers wildly scattered and discontinuous aggregates of points of matter.) Let w_a and w_b be the cube worlds that result from removing all the points of matter not contained within *a* and *b* respectively. Then whereas the haecceitist holds that w_a is identical with w_b if and only if *a* is identical with *b*, the anti-haecceitist holds that w_a is identical with w_b if and only if *a* and *b* have the same shape and size.[16] Thus, we must distinguish between the haecceitist cube worlds on the one hand, and the anti-haecceitist cube worlds on the other. Since the relation having-the-same-shape-and-size-as is an equivalence relation over the set of all aggregates of matter contained within the original cube, it induces an equivalence relation over the set of haecceitist cube worlds, and thus partitions this set of worlds into equivalence classes. The anti-haecceitist cube worlds, then, are in one-to-one correspondence with these equivalence classes.

It is now a simple matter to count the number of anti-haecceitist cube worlds by counting the number of equivalence classes. Recall that there are $\beth_2$ haecceitist cube worlds in all. Each equivalence class contains at most $\beth_1$ members because there are only $\beth_1$ Euclidean transformations of a Euclidean space onto itself. But it follows from the arithmetic of infinite cardinals that if a class of $\beth_2$ members is partitioned into subclasses each containing at most $\beth_1$ members, then there are $\beth_2$ such subclasses. Therefore, there are $\beth_2$ anti-haecceitist cube worlds. Although the haecceitist and the anti-haecceitist may disagree as to how to interpret the various modal beliefs listed at the beginning of this section, they can both agree that those modal beliefs commit them to there being at least $\beth_2$ possible worlds.

How does all this bear upon Proposal 1? Let us make the modest assumption that the possible worlds theory will be sufficiently strong to guarantee that there are cube worlds as described above, and that every cube world is a possible world. Then, whether the theory is haecceitist or not, it will follow from the theory that there are at least $\beth_2$ possible worlds. If Proposal 1 is to provide a faithful translation of the possible worlds theory, it must provide at least $\beth_2$ linguistic entities to serve as substitutes for the possible worlds. But it can't. Since there are only a countable number of sentences of *L*, there are at most $\beth_1$ maximal consistent sets of sentences of *L*. That will not be enough linguistic entities to provide a faithful translation of the possible worlds theory.

6. SECOND PROPOSAL

The reductionist needs a proposal for reduction that can provide more linguistic entities than are provided by Proposal 1. The problem is not that there is a lack of linguistic entities in his ontology: by taking sets of sets of expressions of *L*, $\beth_2$ linguistic entities can be made available. The problem is that, if the proposal is to satisfy the naturalness condition, these entities will have to be able to serve as complete descriptions of possible worlds. But I will show that Proposal 1 can be generalized in a natural way so as to provide $\beth_2$ linguistic entities, and thus circumvent the cardinality argument. In order to motivate such a generalization, it will be helpful to look more closely at how Proposal 1 satisfies the naturalness condition.

The naturalness condition requires that possible worlds be identified with constructions out of *L* that can serve as their complete descriptions, but it does not require that any single sentence of *L* be a complete description of a possible world. For example, Proposal 1 may identify a possible world with a set of sentences none of whose members is a complete description of that world. It is rather the set of sentences as a whole that is to be taken as the complete description of a world; the set of sentences is to be thought of as describing a world at which *all* of the sentences of the set are true. Thus, Proposal 1 satisfies the naturalness condition because we can think of a maximal consistent set of sentences as playing the role, semantically speaking, of the infinite conjunction of all its members. This shows that Proposal 1, in effect, allows the reductionist to make use of the expressive power of a certain *infinitary expansion* of *L*; an infinitary language that adds to the logical apparatus of *L* an infinitary connective of *sentential* conjunction.

This suggests a way of generalizing Proposal 1. An infinitary language that only permits infinite conjunctions of *complete sentences* is quite weak as far as infinitary languages go. Why not allow the reductionist to make use of an infinitary logical expansion of L that permits infinite conjunctions of *open formulas* as well? Let L^* be the infinitary logical expansion of L that permits infinite conjunctions over sets of less than $\beth_2$ open (or closed) formulas; infinitely long sentences of L^* are formed by attaching finite strings of quantifiers to such infinitely long formulas. (Thus, L^* is the infinitary language $L_{\beth_2\beth_0}$, to use a standard nomenclature.) The semantics for the language L^* is developed in the obvious way. In particular, the appropriate clause in the definition of truth and satisfaction reads: an infinite conjunction of formulas is satisfied by an assignment of objects to its free variables if and only if every conjunct is satisfied by that assignment. In general, the language L^* will be richer in expressive power than the original language L. For example, let L contain a predicate for the greater-than relation between real numbers and a name for every (standard) natural number, but no other non-logical constants. Although the Archimedian property of the reals, that every real number is exceeded by some (standard) natural number, cannot be expressed in L, it can be expressed in L^* by: for all reals r, it is not the case that r is greater than 0 and greater than 1 and greater than 2 and . . . . Such examples as this suggest — rightly, as we shall see — that a stronger proposal for reduction will result if L is replaced by L^* in Proposal 1:

PROPOSAL 2. *Possible worlds are maximal consistent sets of sentences of L^*.*

How does Proposal 2 fare with respect to the three conditions for reduction laid down earlier in this paper? It fares at least as well as Proposal 1 with respect to the non-circularity condition. Both proposals are faced with the problem of defining consistency. True, questions about the consistency of sets of sentences of L^* go beyond questions about the consistency of sets of sentences of L in that they require, in effect, judgments about the compossibility of infinite sets of open formulas for their answers. But such judgments seem no more problematical than the judgments about the compossibility of finite sets of open formulas already required by Proposal 1.

Moreover, the constructions out of linguistic entities that are to replace the possible worlds, the maximal consistent sets of sentences of L^*, are all non-circularly available to the reductionist. For the expressions of L are all assumed to belong to the reductionist's ontology, and the sentences of L^* can be defined as set-theoretic constructions out of the expressions of L (perhaps together with some symbols for the infinitary connectives). Indeed, the work of constructing the sentences of L^* has already been done for the reductionist by the logicians who developed the syntax for infinitary languages.[17]

Proposal 2 also satisfies that part of the naturalness condition that requires that possible worlds be replaced by linguistic entities that can serve as their descriptions. (Whether or not the linguistic entities provided by Proposal 2 can serve as *complete* descriptions of possible worlds (for a reasonable language L) will be discussed below.) The sentences of L^* all have a definite meaning as long as the sentences of L do. Indeed, truth conditions for sentences of L^* are completely determined by the semantics for L together with the semantical rules for the infinitary connectives. These truth conditions naturally correlate the maximal consistent sets of sentences of L^* with the possible worlds that they purport to describe.

With respect to the faithfulness condition, Proposal 2 is a definite improvement over Proposal 1; for Proposal 2 can provide enough linguistic entities to faithfully translate the sentence: 'There are at least $\beth_2$ possible worlds'. This will be shown with respect to the cube worlds introduced above. It will suffice to show that, for some appropriate choice of L, distinct cube worlds are L^*-discernible; for then the set of sentences of L^* true at one of the worlds and the set of sentences of L^* true at the other will be distinct maximal consistent sets of sentences of L^*. Since there are $\beth_2$ cube worlds, it follows that Proposal 2 provides at least $\beth_2$ maximal consistent sets of sentences.

First I will show that, if L contains some rudimentary mathematical language, then L^* will contain, in effect, a name for every real number.[18] Thus, let us assume that L contains at least a predicate for the greater-than relation between real numbers, function symbols for the arithmetical operations, and names for the numbers 0 and 1. Then every rational number is designated by some term of L. We can use the fact that every real number is the least upper bound of some set of rational numbers to show that every real number uniquely satisfies some open formula of L^*. For consider any real number r, and let $\{q_i\}$ be a set of rational numbers (indexed by ω) that has r as its least upper bound (the set of all rationals less than r will do). There is a formula of L^* that is satisfied by a real number just in case it is the least upper bound of the q_i; that is, just in case it is greater than q_1 and greater than q_2 and . . . and such that any other real number that is greater than q_1 and greater than q_2 and . . . is greater than it. Such a formula uniquely picks out the real number r.

The names of real numbers contained in L^* can be used in describing the cube worlds if we assume that L has the means to speak of a Euclidean assignment of spatial coordinates to points of matter. For example, L might contain an 8-place predicate that holds between four points of matter and four triples of real numbers just in case the assignment of those four triples to those four points determines a *Euclidean coordinatization* of space, that is, an assignment of triples of real numbers to all points of matter that is in conformity with the Euclidean structure of space. And L might contain a 9-argument function symbol representing the assignment of coordinates (triples of real numbers) to points of matter relative to an initial assignment of coordinates to four points of matter.

Consider now the haecceitist version of the cube worlds. Here we must further assume that L contains names for four points of matter of the original cube, and that these points have been assigned coordinates in such a way as to determine a Euclidean coordinatization of the entire cube. On these assumptions, it is a simple matter to formulate, for any two cube worlds, a sentence of L^* that is true at one of the worlds but not at the other, that is, a sentence of L^* that discerns the two worlds. This is because, since L^* contains a name for every real number, and so (with a modicum of set theory) a name for every triple of real numbers, L^* also contains a name for every point of matter of the original cube: every such point can be picked out by reference to its spatial coordinates. Now, consider any two haecceitist cube worlds. Since the worlds are distinct, there must be some point of matter of the original cube that exists at one of the worlds but not at the other. But then any sentence of L^* that asserts that that point of matter exists is true at one of the worlds but not at the other. Since any two haecceitist cube worlds are thus L^*-discernible, and since there are $\beth_2$ cube worlds in all, it follows that there must be a least $\beth_2$ maximal consistent sets of sentences of L^*.

Let us now turn to the anti-haecceitist version of the cube worlds. Here it does no good to use the names for real numbers to introduce names for the individual points of matter. Rather, the anti-haecceitist uses the names for real numbers to formulate qualitative descriptions of the cube worlds, descriptions of the overall shape and size of the worlds' aggregates of matter. Thus, let T be the set of triples of real numbers that are assigned to points of matter of the original cube under some arbitrary Euclidean coordinatization; and let S be an arbitrary subset of T, and w_S the corresponding (anti-haecceitist) cube world. The world w_S can be described by a sentence of L^* that asserts the following: On some Euclidean coordinatization of space, all and only the coordinates in S are assigned to points of matter existing in the world. For each (anti-haecceitist) cube world, there will be such a sentence of L^* that is true at that world but at none of the others. So here again we see that Proposal 2 can provide $\beth_2$ maximal consistent sets of sentences, and thus enough linguistic entities to undermine the cardinality argument.

Proposal 2 can succeed where Proposal 1 failed because Proposal 2 is based upon a weaker requirement for reducibility to L. Recall that Proposal 1 was based upon the idea that a class of possible worlds is reducible to L if any two discernible worlds from that class are L-discernible. But this condition for reducibility to L, although sufficient, is not necessary. Proposal 2 makes use of a weaker, but still sufficient, condition for reducibility: a class of possible worlds is reducible to L if any two discernible worlds from that class are L^*-discernible. Since, as we have seen, there may be worlds that are discerned by a sentence of L^* but not by a sentence of L, Proposal 2 is essentially more powerful than Proposal 1, and can reduce wider classes of possible worlds.

7. FURTHER GENERALIZATIONS

Proposal 2 can itself be generalized, in at least two directions. First, recall that L^* went infinitary with respect to conjunction, but not with respect to quantification. L^* can be further expanded by introducing a stock of $\beth_2$ individual variables to be used in the construction of formulas, and permitting universal quantification with respect to sets of such variables of cardinality less than $\beth_2$. (This gives the infinitary language $L_{\beth_2}$, to use a standard nomenclature.) Secondly, the bound on the number of formulas that can be conjoined, and the number of variables that can be quantified over, can be extended to arbitrarily high cardinality. For each infinite cardinal κ, there is an infinitary expansion of L, L_κ, which permits conjunctions over any set of less than κ formulas, and universal quantification with respect to any set of less than κ individual variables. These infinitary languages provide, for each infinite cardinal κ, a distinct proposal for reduction:

PROPOSAL 3_κ. *Possible worlds are maximal consistent sets of sentences of L_κ.*

Each of the Proposals 3_κ is based upon the idea that L_κ-discernibility provides a sufficient condition for reducibility to L. Taken together, the series of Proposals 3_κ suggests the following *necessary* and sufficient condition for reducibility to L: a class of possible worlds is reducible to L if *and only if* any two discernible worlds from that class are L_κ-discernible, for some infinite cardinal κ. Corresponding to this necessary and sufficient condition, there is a maximally general proposal for reduction. It can be most easily formulated by introducing the infinitary language L_∞, which is defined as the union of the languages L_κ, for all infinite cardinals κ:

PROPOSAL 4. *Possible worlds are maximal consistent classes of sentences of L_∞.*[19]

Note, however, that the maximal consistent classes of sentences of L_∞ are *proper* classes — they are 'too large' to be sets. Thus, only a reductionist who admits proper classes as well as sets into his ontology can feel free to make use of Proposal 4.

Whether or not the reductionist will need to make use of the full power of Proposal 4 will depend upon which theses about possible worlds he has included in the possible worlds theory. For example, if the reductionist believes that any possible world can be described by giving the distribution of no more than $\beth_1$ qualitative properties over a space-time of no more than $\beth_1$ points, then the cardinality of the set of all possible worlds will be no more than $\beth_2$, and Proposal 2 will succeed if any proposal will. But if the reductionist believes that, for any ordinal number, there is a possible world in which time is composed of a succession of instants having the order type of that ordinal, then there will be no *set* of all possible worlds, and the full generality of Proposal 4 will be needed in any attempt to reduce the *class* of possible worlds to language. But the question now arises: what are the limitations on even this most general proposal for reducing possible worlds?

8. THE OBJECTION FROM DESCRIPTIVE IMPOVERISHMENT

Initially, one might have thought that there were two sorts of limitation that must be overcome by any proposal for reducing possible worlds to a language *L*. Let us assume, as is customary, that the vocabulary of *L* has been divided into two parts: a logical part and a non-logical, or descriptive, part. Then, one might have thought that the prospects for a successful reduction would have been limited, on the one hand, by any impoverishment of the logical apparatus of *L*, and, on the other hand, by any impoverishment of the descriptive apparatus. What I hope that the generalized proposals of the preceding section have shown is that an impoverishment of the logical apparatus does not in fact limit the prospects for a reduction at all. As long as the reductionist has set theory (and perhaps class theory) at his disposal, he can always cook up set-theoretic constructions out of the expressions of *L* that can do all the work that the expressions of any infinitary expansion of *L* can do. Indeed, the standard proposals for reduction such as Proposal 1 already make use of this idea in an implicit way. I have suggested making the idea explicit by simply using sentences of an infinitary expansion of *L* in forming the required set-theoretic constructions. The move is perfectly legitimate; it merely allows the reductionist to make full use of the descriptive resources of the original language *L*.

The problem of an impovershment of the descriptive vocabulary, however, cannot be dealt with as easily. I claim that any language *L* that is appropriate to the task of reducing the possible worlds will have its descriptive vocabulary impoverished in such a way as to present insurmountable difficulties for the reductionist. In brief, the problem is this. If the language *L* is to be able to provide a non-circular reduction of possible worlds to an actualist ontology, then the descriptive resources of L will have to be, in a sense to be illustrated, imprisoned within the actual world. But then only possible worlds that are, in some broad sense, rearrangements of the actual world can be constructed (in a natural way) out of the linguistic entities of L. That will not be all of the possible worlds.

Indeed, not even the cube worlds, it seems to me, can be taken to be rearrangements of the actual world, and thus constructible out of the expressions of a reasonable language. So if the cube worlds are possible worlds, as I have claimed, then no attempt at reducing possible worlds to language can succeed. Let us first see where the problem lies with respect to the haecceitist version of the cube worlds. When I argued above that any two haecceitist cube worlds were L^*-discernible, and thus that the class of such worlds was reducible to L, I had to assume that L contained names for points of matter of the original cube. But if L is required to be a reasonable language, in particular, an actual language used by actual people, then that assumption is unacceptable. Presumably, there does not exist a perfect cube of uniform matter anywhere in the actual world; nor do there exist dimensionless points of matter out of which such a cube might be composed. But providing names for non-actual points of matter of a non-actual cube is beyond the reach of the descriptive apparatus of an actual language. Such points cannot be named by ostension; nor can they be distinguished one from the other by their qualitative properties, or by their qualitative relations to actual existents. It follows that there will be cube worlds that are discernible (according to the haecceitist), but not linguistically discernible with respect to any infinitary logical expansion of L, for any reasonable language L: just take two cube worlds whose aggregates of matter have the same shape and size. But then no linguistic construction out of L can be naturally correlated with one of the two worlds but not the other; no linguistic construction can provide a *complete* description of either of the two worlds. In short: the haecceitist cube worlds are not reducible to L. The generalized proposals for reduction of the preceding section cannot help the reductionist here because the problem arises from an impoverishment of the descriptive vocabulary, not from an impoverishment of the logical vocabulary.

The reductionist with anti-haecceitist leanings runs into difficulties of a somewhat different sort in attempting to reduce the cube worlds. I argued above that, for any anti-haecceitist system of cube worlds, distinct worlds of that system are L^*-discernible. Moreover, no assumptions on L were needed that would make the reduction circular, as was the case with the haecceitist cube worlds. Nevertheless, showing that any two worlds from some *one* system of cube worlds are L^*-discernible shows only that the class consisting of worlds from that one system is reducible to L; it does not show that the class consisting of worlds from *all* the different systems of cube worlds is reducible to L. I claim that there are numerous different systems of cube worlds, differing with respect to the *kind* of matter that their worlds contain. I argue thus. Consider a world that contains *two* kinds of uniformly dense matter, and suppose that the two kinds of matter have all of their qualitative properties in common. However, they are distinguished relative to one another by the fact that matter of different kinds mutually attracts, matter of the same kind mutually repels. Surely, such a world is possible. Call one of the kinds of matter p-matter and the other kind n-matter. Consider the possibility that there exists nothing but a single cube of p-matter, and the possibility that there exists nothing but a single cube of n-matter. I claim that these are distinct possibilities, and thus that the system of cube worlds composed of p-matter is distinct from the system of cube worlds composed of n-matter. This claim does not admit of demonstration; but to deny it would be to hold not only that we cannot specify other worlds by stipulating what individuals they contain, but that we cannot specify other worlds by stipulating what kinds they contain. Such an extreme form of anti-haecceitism is strongly at variance with modal intuitions, and no philosopher to my knowledge has endorsed it.

Now, unless L has the descriptive resources to single out one of these two kinds of matter, there will be distinct worlds from *different* systems of cube worlds that are not L^*-discernible, and so that cannot be assigned distinct linguistic constructions out of L: just take a cube world composed of p-matter and a cube world composed of n-matter whose aggregates of matter have the same shape and size. Could an actual language produce an expression that applied to only one of these kinds of matter? I think not. According to modern science, nothing that exists in the actual world is the stuff of which a cube world is made; so we cannot fix upon either p-matter or n-matter by means of ostension.

Moreover, by the symmetry built into the case, any purely qualitative description that applies to *p*-matter applies to *n*-matter, and vice versa. So, for the anti-haecceitist as well as the haecceitist, the cube worlds resist reduction. The haecceitist's problem with respect to distinguishing individual points of matter rearises for the anti-haecceitist with respect to distinguishing kinds of matter.

By arguing along lines similar to these, I think it can be shown that much of our talk about possible worlds cannot be successfully reinterpreted as disguised talk about linguistic entities. But might the reductionist have done better to have chosen other entities from his nominalist ontology to be the possible worlds, rather than the linguistic entities? That would have been of no avail. The proposals presented in this paper will succeed in reducing possible worlds to a nominalist ontology if any proposal will. For assume that some proposal for reduction makes use of a non-linguistic, actual, concrete entity in constructing the possible worlds. That entity can be uniquely described by some open formula of some infinite expansion of some reasonable language *L*. At any rate, this is true if, as I suppose, every actual, concrete entity can be singled out by means of its spatio-temporal and causal relations to entities with which we are familiar.[20] So the reductionist can just as well use the open formula in constructing the possible worlds as use the concrete entity that the open formula describes. In general, whatever can be reduced to actual, concrete entities can equally be reduced to their descriptions.

I have supposed that the reductionist has a nominalist ontology and ideology. Would it help to provide him in addition with a conceptualist ontology and ideology? It would not. The proposals of this paper have already, in effect, made use of such conceptualist resources. Whatever is conceivable by actual people is describable within a reasonable language, an actual language used by actual people.[21] So whatever is conceivable can be represented by some linguistic construction. Moreover, since the reductionist's ideology has already been allowed to include modal notions such as consistency, there seems to be nothing left for a conceptualist ideology to offer. So, adding a conceptualist ontology or ideology would not improve the reductionist's chances of success. The proposals of this paper already make full use of the combined resources of the nominalist and the conceptualist. The failure of these proposals, then, marks the failure of any nominalist or conceptualist proposal for ontologically reducing possible worlds. These proposals fail, I conclude, because the possible outruns the actual not in number, but in kind.[22]

NOTES

[1] Versions of this paper were presented at Princeton University in 1980 and Rutgers University in 1981. David Lewis has made helpful comments at various stages of its development.

[2] I am skeptical that other types of reduction — for instance, a translation of talk of possible worlds into a language containing modal operators and higher-order quantifiers — can succeed in eliminating an ontological commitment to possible worlds; but I will not attempt to argue this here.

[3] This is the ontology accepted by Quine throughout most of his career. See, for example, Quine [12]: 266—270. Possible exceptions are an early paper [6] with Goodman in which he rejected the abstract entities; and a late paper [14] in which he seems to reject the concrete ones.

[4] Of course, when I speak of language, it will always be an *interpreted* language that I have in mind; uninterpreted linguistic entities do not purport to describe possible worlds, or anything else.

[5] Some would say that no natural language has a significant sublanguage satisfying all of the above conditions, even if the natural language is enriched by the languages of science, mathematics, and formal logic. This claim is certainly controversial; but there is no space to discuss it here.

[6] Here and throughout, what 'discernible' means is relative to the possible worlds theory chosen by the reductionist. Strictly speaking, it is *relative discernibility* with respect to the predicates of the possible worlds theory (excluding identity). But if we assume that the possible worlds theory has a binary predicate 'is discernible from', then relative discernibility with respect to the predicates of the theory will correspond to a notion of metaphysical discernibility (although, as we shall see, it will not correspond to a notion of *qualitative* discernibility if the theory is haecceitist). On the distinction between relative and absolute discernibility, see Quine [12]: 230.

[7] The two conditions required thus far are in rough agreement with the conditions given by Quine in, for example, [13]: 26—29. Not so for the third condition now to be discussed.

[8] Note that, to use Quine's terminology, L-discernibility is, in effect, *absolute* discernibility with respect to the predicates of L.

[9] Roper's paper came to my attention after this paper was completed. Roper has noticed (p. 51) that Proposal 1 can be generalized so as to meet the cardinality objection, although he provides few details. He says nothing to defend the proposal against the objection from descriptive impoverishment. Perhaps he is not requiring that the possible worlds be reduced to what I call a reasonable language. That would help to explain his puzzling conflation of reductions to sentences with reductions to propositions. See n. 22 below.

[10] In Carnap [3]: 9.

[11] Cf. Carnap's discussion in [4]: 70—76.

[12] For further discussion of Carnap's proposal, and a more detailed comparison with Proposal 1, see Bricker [2]: 194—200.

13 The argument of this section is derived from a similar argument in Lewis [11]: 90. I have presented the argument in such a way as to make it more neutral with respect to controversial metaphysical issues about identity over time, and identity across possible worlds.
14 I use the terms 'haecceitism' and 'anti-haecceitism' roughly in accordance with the usage of David Kaplan [9]. But there is no general agreement as to exactly how to use these terms.
15 Of course, I am speaking of an anti-haecceitist not only with respect to matter, but also with respect to points of space. Otherwise, he counts the cube worlds like the haecceitist with respect to matter: w_1 and w_2 are distinct cube worlds because their respective blocks of matter occupy different points of space.
16 Nothing compels the anti-haecceitist to use same shape and size as his criterion for individuating the cube worlds. He could use same shape alone; or he could use something even weaker such as same topological structure. It depends upon whether or not he believes that there are absolute notions of shape and size that can support transworld comparisons. A discussion of this issue is beyond the scope of the present work.
17 For example, see Karp [10].
18 That is, L^* will have the expressive power of a language that contains a name for every real number. Such names can be provided by introducing into L, and thus into L^*, a description operator that is contextually defined *à la* Russell.
19 The languages L_κ and the language L_∞ are discussed, for example, in Dickmann [5].
20 The most plausible exception would be a concrete part of the actual world that was spatio-temporally and causally disconnected from the part we inhabit.
21 Although, on some views, the public natural language would have to be supplemented with private languages, one for each creature capable of conception.
22 What about proposals to reduce possible worlds to maximal consistent sets of *propositions*, rather than sentences (as, for example, in Adams [1])? If the proposal does not require that the propositions all be expressible within (an infinitary expansion of) a reasonable language, then my objections do not apply. I have argued against such proposals along different lines in Bricker [2]: 220—229.

REFERENCES

[1] Robert M. Adams, 'Theories of actuality', *Noûs 8* (1974): 211—231.
[2] Phillip Bricker, *Worlds and Propositions: The Structure and Ontology of Logical Space* (doctoral dissertation, Princeton University, 1983).
[3] Rudolf Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*, 2nd ed. (Chicago: The University of Chicago Press, 1956).
[4] Rudolf Carnap, *Logical Foundations of Probability* (Chicago: University of Chicago Press, 1950).
[5] M. A. Dickmann, *Large Infinitary Languages: Model Theory* (Amsterdam: North-Holland Publishing Corp., 1975).
[6] Nelson Goodman and W. V. Quine, 'Steps towards a constructive nominalism', *Journal of Symbolic Logic 12* (1947): 105—122.
[7] Frank Jackson, 'A causal theory of counterfactuals', *Australasian Journal of Philosophy 55* (1977): 3—21.
[8] Richard C. Jeffrey, *The Logic of Decision* (New York: McGraw-Hill, 1965).
[9] David Kaplan, 'How to Russell a Frege-Church', *Journal of Philosophy 72* (1975): 716—729.
[10] C. R. Karp, *Languages with Expressions of Infinite Length* (Amsterdam: North-Holland Publishing Corp., 1964).
[11] David Lewis, *Counterfactuals* (Oxford: Basil Blackwell, 1973).

[12] W. V. Quine, *Word and Object* (Cambridge, Mass.: The MIT Press, 1960).
[13] W. V. Quine, 'Ontological relativity', *Ontological Relativity and Other Essays* (New York: Columbia University Press, 1969).
[14] W. V. Quine 'Whither physical objects', *Essays in Memory of Imre Lakatos*, R. S. Cohen *et al.*, eds. (Dordrecht: D. Reidel, 1976).
[15] Andrew Roper, 'Toward an eliminative reduction of possible worlds', *The Philosophical Quarterly 32* (1982): 45—59.

Department of Philosophy,
P.O. Box 3650 Yale Station,
Yale University,
New Haven, CT 06520,
U.S.A.