WORLDS AND PROPOSITIONS:

The Structure and Ontology of Logical Space

Phillip Bricker

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CONTENTS

ABSTRACT

In sections 1 through 5, I develop in detail what I call the _standard theory_ of worlds and propositions, and I discuss a number of purported objections. The theory consists of five theses. The first two theses, presented in section 1, assert that the propositions form a Boolean algebra with respect to implication, and that the algebra is complete, respectively. In section 2, I introduce the notion of logical space: it is a field of sets that represents the propositional structure and whose space consists of all and only the worlds. The next three theses, presented in sections 3, 4, and 5, respectively, guarantee the existence of logical space, and further constrain its structure. The third thesis asserts that the set of propositions true at any world is maximal consistent; the fourth thesis that any two worlds are separated by a proposition; the fifth thesis that only one proposition is false at every world.

In sections 6 through 10, I turn to the problem of reduction. In sections 6 and 7, I show how the standard theory can be used to support either a reduction of worlds to propositions or a reduction of propositions to worlds. A number of proposition-based theories are developed in section 6, and compared with Adams's world-story theory. A world-based theory is developed in section 7, and Stalnaker's account of

the matter is discussed. Before passing judgment on the proposition-based and world-based theories, I ask in sections 8 and 9 whether both worlds and propositions might be reduced to something else. In section 8, I consider reductions to linguistic entities; in section 9, reductions to unfounded sets. After rejecting the possibility of eliminating both worlds and propositions, I return in section 10 to the possibility of eliminating one in favor of the other. I conclude, somewhat tentatively, that neither worlds nor propositions should be reduced one to the other, that both worlds and propositions should be taken as basic to our ontology.

INTRODUCTION

I take it that propositions and possible worlds will have a role to play in any systematic reconstruction of our conceptual scheme. That is not to say, however, that one must be a realist with respect to propositions and possible worlds, that one must take propositions and possible worlds as basic to one's ontology. Perhaps both propositions and possible worlds can be reduced to entities considered ontologically more respectable. Or, failing this, perhaps the ontologically less respectable of the two can be reduced to the other. These questions of reduction will be the chief concern of the latter half of this work, sections 6 through 10. Such questions can only be answered relative to a specification of the theory of propositions and possible worlds that a reduction would be required to preserve. To this end, I develop in the first half of this work, sections 1 through 5, what I call the <u>standard theory</u> of propositions and possible worlds, and I examine some of the theory's metaphysical implications. Before turning to this theory, however, I need to make some preliminary remarks about the notion of proposition with which the theory is concerned.

What I mean by the term 'proposition' is to be taken from the theses I explicitly adopt, not from whatever preconceived notions one may have about the term. But I have not shied away from using an old, imprecise term to designate a more precise, even technical concept, so as not to lose sight of how that concept roughly fits into the general scheme of things: understanding is not fostered in a vacuum. Whoever appropriates an old term in this way has the responsibility of making clear just which of the old senses he is trying to explicate, so that there will be no confusion as to what is not meant by the term. With 'proposition' the problem is especially acute. The term has been used in so many different ways throughout its checkered career that it is multiply ambiguous, and in a devious way that has often gone unrecognized.[1] No wonder, since for most practical purposes, and many philosophical ones, the different senses need not be distinguished; to do so would be tedious. But undergoing some such tedium would be well-advised for the philosophical task I have before me, lest I be accused of equivocating in a place where it matters.

First, let me set aside some conceptions of proposition that will not be invoked in this work. Propositions have sometimes been identified with sentences, either interpreted or uninterpreted, either tokens or types. On the present conception, I will say that propositions are expressed by sentences, but the propositions are to be kept distinct from the sentences that express them.[2] Are all propositions expressed by

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[1] For a short history of uses of the term 'proposition', see Alonzo Church, The Problem of Universals (Notre Dame, Ind.: University of Notre Dame Press, 1956), pp. 3-11.

[2] Actually, such talk of sentences expressing propositions must be taken with a grain of salt. Most (if not all) sentences of a natural language only express propositions, in my sense, relative to a context of utterance, and even then, only if all vagueness and ambiguity has been eliminated.

the sentences of any one language? Not for anything ordinarily called a language: there will not be enough sentences to do the job.[3]

Propositions have sometimes been identified with ideas in the mind, or with other mental entities. On the present conception, these mental entities at most represent propositions; and I know of no reason for believing that any mind that ever was or will be is capable of representing all the propositions. Moreover, I will argue in sections 5 and 8 that propositions, in my sense, cannot be reduced to linguistic entities or to mental entities, in other words, that propositions cannot be constructed out of such entities. In this sense, propositions are independent of language and of thought.

Propositions have sometimes been identified with the meanings of sentences, or of declarative sentences. I will call such a conception of proposition a semantic conception; there are as many semantic conceptions as there are ways of individuating meanings. I take it that propositions on a semantic conception have some sort of grammatical structure, perhaps the surface grammatical structure of the sentences that express them, perhaps some hypothetical deep structure or logical form. In any case, propositions of a semantic variety might differ because they differ in structure, even though they agree in factual content.

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[3] See sections 1 and 8 for relevant arguments.

That does not happen on what I will call a metaphysical conception of proposition, the sort of conception here being invoked. On a metaphysical conception, propositions are individuated solely by factual content, not by structure or form:[4] distinct propositions always differ in factual content. There are as many metaphysical conceptions of proposition as there are ways of explicating the notion of factual content. I can say something about the notion of factual content that I have in mind by relating propositions to possible worlds.

Propositions are linked to worlds via a binary relation of truth: I will say that a proposition is true at a world.[5] I will also say that a proposition characterizes or describes any world at which it is true. In brief: propositions are properties of worlds. This excludes from consideration those conceptions according to which propositions only characterize worlds relative to something else, such as a time or a place. For example, tensed propositions will not be propositions in my sense: a tensed proposition is true or false at a world state, perhaps a world-time pair, not at a world simpliciter. Similarly, egocentric propositions will not be propositions in my sense: an egocentric proposition is true or false at what Quine calls a centered world,[6] in effect, a world-individual pair, not at a world simpliciter. Something

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[4] Which is not to say that they have no (internal) structure or form; that is a separate question.

[5] Throughout this work I use 'possible world' and 'world' interchangeably, except when proposals involving impossible worlds are under discussion.

[6] In "Propositional Objects", Ontological Relativity and Other Essays (New York: Columbia University Press, 1969), p. 154.

like tensed propositions and egocentric propositions are needed as objects of the so-called propositional attitudes: belief, desire, and such.[7] For this reason, a conception of proposition that takes the propositions to be (all and only) the objects of the attitudes must be kept distinct from the conception of proposition being developed in this work. A conception that takes propositions to be the objects of the attitudes will be called epistemological.

What about the mathematical propositions, and other brands of noncontingent proposition? These have not been excluded. They can be taken to characterize worlds, although in a rather trivial way: necessary propositions characterize all worlds; impossible propositions characterize none. In section 5, I will have reason to restrict further the conception of proposition being discussed so as to require that there be only one necessary proposition and only one impossible one; but for now no such restrictions are in force.

Although I recognize metaphysical conceptions that allow for a plurality of true mathematical propositions, I am not so liberal as to recognize metaphysical conceptions that allow for a plurality of propositions that are logically true. All logical truths have the same factual content: none at all. Logically true sentences all express the same proposition. These claims are not intended to be controversial, but merely explicative of the notion of factual content that I have in

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[7] For the arguments, see Lewis, "Attitudes De Dicto and De Se", Philosophical Review, 88 (1979), pp. 513-543. Lewis, however, assimilates all such propositions to what he calls properties: sets of possibles.

mind.

What more I mean by 'proposition' and by 'world' is best taken from the five theses presented below, and from the discussion that accompanies them. The first four theses I take to be analytic in the following sense: whoever denies them cannot mean by 'world' and 'proposition' what I mean by these terms. Purported objections to the truth of these theses, I will claim, rest upon confusions between different conceptions of world and proposition. Only the fifth thesis will engender any genuine controversy.[8] But all five theses are in need of justification nonetheless. Metaphysical inquiry, in large part, is the attempt to frame the right definitions, to latch on to the most fruitful concepts. The five theses must be judged according to their success or failure in capturing concepts that are fit to play a foundational role in philosophical analyses. I believe that the notions of world and proposition captured by the standard theory have such a foundational role to play. The evidence for this will not be presented herein; I refer the reader to relevant recent work in philosophy, especially in philosophical logic and the philosophy of language. My goal in this work is to elaborate in detail a theory that embodies suitable conceptions of world and proposition, to compare this theory with alternative theories that embody alternative conceptions, and then to use these theories as a backdrop against which to discuss fundamental questions of ontology. The theory I examine is only a fragment of a

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[8] See section 5, subsection "The Modal Objection".

complete account of worlds and propositions; but an important fragment, I think, and a necessary beginning.

SECTION 1

PROPOSITIONS AND BOOLEAN ALGEBRA

First Thesis.

Whatever may be controversial about the nature of the propositions metaphysically conceived, this much at least seems certain: they tend to band together and form a Boolean algebra.[1] It is of the nature of propositions to conjoin and disjoin, to complement and imply one another, and in general to interact in all the familiar Boolean ways. The thesis that the propositions are Boolean in structure can be given any of a large number of distinct formulations, formulations differing as to which concepts are taken as primitive, which as defined, and as to which statements are taken as axioms, which as theorems. The formulation presented below is distinguished in part by its economical conceptual basis: it makes use of but a single primitive concept, the Boolean less-than-or-equal relation, which is to be interpreted as (some type of) implication between propositions. The first thesis to be discussed, then, is that:

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[1] It seems that not everyone agrees. Some objections will briefly be discussed later in this section. See also the discussion of Adams's theory in section 6.

THESIS 1. The propositions form a Boolean algebra with respect to implication.

Before listing the ten subtheses that constitute Thesis 1, let me say a few words about the notion of implication in question. Intuitively, a proposition p implies a proposition q just in case p says everything that q says, perhaps more. Whenever p implies q, there is a sense in which q is contained in p: to assert both would be redundant; p alone suffices. Thus, if p implies q, it is natural to say that p is <u>at least as strong as</u> q (or that q is <u>at least as weak as</u> p). Let us say that p <u>properly implies</u> q whenever p implies q but q does not imply p. In this case I will say that p is <u>stronger than</u> q (or that q is <u>weaker than</u> p).

Unfortunately, such informal remarks in terms of 'what is said' and the like, although helpful in distinguishing propositional implication from various other propositional relations, can never by themselves single out the particular relation of implication here intended. Each of the different conceptions of proposition mentioned in the introduction has its own peculiar relation of implication. Since the use of phrases like 'what is said' and the use of the term 'proposition' are shifty in parallel ways, such phrases cannot be used to distinguish the particular metaphysical relation of implication here intended from other metaphysical relations of implication, or from various semantic and epistemological relations. Nevertheless, whenever phrases like 'what is said' are used in what follows, it will always be an appropriate metaphysical sense that I have in mind. That sense is perhaps best clarified by attending to the theses about propositions that are accepted in this and the following sections.

Thesis 1 is taken to comprise the ten postulates listed below.[2] The relation of implication will be symbolized throughout by '→'; the variables 'p', 'q', 'r', and 's' range over propositions. The first three postulates assert that the propositions are _partially ordered_ by implication:

(1.1) For all p, p→p.

(1.2) For all p, q, and r, if p→q and q→r, then p→r.

(1.3) For all p and q, if p→q and q→p, then p=q.

These postulates assert that implication is _reflexive_, _transitive_, and _anti-symmetric_, respectively.

The next two postulates assert that, in addition, the propositions under implication form a _lattice_: every pair of propositions has both a greatest lower bound and a least upper bound.

(1.4) For all p and q, there is an r such that r→p and r→q, and such that, for all s, if s→p and s→q, then s→r.

(1.5) For all p and q, there is an r such that p→r and q→r, and such that, for all s, if p→s and q→s, then r→s.

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[2] The postulates are standard and so my presentation will be brief. For a fuller discussion of these postulates, and an examination of some of their consequences, one can consult almost any exposition of Boolean algebra. G. Birkhoff and S. MacLaine, _A Survey of Modern Algebra_, 3rd ed. (New York: Macmillan, 1965), Ch. XI is a good introduction to the approach taken here. R. Sikorski, _Boolean Algebras_, 2nd ed. (New York: Academic Press, 1964) contains almost any fact about Boolean algebra that will be needed in what follows, and will be cited accordingly.

By (1.3) these bounds are unique. We may thus define the <u>conjunction</u> of p and q, symbolized 'p&q', to be the greatest lower bound of p and q; and the <u>disjunction</u> of p and q, symbolized 'pvq', to be their least upper bound:

(D1) For all p, q, and r, p&q=r if and only if r→p and r→q and, for all s, if s→p and s→q, then s→r.

(D2) For all p, q, and r, pvq=r if and only if p→r and q→r and, for all s, if p→s and q→s, then r→s.

The next two postulates add that the lattice of propositions is a <u>distributive lattice</u>:

(1.6) For all p, q, and r, p&(qvr)=(p&q)v(p&r).

(1.7) For all p, q, and r, pv(q&r)=(pvq)&(pvr).

(1.6) and (1.7) are called the distributive laws. They can be rewritten in primitive notation using the definitions (D1) and (D2).

The existence of a maximal and a minimal proposition is assured by the following two postulates:

(1.8) There is a p such that, for all q, q→p.

(1.9) There is a p such that, for all q, p→q.

By (1.3) these propositions are unique. The first, called the <u>universal</u> proposition, will be designated by 'T'; the second, called the <u>null</u> proposition, will be designated by '∅'. They are defined by:

(D3) For all p, p=T if and only if, for all q, q→p.

(D4) For all p, p=∅ if and only if, for all q, p→q.

The final postulate guarantees that every proposition has a complement, or negation:

(1.10) For all p, there is a q such that $p \vee q = T$ and $p \& q = \emptyset$.

(1.10) can be rewritten in primitive notation using definitions (D3) and (D4). In any distributive lattice, complements are unique. We may thus define the complement, or negation, of p, symbolized '-p' as follows:

(D5) For all p and q, $-p=q$ if and only if $p \vee q = T$ and $p \& q = \emptyset$.

If all of the postulates (1.1) through (1.10) hold, then the propositions with respect to implication form an ortho-complemented, distributive lattice; that is, a Boolean algebra.

One further definition will be useful. The operation that takes two propositions, p and q, and forms their material conditional, symbolized '$p \supset q$', can be defined by:

(D6) For all p and q, $p \supset q = -p \vee q$.

One must be careful to distinguish the relation $\rightarrow$ from the operation $\supset$. They are related, of course, as follows: $p \supset q = T$ if and only if $p \rightarrow q$.

Although the official version of Thesis 1 consists of postulates (1.1) through (1.10), all written in primitive notation, it will sometimes be convenient to consider an equivalent but less economical formulation. Let P^* be the structure (or ordered octuple) consisting of the class of propositions, the relation $\rightarrow$, the operations &, v, -, and $\supset$, and the constants T and $\emptyset$. Then, THESIS 1*. P* is a Boolean algebra; that is, P* satisfies (1.1) through (1.10) and (D1) through (D6).

Clearly, if Thesis 1* holds, then so must Thesis 1. Moreover, if Thesis 1 holds, then Thesis 1* will hold under the appropriate definitions. The only difference between the two theses is in what is taken to be an axiom, and what a definition. But since Thesis 1 is to be taken as determining in part what is meant by the term 'proposition', the difference between axioms and definitions can be ignored as long as parsimony is not at issue.

<u>First Reformulation</u>: <u>The Laws of Propositional Logic</u>.

Boolean algebra, everyone knows, has something to do with logic. One might wonder, therefore, to what extent Thesis 1 can be reformulated in more familiar logical terms. Two such attempts at reformulation naturally suggest themselves and will be developed below. These reformulations, I hope, will serve to shed light on the content of Thesis 1; more importantly, they show how the algebraic approach used throughout this work relates to alternative approaches by way of logic or semantics.

Thesis 1 is closely related to the following claim:

(A1) The propositions satisfy the laws of (classical) propositional logic.

In order to say how close, I need to specify just what is to be meant by the laws of propositional logic. If there are to be any such things at all, propositional logic will have to be formulated as a _theory_; (A1) will then assert that this theory holds true of the propositions. The theory of propositional logic, I take it, should hold true when interpreted as applying to sentences as well as when interpreted as applying to propositions.[3] The satisfaction of this desideratum, we shall see, results in (A1) being weaker than Thesis 1.

To facilitate the comparison between Thesis 1 and (A1), I will formulate propositional logic as a _first-order_ theory. The language of the theory can be described as follows:

(1) What are sometimes taken to be the basic sentence letters, 'p', 'q', and so on, are instead taken to be individual variables ranging over propositions.

(2) What are sometimes taken to be the sentential connectives, '&', 'v', '-', and '⊃', are instead function symbols to be interpreted as proposition-forming operations on propositions; 'T' and '∅' are constant symbols to be interpreted as denoting propositions; '→' is a relation symbol to be interpreted as a relation between propositions. The language also has a one-place predicate, 'Taut', to be interpreted as a property of propositions. 'Taut', however, is not a primitive symbol of the language; all occurrences of 'Taut' are to be eliminated from the final theory

[3] Quine, for example, allows that the variables of the theory of propositional logic might range over either propositions (what sentences denote) or the sentences themselves. Needless to say, Quine prefers the latter approach. See W. V. Quine, "Ontological Remarks on the Propositional Calculus", in _The Ways of Paradox_ (Cambridge, Mass.: Harvard, 1966), pp. 265-272.

in accordance with the definition given below.

(3) The expressions involving variables, function symbols, and constant symbols that are sometimes taken to be the well-formed formulas are instead terms of the language to be interpreted as ranging over propositions.

Which sentences of the language belong to the theory of propositional logic? Consider any argument-form that is classified as valid by, say, the truth-table method. From the point of view of the present theory, it consists of zero or more premise-terms and a single conclusion-term. Presumably, there is a law of propositional logic asserting, roughly, that whenever all the premise-terms denote something tautologous, the conclusion-term denotes something tautologous as well. This law can be expressed within the first-order theory by making use of the predicate 'Taut'. Explicitly: if there are one or more premise-terms, take the universal closure of the material conditional whose antecedent is the conjunction of the premise-terms each prefixed by 'Taut', and whose consequent is the conclusion-term prefixed by 'Taut'; if there are no premise-terms, just prefix the conclusion-term by 'Taut' and take its universal closure. Let VAL be the set of all such sentences, one for each valid argument-form.[4] For example, the following two sentences are in VAL, representing familiar argument-forms of zero and two premise-terms, respectively: for all p, Taut(pv-p); for all p and q, if Taut(p) and Taut(p⊃q), then Taut(q).

[4] The sentences of VAL, of course, are largely redundant, but I am not concerned with providing a succinct formulation of the laws of propositional logic.

How should the predicate 'Taut' be defined? Since the laws of propositional logic are to hold true when the terms of the theory denote sentences as well as when they denote propositions, the laws should not imply that all tautologous terms have the same denotation as 'T'; when the theory is applied to sentences, 'T' may denote just one tautologous sentence among others. Thus, the predicate 'Taut' should not simply be defined as '=T'. Instead, I will make use of the fact that according to propositional logic something is tautologous just in case it is implied by everything, and take this as the definition of 'Taut':

(T) For all p, Taut(p) if and only if, for all q, $q \rightarrow p$.

Finally, we need to further specify the interpretation of '$\rightarrow$'. One of many ways to do this is as follows:

(I) For all p and q, $p \rightarrow q$ if and only if Taut($p \supset q$).

VAL plus (T) plus (I) provide us with a formulation of the theory of propositional logic, and so a precise explication of (A1):

(A2) The structure P* is a model of VAL plus (T) plus (I).

The relation between (A1) and Thesis 1 can now be established by comparing (A2) with Thesis 1*. It is routine to check that Thesis 1* entails (A2): whatever is a Boolean algebra satisfies the laws of propositional logic, that is, VAL plus (T) plus (I). The converse, however, fails to hold because (A2) does not entail that equivalent propositions are identical; that is, (A2) does not entail (1.3). Moreover, the eight other postulates containing '=' hold only in a weakened form in which identity is replaced by mutual implication. (A1) and Thesis 1 would be equivalent if we took the laws of propositional logic to include (1.3). But that, I have claimed, would be a mistake. The set of sentences of a language would normally be said to satisfy the laws of propositional logic (under the obvious interpretation), although they fail to satisfy (1.3). (A1) is thus genuinely weaker than Thesis 1.

Postulate (1.3) plays an important role in distinguishing metaphysical conceptions of propositions from various semantic and epistemological conceptions; for propositions of the semantic variety can, in general, imply one another without being identical, and perhaps this is true as well for propositions of an epistemological variety that are suitable as objects of the attitudes. Indeed, (1.3) embodies the feature by which I distinguished metaphysical conceptions of proposition from other conceptions: propositions that agree in content are one and the same. Does (1.3) also rule out any metaphysical conception of proposition according to which there is more than one necessary proposition? That depends upon how the relation of implication is interpreted. If there is a meaningful notion of implication satisfying the ten postulates and according to which not all necessary propositions imply one another (as I believe to be the case), then (1.3) by itself does not require that there be only one necessary proposition. The universal (or tautologous) proposition T would then be one necessary proposition among others. For example, the mathematical propositions might satisfy Thesis 1 with respect to an appropriate implication relation without being reduced to two in number, one true and one false.[5]

Although we have seen that (A1) is weaker than Thesis 1, it can easily be strengthened so that the two become equivalent. We need only redefine 'Taut' by the stronger:

(T') For all p, Taut(p) if and only if p=T.

When (T) is replaced by (T'), a stronger theory results: the theory of propositional logic asserts only that all tautologous propositions are equivalent to T; the strengthened theory asserts in addition that all tautologous propositions are identical with T, and thus that there is only one tautologous proposition. Postulate (1.3) can easily be shown to follow from the strengthened theory, and thus Thesis 1 can be equivalently reformulated as:

(A3) The structure P* is a model of VAL plus (T') plus (I).

Although (A3) is equivalent to Thesis 1, it is no longer an acceptable explication of (A1): the laws of propositional logic do not quite by themselves determine the Boolean structure of the propositions.

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[5] But the mathematical propositions will have a more serious run-in with Thesis 5 in section 5 below. See the subsection entitled "The Mathematical Objection".

Second Reformulation: The Lindenbaum-Tarski Algebra.

The above reformulation of Thesis 1 treats propositional logic, in effect, as a metaphysical theory about propositions. But it is also possible, and perhaps more customary, to treat propositional logic as a semantic theory about the logical relationships among sentences of a specified formal language. I will examine this semantic theory from an algebraic perspective, so as to facilitate a comparison with Thesis 1. Let us now take the countably many basic sentence letters 'p', 'q', and so on, and the expressions that can be built up from them in the usual way by means of '&', 'v', '-', '⊃', 'T', and '∅', to be sentences of the language of propositional logic, rather than terms as before.[6] Two sentences can be checked for logical equivalence by, say, the truth-table method. Since logical equivalence is an equivalence relation, it partitions the set of sentences into equivalence classes. The logical relations between sentences determine, in a natural way, corresponding relations between the equivalence classes. For example, one equivalence class can be said to logically imply another if and only if every sentence in the first class logically implies every sentence in the second class, where logical implication between sentences can be checked by the truth-table method. It is straightforward to show that the equivalence classes form a Boolean algebra with respect to logical implication; that is, they satisfy the ten postulates of Thesis 1 (reading 'equivalence class' for 'proposition' and 'logical implication'

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[6] Context will always decide whether the above symbols apply to the algebra of propositions, or, as here, to the language of propositional logic.

for 'implication'). Moreover, using the six definitions, the six Boolean operations and constants can be defined so as to apply to the equivalence classes of sentences. The resulting algebra is called the Lindenbaum-Tarski algebra for propositional logic (with respect to the chosen language); let us call it LT for short.

One might wonder whether the algebra LT can tell us anything about the algebra of propositions P*, thereby shedding new light on Thesis 1. Certainly, there is a connection between the equivalence classes of LT and the propositions. The sentences of propositional logic, when interpreted, express propositions. Since logically equivalent sentences presumably express the same proposition on the metaphysical conception,[7] each equivalence class can be thought of as representing a unique proposition, the proposition expressed by all its members. Perhaps the entire algebra LT can similarly be taken to represent the algebra of propositions. Since one algebra represents another in virtue of having the same structure, we could then replace Thesis 1 by:

(B1) P* is isomorphic to LT.

Indeed, (B1) entails Thesis 1, since LT is a Boolean algebra; but (B1) entails much more besides. LT is a countable, atomless Boolean algebra, so it follows from (B1) that the Boolean algebra of propositions is countable and atomless as well. Can these consequences of (B1) be accepted?

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[7] See the brief discussion in section 5, subsection "The Mathematical Objection".

Surely the algebra of propositions is not countable. For example, for every nonempty set of natural numbers, there is a distinct proposition saying that the number of hairs on my head is a member of that set. Since there are uncountably many sets of natural numbers, it follows that there are uncountably many propositions. On the other hand, there are only countably many sentences in the language of propositional logic; so not all propositions will be expressed by sentences no matter how those sentences are interpreted. But then not all propositions will be represented by equivalence classes in LT. In imputing the structure of LT to the propositions, the holder of (B1) tacitly assumes that all propositions can be expressed within the language of propositional logic. Since this assumption is false, the most one can assert about LT is that it represents, with respect to a given interpretation, the algebra of those propositions *expressible within the language*. But why should the expressible propositions have the same structure as the propositions generally? The Lindenbaum-Tarski algebra for a language cannot be expected to represent the propositions if not all propositions are expressible within the language.

There is a way to preserve the assumption that all propositions are expressible within a single language, as long as we are not too choosy as to what to count as a language. We need only allow propositional logic to be based upon uncountable languages, for example, languages with uncountably many basic sentence letters. Then, for any infinite cardinal number α, we can define the Lindenbaum-Tarski algebra for a language with α basic sentence letters similarly to the way LT was defined above. Call this algebra $LT\alpha$. Since, for some α, there will be an interpretation of the α basic sentence letters under which all propositions are represented by equivalence classes in $LT\alpha$,[8] we can avoid the earlier objection to (B1) by replacing it with the weaker:

(B2) P^* is isomorphic to $LT\alpha$, for some infinite cardinal α.

(B2) no longer has the objectionable consequence that there are only countably many propositions.

But it still has the consequence that the algebra of propositions is atomless, that is, that, for every nonnull proposition, there is another nonnull proposition that properly implies it. For each infinite α, $LT\alpha$ is atomless: every nonnull equivalence class can be conjoined with the equivalence class of some basic sentence letter to get a second nonnull equivalence class that properly implies the first. Two features of $LT\alpha$ conspire to prevent atoms from forming:

(1) There is a set of elements of $LT\alpha$ -- namely, the set of equivalence classes of basic sentence letters -- that is independent: no member of the set or its negation is logically implied by any nonnull Boolean combination of the other members.

(2) Moreover, this independent set generates the entire Boolean algebra: all elements of the algebra are Boolean combinations of members of the set.

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[8] This is a good place to note that I am assuming throughout that the class of propositions is not "too large" to be a set. Otherwise none of the $LT\alpha$ will have enough sentences to express all the propositions. See section 8, subsection "Further Generalizations" for a brief discussion.

Whenever these two conditions are met, the algebra is called a free Boolean algebra. The LTα are, up to isomorphism, all the free, infinite Boolean algebras; and all free, infinite Boolean algebras are atomless.[9]

Now, if (B2) can legitimately be used to establish a connection between the algebra of propositions and some one of the LTα, then Thesis 1 can be strengthened by adding atomlessness and freedom to the more basic Boolean properties that characterize the propositions. But the argument for (B2) adumbrated above establishes no such connection. Roughly, the argument was this: once an interpretation has been chosen, the structure of the LTα in question is projected by way of the interpretation onto the propositions; therefore, the propositions have the same structure as the LTα in question. This argument requires that the interpretation be such that distinct equivalence classes of the LTα always represent distinct propositions; else the structure might not be projected unaltered. Moreover, the relation of logical implication between equivalence classes must be projected onto the appropriate relation of propositional implication between propositions, the relation relevant to the structure P*. What grounds are there for believing that there exists any such interpretation? Such an interpretation would have to map the set of basic sentence letters onto a set of independent propositions that generates the entire algebra, where independence is here defined in terms of propositional implication. Thus, the argument simply requires what was to be demonstrated, namely, that the algebra of propositions is a free Boolean algebra. In fact, it will follow from

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[9] For more on free Boolean algebras, see Sikorski, op. cit., pp. 42-45.

the five theses of the standard theory that the algebra of propositions is not free, indeed, that it is atomic.[10] It suffices for now to note that (B2) makes a substantive metaphysical claim about the propositions that could not plausibly be supported by semantic and logical considerations alone.

The assumption that one of the $LT\alpha$ represents the algebra of propositions is called into question as soon as one realizes that sentences were grouped into equivalence classes according to logical implication, not propositional implicaton. But what if we alter the $LT\alpha$ so as to reflect the relation of propositional implication? Note that, to capture propositional implication, one need not subdivide any equivalence class of the $LT\alpha$ in question, because whenever two sentences are logically equivalent, they are propositionally equivalent as well. This means that propositional equivalence forms a coarser partition upon the sentences than logical equivalence; it thus can be represented by forming equivalence classes of equivalence classes, grouping together those equivalence classes whose members, under the given interpretation, are propositionally equivalent and so express the same proposition. All the other Boolean relations and operations can be readjusted in the obvious way so as to reflect the metaphysical relations of the algebra of propositions, rather than the logical relations of the $LT\alpha$. The resulting structure is called a quotient algebra of the original algebra; in the present case, identifying by propositional equivalence results in a quotient algebra of the $LT\alpha$ in question.[11] We now see how

[10] Proposition (5.4) in section 5.

to restate (B2), asserting only as much of a link between metaphysics and semantics as the present discussion permits:

(B3) P* is isomorphic to a quotient algebra of LT_α, for some infinite cardinal α.

(B3) no longer rests upon the two dubious assumptions that underlay (B1): not all propositions need be expressible within a countable language; and propositional implication need not impose the same structure upon the propositions that logical implication imposes upon the sentences of the language of propositional logic.

But (B3) is just a roundabout way of asserting Thesis 1; every Boolean algebra is (isomorphic to) a quotient algebra of some free, infinite Boolean algebra, that is, a quotient algebra of one of the LT_α. Perhaps some thesis intermediate in strength between (B2) and (B3) can be justified; this might be so, for example, if some version of logical atomism were true.[12] But the merits of such a proposal would have to be decided on metaphysical grounds, not on the chiefly semantic and logical

[11] More precisely, the elements of the quotient algebra are equivalence classes of elements of the LT_α under the equivalence relation propositional equivalence. One element implies another in the quotient algebra if and only if some member of the one propositionally implies some member of the other. A necessary and sufficient condition for the resulting structure to be a Boolean algebra is that the set of elements of the LT_α that express the universal proposition form a filter over the LT_α: (1) the conjunction of any two members is again a member; and (2) any element implied by a member is again a member. These conditions are clearly satisfied for the case at hand. For more on quotient algebras see Sikorski, op. cit., sec. 10.

[12] One such version: there is a set of independent propositions out of which all atomic propositions are (finite or infinite) Boolean combinations.

considerations that have been adduced in developing (B3). There is no easy route to metaphysics by way of semantics or logic.

Objections to the First Thesis.

We have twice rearrived at Thesis 1 by taking two quite different detours through classical propositional logic. This suggests that Thesis 1 stands or falls with classical logic, and that arguments for accepting some alternative, "deviant" logic are likewise arguments for rejecting Thesis 1. Indeed, proponents of deviant logics -- relevance logicians, intuitionists, and quantum logicians, to name but a few -- have often uttered words that, on the face of it, directly contradict Thesis 1. But the discussion in the introduction, I hope, has made us wary of apparent disagreements over the nature of propositions. If the deviant logician's notion of proposition differs from the metaphysical notion of proposition invoked in Thesis 1, the disagreement vanishes. Thesis 1 need not be incompatible with the acceptance of a deviant logic, as long as that logic has no metaphysical aspirations.

Take the case of the relevance logician who asserts that the propositions with respect to entailment (or relevant implication) do not form a Boolean algebra.[13] He bases this assertion on the fact that his semantic notion of entailment imposes a non-Boolean structure on the equivalence classes of sentences (modulo coentailment), that is, the fact that the Lindenbaum-Tarski algebra for entailment is not a Boolean algebra. Given that he simply identifies the propositions with the

[13] See, for example, Anderson and Belnap, Entailment (Princeton: Princeton University Press, 1975), ch. 18, especially pp. 193-194.

equivalence classes of sentences, it is far from clear what, if anything, he thinks can be inferred about the propositions metaphysically conceived. What is clear is that he cannot infer that the metaphysical propositions have a non-Boolean structure. The situation is similar to the one before with respect to logical implication. One and the same metaphysical proposition can be expressed by distinct sentences that do not entail one another. For example, all logically true sentences express the same metaphysical proposition because distinctions between logically true sentences are irrelevant to the propositions metaphysical role, no matter how relevant they may be to the semantic or epistemological roles that the propositions play. But then the most that can be inferred is that the metaphysical propositions have the structure of some quotient algebra of the Lindenbaum-Tarski algebra for entailment. That does not contradict Thesis 1: some quotient algebras of the Lindenbaum-Tarski algebra for entailment <u>are</u> Boolean algebras. Indeed, the Lindenbaum-Tarski algebra for classical propositional logic is (up to isomorphism) one such quotient algebra, as follows from the fact that the implication relation of classical logic is an extension of the entailment relation. Thus, the relevance logicians's semantic views about entailment yield no substantial consequences for metaphysical conceptions of propositions. One is free to accept relevance logic on semantic grounds, should one be so inclined, without thereby having to abandon Thesis 1.

The same treatment can be applied to other cases of deviant logic. Thus, the proponent of intuitionist or verificationist logic, because of his aversion to metaphysics, links the notion of a proposition, and the definitions of the propositional operations, to what can be or is already known. It is not surprising that the propositional structure when developed along such epistemological lines might turn out to be non-Boolean; the upholder of Thesis 1 can agree to that. Of course, the intuitionist or verificationist will probably look disdainfully upon Thesis 1 in accordance with his wholesale dismissal of metaphysics. But that is more a philosophical attitude than a philosophical argument, and requires no argument in response.

The case of quantum logic is, perhaps, more controversial. Unlike the previous two cases, the quantum logician's notion of proposition need be neither semantic nor epistemological, since the uncertainty principle may well be an objective feature of the world. But he limits the application of the term 'proposition' to certain physically realizable yes-no experiments; and the propositional operations are defined operationally in terms of possible or actual measurements. He then discovers, _empirically_, that the propositional structure so-defined is non-Boolean. This discovery, surely, is a contingent fact about the way the world is; as such, it has no bearing upon the notion of proposition appropriate to Thesis 1, a notion not to be tied down to any one world.[14]

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[14] For a description of the quantum logician's notion of a proposition, see Jauch, _Foundations of Quantum Mechanics_ (Reading, Mass.: Addison-Wesley, 1968). Jauch emphasizes the compatibility of "quantum logic" and classical logic (p. 77).

I think that most if not all cases of deviant logic can be dealt with along these lines. But what if someone persists in dissenting from Thesis 1, insisting all along that he means by 'proposition' what I mean by 'proposition'? My answer has already been stated above. Thesis 1 cannot be coherently denied because it determines in part what is here meant by the term 'proposition'. Whoever verbally dissents from Thesis 1 succeeds only in providing evidence that he does not mean by 'proposition' what I mean; attempts to deny Thesis 1 thus result either in a change of subject, or in utter nonsense. We have examined some cases of the former above; we are now contemplating a case of the latter.[15]

That, at any rate, is the short reply. But an extended defense of Thesis 1 would require much more. What is to prevent the metaphysician from defending any thesis for which he has no argument by labelling it true in virtue of meaning? Nothing ever comes of nothing. If a metaphysician bases his theory upon such analytic theses, and analytic theses are devoid of content, then it is with justice that he would be accused of fashioning castles in the air. At this point the metaphysician can turn formalist and embrace the consequence that his theory has no extra-logical content; he then deserves no further hearing. Somewhat more plausibly, the metaphysician can retract the analytic status of his theses and accordingly limit the scope of his enterprise: he can claim that his project is merely to frame

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[15] This general strategy for dealing with deviant logics has been championed by Quine in Philosophy of Logic (Englewood Cliffs, N. J.: Prentice-Hall, 1970), ch. 6.

hypotheses, without regard to their truth or falsity, and then to follow out their logical consequences. But that is to confuse metaphysics with logic. As a third option, he can deny that analytic theses need be devoid of content, and accept responsibility for providing arguments for the existence and, where relevant, the uniqueness of his purported subject matter. That is the general approach taken here, although lack of space will prevent me from attempting to fulfill the responsibility in question.

It might help to clarify the notion of analytic being used here to note an analogy with the foundations of mathematics. The axioms upon which a mathematical theory is based can usually be divided, at least roughly, into those that are analytic and those that are not. For example, the axiom of extensionality in set theory is clearly analytic, determining in part what is meant by 'set'. But the continuum hypothesis, on the other hand, does not seem to have this status, and its truth or falsity can be meaningfully discussed without merely changing the subject. The axiom of choice is probably a borderline case. Now, with regard to this analogy, Thesis 1 is like the axiom of extensionality, as are Theses 2 through 4 to follow. Only Thesis 5 is genuinely controversial, more like the continuum hypothesis or the axiom of choice.

Both metaphysicians and mathematicians, it seems to me, require arguments to secure their subject matter. In the case at hand, arguments for the existence of propositions in a sense appropriate to Thesis 1 are required. It would take me too far afield to detail the arguments that can be given, although perhaps it should be noted that such arguments tend to rest ultimately upon pragmatic considerations. In rough outline, they go as follows: one argues that the best total theory makes ostensible reference to the notion of proposition in question, that such reference is ineliminable, and that the entities ineliminably referred to by the best total theory should be given credence. Only the second of these three steps in the argument -- the question of ontological reduction -- will play a major role in the sections to follow. Such questions of ontological reduction, however, bear not so much upon the truth of Thesis 1 as upon the identification of its subject matter. Those parts of the argument that bear directly upon the truth of Thesis 1 will henceforth be assumed without further ado.

Second Thesis.

The propositions, I have said, have work to do in characterizing the worlds. Their *modus operandi* favors economy of means over economy of number. Whenever a task can be performed by some finite number of propositions working in Boolean combination, there is always a single proposition capable of performing that task alone. Mathematically speaking, the propositions are *closed* under the Boolean operations.

Conjunction is a case in point. One of the Boolean postulates (1.4) assures us that whatever characteristic of worlds can be picked out by two propositions, p and q, working conjunctively, can be picked out by a single proposition, p&q, working alone. It is that proposition which, intuitively speaking, says everything that p says and everything that q says, but not a bit more. With the aid of mathematical induction, this can be extended to any finite number of propositions, $p_1, \ldots, p_n$: there is a unique proposition that is at least as strong as each of the p_i, and such that any other proposition at least as strong as each of the p_i is stronger than it. In other words, every finite set of propositions has a greatest lower bound. But there is no way, given only the axioms for Boolean algebra, to extend this closure property into the transfinite. In an arbitrary Boolean algebra, an infinite set of elements may fail to have a greatest lower bound. But such a Boolean algebra could not possibly represent the propositions. For surely, whether we consider a finite set or an infinite set of propositions, all the members taken together must say something if each of the individual members does, and that something is itself a proposition.

The property in question is customarily called <u>completeness</u>. A Boolean algebra is <u>complete</u> if every subset of the algebra has a greatest lower bound. Thanks to duality, it follows from completeness that every subset also has a least upper bound. In order to exclude from consideration Boolean algebras that are not complete, I will add to Thesis 1:

THESIS 2. Every set of propositions has a greatest lower bound under implication.

Let us introduce the symbols 'Λ' and 'V' for the operations of set-conjunction and set-disjunction, respectively; and let us use capital letters such as 'P' and 'Q' to range over sets of propositions. Thus, ΛP, for example, is the proposition that is the conjunction of all the members of P. These operations are officially defined in terms of the greatest lower bound and the least upper bound under implication as follows:

(D7) For any proposition p and set of propositions Q, $\Lambda Q=p$ if and only if $p\rightarrow q$ for all $q\varepsilon Q$, and, for any proposition r, if $r\rightarrow q$ for all $q\varepsilon Q$, then $r\rightarrow p$.

(D8) For any proposition p and set of propositions Q, $VQ=p$ if and only if $q\rightarrow p$ for all $q\varepsilon Q$, and, for any proposition r, if $q\rightarrow r$ for all $q\varepsilon Q$, then $p\rightarrow r$.

Most garden-variety Boolean algebras are complete. Finite Boolean algebras are of course complete. Given any set, the full subset algebra based upon that set, ordered by inclusion, is a complete Boolean algebra, since the intersection of any set of subsets is itself a subset, and the greatest lower bound of that set of subsets. The Lindenbaum-Tarski algebra for propositional logic considered above is a complete Boolean algebra, as is, more generally, any freely generated Boolean algebra. This is because the only infinite ascending chains of elements that ever get generated are bounded above only by the unit of the algebra, and so no set of elements other than singleton unit can have an infinite sequence of greater and greater lower bounds. Note that the set of generators itself, if infinite, will just have the zero of the algebra as its greatest lower bound.

Objections to the Second Thesis.

Not all Boolean algebras are complete however. What grounds could one have for claiming that the propositions lack this property, and thus for denying Thesis 2? The only such arguments that I can conceive of involve, as with Thesis 1, an equivocation between the notion of proposition here intended and some other, more restricted, notion of proposition. Let us consider just one such argument. A denial of Thesis 2 might be based upon a wrong identification of the propositions with the propositions expressible within some (finitary) language. For indeed the expressible propositions of some languages, though they form a Boolean algebra, do not form a complete Boolean algebra, as the following example will illustrate.

Consider any countable language rich enough to express quantitative propositions about the sizes of ordinary physical objects: a numerically enriched English will do, if we ignore the vagueness. I will assume that the language has names for all the rational numbers, say in terms of fractions and decimals; but since only countably many of the real numbers are expressible in a countable language, there must be real numbers, and indeed real numbers between 0 and 1, that are not expressible in the language. Let one such number be r; and let the infinite decimal expansion of r be given by $.d_1d_2d_3 \ldots$, where each d_i is standing in for an integer between 0 and 9 inclusive. Now consider the infinite set of propositions $P = \{p_1, p_2, \ldots\}$, where p_i is the proposition expressed by the sentence: the tallest man that ever was or will be is more than $.d_1d_2 \ldots d_i$ meters tall. Each of the propositions

p_i is expressible in the language, as well as saying something quite definite, and true at the actual world. But in the algebra of expressible propositions, the set P has no greatest lower bound. For consider the infinite sequence of expressible propositions q_1, q_2, ..., where q_i is expressed by the sentence: the tallest man that ever was or will be is more than $.d_1d_2 \ldots d_i + 1/10^i$ meters tall. This forms a sequence of greater and greater lower bounds of P which is not bounded above by any lower bound of P. But this is only to say that the infinite conjunction of the members of P is not expressible in the language; for all the p_i taken conjunctively still say something quite definite, and true at the actual world -- namely, that the tallest man that ever was or will be is r meters tall or more. Propositions don't just go away if we lack the means to express them. Thus, this argument that the <u>expressible</u> propositions are not complete in no way casts doubt upon the thesis that the propositions, conceived of as entities existing independently of language, form a complete Boolean algebra.

SECTION 2

LOGICAL SPACE

Logical Space as a Field of Sets.

I am now in a position to develop one of the central notions of this work, the notion of logical space. The term 'logical space' has been used in different ways by different people; the notion developed here is intended to be a generalization of just one such usage, though perhaps it is also compatible with some of the others.[1] By generalizing the notion, it will be possible to interpret various theses about worlds and propositions as theses about the structure of logical space. It should be noted, however, that one could generalize along any of various dimensions; the notion developed here, although convenient for present purposes, is not the only one possible. Some alternative conceptions will be briefly discussed at the end of this section.

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[1] For the notion being generalized, see Lewis "Attitudes De Dicto and De Se, Philosophical Review, 88 (1979), especially pp. 517-519. The first use of the term 'logical space' that I know of is in Wittgenstein's Tractatus Logico-Philosophicus. The notion developed here seems to me to be compatible with Wittgenstein's use of the term, except that he speaks of states of affairs where I will speak of worlds. See, for example, remarks 2.11 and 3.4. But these are murky waters indeed.

An explication of the notion of logical space should satisfy two desiderata; they can be summarized by saying that logical space should be a material representation of the propositional structure posited by Theses 1 and 2. Let me explain what I mean first by a representation, and then by a representation being material.

A representation of the Boolean algebra of propositions is simply another algebra that is isomorphic to it. Thus, it is required that the Boolean structure of the propositions be accurately reflected in the structure of logical space; no Boolean information about the propositions is lost. But if logical space is to have any independent interest, some new information will have to be gained: logical space will be richer in structure or content than the Boolean algebra of propositions. What type of representation could provide an appropriate candidate for logical space?

A fundamental theorem about Boolean algebras assures us that at least one appropriate type of representation will be at our disposal: every Boolean algebra can be represented by -- that is, is isomorphic to -- a field of sets.[2] A field of sets is an ordered pair whose first member is a nonempty set and whose second member is a family of subsets over that set which is closed under (finite) unions, (finite) intersections, and complements. The set in question is called the space of the field, and its members are called points of the space. Now, if we apply the above-mentioned representation theorem for Boolean algebras to the case at

[2] See R. Sikorski, Boolean Algebras (New York: Academic Press, 1964), sec. 8 for a sharper formulation of the theorem and its proof. The theorem is due to Stone.

hand, we have: the Boolean algebra of propositions is isomorphic to a field of sets; that is, there exists a field of sets such that there is a one-to-one mapping of the set of propositions onto the family of subsets of the field which carries the relation of implication into the relation of set-inclusion. This suggests that we might find a suitable candidate for logical space among the fields of sets that are isomorphic to the Boolean algebra of propositions.

But if we are to further narrow down the candidates for logical space, we shall have to make the discussion less abstract. Since a field of sets has more structure than the Boolean algebra that it represents, two fields of sets can be isomorphic to the same Boolean algebra without being isomorphic to each other.[3] Thus, the requirement that logical space be a representation of the Boolean algebra of propositions does not even characterize logical space up to isomorphism. To accomplish this, something will have to be said about what the points of logical space should be. This is where the requirement that the representation be <u>material</u> comes in. I will call the representation <u>material</u> if, in some sense, the points of the space provide the subject matter of the propositions, so that the propositions are mapped by the isomorphism into the entities that they are about. To provide an illustration from another domain: the Frege-Russell definition of number is an attempt to provide a material representation of the algebra

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[3] For example, let N be the set of natural numbers and R the set of reals. The fields of sets <N, {N, {}}> and <R, {R, {}}> are both isomorphic to the two-element Boolean algebra, but they are not isomorphic to each other since their spaces are of different cardinality.

of numbers; the von Neumann or Zermelo definitions of number, though they also provide representations of the algebra of numbers, cannot plausibly be taken to provide material representations in the sense here intended.

Now, whatever else propositions can be taken to be about, they can always be taken to be about worlds. For example, the proposition that there are pink flamingoes in Princeton may, in some sense, be about any or all of the following: some or all pink things, some or all flamingoes, Princeton, pinkness, flamingohood, containment, existence, and so on. But it is also a proposition about worlds: it asserts that the world is a having-pink-flamingoes-in-Princeton world. Indeed, it is because propositions can always be taken to be about worlds that I have been able to speak of propositions as world-characterizers. Now, what this suggests is that the material representation of the propositions that we are looking for should be a field of sets with the worlds among the points of its space.

But here we are faced with a choice. Should we say that only worlds can be the points of logical space? Or should we leave it open whether or not some other kind, or kinds, of entity might be needed as well for providing the subject matter of the propositions? It will follow from the five theses discussed in this chapter -- what will be called the standard theory -- that the worlds alone suffice for representing the propositional structure. Moreover, if we did not put strong restrictions upon what entities could be points of logical space, there would be no hope of singling out a unique field of sets to represent the propositional structure. One can always throw extraneous entities into the space without disturbing the isomorphism between it, considered as a Boolean algebra, and the Boolean algebra of propositions. So it seems best to stipulate at the outset: no extraneous entities allowed in logical space.[4]

This stipulation does, however, pose a slight problem. I will want to consider certain nonstandard conceptions of the propositions which require that the points of logical space include entities which are not worlds; only then would logical space be able faithfully to mirror the propositional structure. How can we discuss such nontrivial, nonstandard conceptions if we simply stipulate that the points of logical space must be worlds? There is a way. We can always say that the nonstandard conception is introducing nonstandard worlds alongside the worlds, properly so-called, to serve as the points of logical space. So the stipulation that the points of logical space be worlds need not limit the scope of our inquiry in any way, as long as we allow some flexibility in the interpretation of 'world' when discussing the nonstandard conceptions.

Summing up what we have so far: logical space is to be a field of sets isomorphic to the Boolean algebra of propositions, and the space of this field is to contain all and only the worlds. It follows that propositions will be mapped into sets of worlds under the isomorphism. But which sets of worlds will be images of propositions? This will as

[4] To emphasize this choice, it might be better to call the notion being developed "modal logical space"; but I will stick with the more usual term.

yet be underdetermined (unless logical space is finite); there will be distinct isomorphisms satisfying the above conditions, and so distinct fields of sets left as suitable candidates for logical space. We need a further condition on logical space, a condition that is already implicit in the requirement of materiality discussed above: the isomorphism in question must be a natural one, mediated by some natural relation between propositions and worlds. Such a relation is close at hand: the binary relation of truth.[5] The further condition is that the isomorphism must map a proposition into the set of worlds at which it is true. Indeed, without this further condition it is hard to see what significance there would be to the fact that logical space provided a representation of the propositional structure.

Putting this all together, we arrive at the following definition of logical space. Logical space is the unique field of sets over the space of worlds which is isomorphic to the Boolean algebra of propositions under the mapping that takes propositions into the sets of worlds at which they are true. I should add: if any such field of sets exists. Nothing said thus far guarantees either the existence or the uniqueness of logical space, as defined above. It is the purpose of the theses of the next three sections to determine whether a unique logical space exists, and, if so, what its structural characteristics might be.

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[5] Could there be another natural relation that preserved the Boolean structure? I can't think of one. (Falsity is a natural relation, but it gets the set-inclusion relation backwards.)

Alternative Conceptions of Logical Space.

I conclude this section by mentioning four alternative conceptions of logical space. First, there is a further generalization of the notion of logical space that will be useful in what follows; it results from dropping the requirement that logical space be isomorphic to the Boolean algebra of propositions. Thus we might define modal space to be the unique family of subsets over the space of worlds whose members are the images of propositions with respect to the mapping that takes propositions into the sets of worlds at which they are true. Nothing in this definition requires that modal space be a field of sets, or that it represent the propositional structure. On the other hand, the existence of modal space is immediate from the definition, in contrast to the existence of logical space. If logical space does exist, then logical space and modal space coincide, and modal space also represents the Boolean algebra of propositions; to distinguish between the two would then be superfluous. But modal space provides a convenient framework within which to discuss theses about worlds and propositions prior to, or in lieu of, a commitment to the existence of logical space.

Although the notion of logical space defined above may not seem general enough for some purposes, it may seem too general for others. That there is a plausible narrower conception of logical space can be seen by considering the notion of completeness. Since logical space preserves the Boolean structure, and the Boolean algebra of propositions is complete, does it follow that logical space is complete? Yes and no. Of course, when considered as a Boolean algebra, logical space is a complete Boolean algebra. But, _when considered as a field of sets_, logical space need not be complete; that is, it need not be the case that, for every set of subsets in the field, its intersection is also in the field. This is because the isomorphism between the Boolean algebra of propositions and logical space need not preserve infinite conjunctions; it need not map an infinite conjunction into the intersection of the images of the conjuncts. So one could add an additional requirement to the definition of logical space: one could require that logical space be a _complete_ field of sets. Whatever the merits of this proposal as an explication of the notion of logical space, I will only have occasion to use the more general notion in what follows. What should be noted here is that Thesis 2 does not guarantee that logical space, as defined above, is complete; that will have to be established, if at all, by the positing of further theses.

Thirdly, it might be of interest to note an alternative conception of logical space according to which it is not a field of sets, but rather a topological space. A _topological space_ is an ordered pair whose first member is a set, called the _space_, and whose second member is a family of subsets of the space, called the _open sets_ of the topological space (or in the topology). The family of open sets satisfies the following two properties: (1) The family is closed under finite intersections; and (2) The family is closed under arbitrary unions. Every field of sets is associated with a unique topological space: the space of the topological space is the space of the field of sets; and the open sets of the topological space are all and only those sets that are unions of subfamilies of sets in the field -- that is, the field of sets is an open basis for the topological space. Thus, if logical space exists considered as a field of sets, then an associated topological space exists which might be called topological logical space. Every structural feature of logical space determines a corresponding structural feature of topological logical space; so, to speak of both is, in a sense, to say things twice. In what follows, I will focus the discussion upon logical space, and restrict mention of the topological analogies to an occasional footnote.

Finally, there are alternative conceptions of logical space according to which logical space has additional structure that is not definable in terms of the usual set-theoretic relations and operations over the field. Such an alternative conception would have been called for if I had chosen to build structure into the algebra of propositions that was not definable in terms of the Boolean relations and operations, and had continued to require that logical space provide a representation of that algebra. By restricting the algebra of propositions to Boolean relations and operations, I did not intend to suggest that the propositions have no further metaphysical (or logical) structure. On the contrary, I believe that many of the relations between propositions that are suggested by the logic of quantification have metaphysical analogues that are not reducible to Boolean relations between propositions. Thus, I believe it is meaningful on the metaphysical conception of propositions to distinguish between general and singular propositions, and to speak of a singular proposition being an instantiation of a general proposition, even though propositions have no grammatical structure by means of which these quantificational notions can easily be characterized.[6]

How would such quantificational notions best be incorporated within the algebra of propositions? The Boolean algebra of propositions could be extended to a Boolean algebra of propositions, properties, and relations; then a monadic operator representing either an existential or a universal quantifier could be introduced and required to satisfy certain familiar postulates.[7] If logical space is to represent this extended algebra, it will presumably have to have not only possible worlds, but possible objects, in the space of the field; and the field will have to have an additional operator corresponding to the quantifier.

I have refrained from taking this further step, in part because I wanted to keep the presentation as simple as possible from a technical standpoint, in part because the issues I discuss involving the reduction of worlds and propositions are not, I believe, seriously affected by the simplification (although a number of other metaphysical issues are affected). Be that as it may, I will make no official use in what follows of the extended algebra of propositions, or of the corresponding extended conception of logical space.

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[6] Indeed, I will make unofficial use of these notions in section 5, subsection "The Modal Objection".

[7] Basically, this is the technique used in doing algebraic logic for quantification theory.

SECTION 3

MAXIMAL CONSISTENT SETS OF PROPOSITIONS

Third Thesis.

The theses posited thus far have spoken of propositions, and of the structure imposed upon them by the relation of implication. In terms of that structure, the universal and the null proposition have been singled out, and the operations of conjunction, disjunction, negation, set-conjunction, and set-disjunction have been defined. As yet the theses have said nothing of truth, or of worlds. Thus, nothing has been said to guarantee that the above propositional notions will have their customary truth-functional properties. On the present approach, this will have to be posited independently, as will be the task of Thesis 3 below.

For now let us simply assume that conjunction, negation, and the rest, are to be truth-functionally standard at every world; later in this section I will ask how this might be challenged. To be more precise, let us say that a theory of worlds and propositions is truth-functionally standard if and only if, for every world w, all of the following standard truth-functional relationships hold:

(3.1) T is true at w.

(3.2) $\emptyset$ is false at w (that is, not true).

(3.3) For all p and q, if $p \rightarrow q$ and p is true at w, then q is true at w. (Thus, if one proposition implies another, then it materially implies the other at every world.)

(3.4) For all p and q, p&q is true at w if and only if p is true at w and q is true at w.

(3.5) For all p and q, pvq is true at w if and only if p is true at w or q is true at w.

(3.6) For all p, p is true at w if and only if -p is false at w. (Reading from left to right, we have that the law of contradiction holds at every world; reading from right to left, we have that the law of excluded middle holds at every world.)[1]

(3.7) For any set of propositions P, $\bigwedge P$ is true at w if and only if every member of P is true at w.

(3.8) For any set of propositions P, $\bigvee P$ is true at w if and only if some member of P is true at w.

It should perhaps be emphasized that (3.1) through (3.8) are here theses about the relation between the propositional structure and worlds, not definitions of the propositional operations and constants; the operations and constants occurring in (3.1) through (3.8) have all been previously defined independently of worlds in section 1.

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[1] At least on one account of these two laws. Another account would require only, respectively, that p&-p be false at every world, and that pv-p be true at every world, for all propositions p. This second account equates the law of contradiction with (3.2) and the law of excluded middle with (3.1). Given (3.4) and (3.5), the two accounts are equivalent.

Although I want the theory of worlds and propositions being developed to be truth-functionally standard, I need not adopt all of the above principles as basic theses of the theory; most of them are redundant, owing to the interrelationships among the Boolean concepts.[2] Instead, I will extract from the above principles a single thesis that, by itself, has all the content of (3.1) through (3.8) combined.

First, I will need the notion of a consistent set of propositions. A set of propositions P is consistent if and only if its conjunction is not the null proposition; that is, if and only if $\bigwedge P \neq \emptyset$. By extension, a proposition is consistent if and only if it is distinct from the null proposition. Now, by the truth-functional standardness of set-conjunction, (3.7), the conjunction of the set of all propositions true at a world, for any world, is itself true. Since, by (3.2), $\emptyset$ is false at every world, it follows that this conjunction is not identical with $\emptyset$; that is, for any world, the set of true propositions is consistent.

A consistent set of propositions that is not properly included in any other consistent set of propositions is called maximal consistent. It follows from truth-functional standardness that, for any world, the set of propositions true at that world is a maximal consistent set of propositions; in particular, the maximality of the set follows from the assumption that the law of excluded middle holds at every world. Proof: Consider any world w, and let Pw be the set of propositions true at w.

[2] But note that, although all Boolean operations are definable in terms of implication, it is not the case that all the truth-functional principles follow from (3.3), the principle that relates implication and the material conditional. One also needs (3.6) and (3.7), as will be seen below.

We have already seen above that Pw is consistent. If Pw is properly included in a set of propositions Q, then there is some proposition q that is in Q but not in Pw. By the law of excluded middle (half of (3.6)), -q is in Pw, and so also in the larger set Q. But if both q and -q are in Q, then Q is not consistent; for the greatest lower bound of Q, ΛQ, implies every member of Q, including q and -q, and the only such proposition is $\emptyset$. Since there is no consistent set of propositions which properly includes Pw, Pw is a maximal consistent set of propositions.

Thus, from the assumption of truth-functional standardness, one is lead to accept:

THESIS 3. For any world, the set of propositions true at that world is a maximal consistent set of propositions.

Conversely, Thesis 3 is all that need be posited in order to guarantee the truth-functional standardness of the theory. To show this, it suffices to show that (3.1) through (3.8) can all be derived from Thesis 3. These derivations are much facilitated by the following basic lemma:

(3.9) For any maximal consistent set of propositions M and any proposition p, $p \varepsilon M$ if and only if $\Lambda M \rightarrow p$.

The proof of Lemma (3.9), and the derivations of (3.1) through (3.8), can be found in the first appendix.

The content of Thesis 3 can be expressed in terms of the notion of modal space developed above. Thesis 3 asserts: modal space is a complete field of sets. That modal space is a field of sets follows from (3.4), (3.5), and (3.6), which guarantee respectively that the field is closed under (finite) intersections, (finite) unions, and complements. That modal space is complete follows from (3.7), which guarantees closure under infinite intersections as well. Though it follows from Thesis 3 that modal space is a field of sets, it does not follow that modal space represents the Boolean algebra of propositions; indeed, for all that Thesis 3 asserts, the same maximal consistent set of propositions could be true at every world, and modal space would have only two members regardless of the number of propositions.

Atomic Propositions.

Before examining in more detail the content of Thesis 3, I want first to establish a basic result about maximal consistent sets of propositions that will be important to the discussion that follows. Every maximal consistent set of propositions is associated in a natural way with a single proposition, namely, the proposition that is the conjunction of all its members. This associated proposition is nonnull because the set is consistent. Moreover, it is a strongest nonnull proposition: nothing could be added to it without falling into contradiction. It is called an atomic proposition, or an atom, for short.[3] More precisely, a proposition a is an atomic proposition if and

[3] Warning: two different senses have been given to the phrase 'atomic proposition', the one stemming from mathematical, the other from philosophical, usage. It is always the mathematical usage that I will

only if $a \neq \emptyset$ and, for all p such that $p \rightarrow a$, either $p=\emptyset$ or $p=a$.

Not only is the conjunction of any maximal consistent set of propositions an atom, but conversely every atom is the conjunction of one and only one maximal consistent set of propositions. Thus the maximal consistent sets of propositions are in one-to-one correspondence with the atomic propositions. To establish this correspondence, it suffices to prove the following three simple lemmas. The proofs make no use of Thesis 3; the correspondence holds independently of the relation between maximal consistent sets of propositions and worlds. Thesis 1 is used throughout the proofs in the guise of familiar facts about Boolean algebra; these facts are simple consequences of the Boolean postulates and will not individually be derived. Thesis 2 assures us that set-conjunction is always defined, but its use in these proofs is eliminable as will be noted below.

LEMMA (3.10). Let M be any maximal consistent set of propositions. Then, ΛM is an atom.

<u>Proof</u>. It suffices to show that the only propositions that imply ΛM are $\emptyset$ and ΛM itself. Let p be any proposition such that $p \rightarrow \Lambda M$. If $p \varepsilon M$, then $\Lambda M \rightarrow p$, and so $\Lambda M = p$ by (1.3). If it is not the case that $p \varepsilon M$, then $-p \varepsilon M$ because M is <u>maximal</u> consistent. So $\Lambda M \rightarrow -p$, whence $p \rightarrow -p$ by (1.2), and $p=\emptyset$. Thus, ΛM is an atom.

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have in mind, according to which atomic propositions are the atoms -- strongest nonnull elements -- of the Boolean algebra of propositions. According to the philosophical usage, atomic propositions, if they exist, are propositions that in some sense independently generate the entire Boolean algebra of propositions.

LEMMA (3.11). Let M and N be maximal consistent sets of propositions. If $M \neq N$, then $\Lambda M \neq \Lambda N$.

Proof. Assume M is a proper subset of N (otherwise, N is a proper subset of M, and we can switch 'M' and 'N' throughout the proof). Let $p \varepsilon N$ but not $p \varepsilon M$. Because M is maximal consistent, $-p \varepsilon M$. So, $\Lambda N \rightarrow p$ and $\Lambda M \rightarrow -p$. But if $\Lambda M = \Lambda N$, then $\Lambda N \rightarrow p \& -p$, and $\Lambda N = \emptyset$, which contradicts the consistency of N. Therefore, $\Lambda M \neq \Lambda N$.

LEMMA (3.12). For any atom a, there exists a maximal consistent set M such that $\Lambda M = a$.

Proof. Let $M = \{q: a \rightarrow q\}$. M is consistent. For, by definition, a is a lower bound of M, and so a implies the greatest lower bound of M; that is, $a \rightarrow \Lambda M$. But $a \neq \emptyset$ because a is an atom. So, by Boolean algebra, $\Lambda M \neq \emptyset$, and M is consistent. Moreover, M is maximal consistent. To show this, it suffices to show that, for any p, either $p \varepsilon M$ or $-p \varepsilon M$. Assume it is not the case that $p \varepsilon M$. Then, by Boolean algebra, $a \& -p \neq \emptyset$. But since $a \& -p \rightarrow a$, it follows from the atomicity of a that $a \& -p = a$. Hence, again by Boolean algebra, $a \rightarrow -p$, and $-p \varepsilon M$. So, M is maximal consistent. Finally, since $\Lambda M \rightarrow a$, it follows from (1.3) that $\Lambda M = a$.

Let the set of all maximal consistent sets of propositions be symbolized by MAX, and the set of all atomic propositions by ATM. It is an immediate consequence of Lemmas (3.10), (3.11), and (3.12) that:

THEOREM (3.13). The operation of set-conjunction maps the maximal consistent sets of propositions one-to-one, onto the atomic propositions: Λ: MAX <--> ATM.

Proof. By Lemma (3.10), for every $M \varepsilon MAX$, $\Lambda M \varepsilon ATM$. By Lemma (3.11), the mapping is one-to-one. By Lemma (3.12), the mapping is onto.

The proof of Theorem (3.13) makes tacit use of Thesis 2 by assuming that set-conjunction is always defined. But Thesis 2 plays no essential role in the proof. Even if the Boolean algebra in question is not complete, and so has sets with no greatest lower bound, it is still the case that all maximal consistent sets have a greatest lower bound (where consistency is now defined simply as having some nonnull lower bound). Thus, although set-conjunction in general is only a partial function on sets, it is total with respect to maximal consistent sets, and in any Boolean algebra maps the maximal consistent sets one-to-one, onto the atomic propositions.

Finite Consistency.

With Theorem (3.13) behind us, let us return to the discussion of Thesis 3. The notion of consistency that was used in developing Thesis 3 must not be confused with a related notion which might be suggested by the logician's use of the term 'consistency' in connection with the study of logical systems. Both of these notions could loosely be said to apply to a set of propositions just in case a contradiction cannot be derived from that set. But if one is only permitted to take into account *finitary* truth-functional relationships in deriving a contradiction, then it is not consistency, as here defined, but what might be called *finite consistency* that is at issue. A set of propositions is *finitely consistent* if and only if every finite subset of that set is consistent.

If a set of propositions is consistent, it is finitely consistent. But the converse need not hold, as the following example will show. Consider the ennumerably infinite set of propositions P = {p_1, p_2, ...}, where p_i is the proposition expressed by: the number of molecules in the universe is finite and greater than i. Every finite subset of P is consistent; for its conjunction says only that the number of molecules in the universe is finite and greater than n, where 'n' is the largest subscript of a name of a member of the finite subset. Thus, P is finitely consistent. But it is not consistent, since the conjunction of all the members of P is the contradiction that there are and are not finitely many molecules in the universe.

One thing suggested by this example is that it is consistency, not finite consistency, that we ordinarily look for in evaluating a person's system of beliefs. If a person were to maintain all of the propositions in the set P, he would contradict himself as surely as if he had explicitly maintained both p and -p, for some proposition p.

More importantly, the example suggests that it is consistency, not finite consistency, that is relevant to the task of characterizing worlds in terms of the sets of propositions true at them. To see this, it will be helpful to introduce some notions and results that will be more fully developed later, once all five theses are in hand. Call a set of propositions <u>realizable</u> if there is a world at which every member of the set is true; and call it <u>finitely</u> <u>realizable</u> if every finite subset is realizable. A single proposition will be called realizable just in case there is a world at which it is true. I will say that logical space is compact if and only if every set of propositions that is finitely realizable is realizable.[4] It will follow from the five theses that logical space is not compact (unless it is finite).[5] Indeed, the example given above can be used to provide a counterexample to compactness. For surely, for arbitrarily large finite n, there are possible worlds with at least n molecules, in which case the set P is finitely realizable. But it is not realizable, since no possible world both has and does not have a finite number of molecules. So, logical space is not compact. Since it is realizability, and not finite realizability, that serves to delimit the scope of the possible, the noncompactness of logical space shows that sets of propositions that are finitely realizable but not realizable play no particular role in the task of characterizing worlds. What does this have to do with consistency and finite consistency? With the aid of Thesis 5 below, the following result can be established for logical space: a set of propositions is consistent if and only if it is realizable.[6] Thus, it follows that sets of propositions that are finitely consistent but not consistent also are of no particular interest to our metaphysical inquiry -- no more interest, that is, than any other set of contradictory propositions.

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[4] In algebraic terminology, logical space is then a perfect field of sets. Terminology to the contrary notwithstanding, perfection is not particularly praiseworthy from a metaphysical perspective.

[5] See proposition (5.5) in section 5.

[6] See proposition (5.3) in section 5 and the discussion that follows.

We can arrive at this same conclusion from a somewhat different direction by asking: what difference would it make if we replaced consistency by finite consistency in Thesis 3? We would then have:

THESIS 3*. For any world, the set of propositions true at that world is a maximal finitely consistent set of propositions.

From Theses 1, 2, and 3* we could still derive (3.1) through (3.6), but (3.7) and (3.8) need no longer hold.[7] That is, Thesis 3* would not guarantee the truth-functional standardness of set-conjunction and set-disjunction. For all that Thesis 3* would require, there might be a world at which the conjunction of all true propositions was not itself true -- indeed, was a contradiction. If such deviant worlds are to be prohibited, we cannot afford to replace Thesis 3 by the weaker Thesis 3*. That is the lesson of the example presented above.[8]

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[7] In algebraic terminology: maximal finitely consistent sets of propositions are ultrafilters over the Boolean algebra of propositions; maximal consistent sets of propositions are <u>principal</u>, as follows from Lemma (3.9).

[8] A theory that replaces Thesis 3 by Thesis 3* will be discussed in more detail below. See section 5, subsection "The Compact Theory".

Objections to the Third Thesis.

But why accept the assumption, embodied in Thesis 3, that the propositions are truth-functionally standard at every world? In discussing this question, I will focus upon views that accept Thesis 1, and thus take the standard Boolean approach to the propositions. Views that posit a non-Boolean structure for the propositions cannot rightly be evaluated with respect to Thesis 3, since the notion of a maximal consistent set of propositions presupposes the Boolean structure for its definition (although analogues of Thesis 3 can be developed for some non-Boolean propositional structures). Thus, Thesis 3 cannot be used in determining whether or not views that involve a nonstandard conception of the propositions also involve a nonstandard conception of worlds. They need not: it may be that the nonstandard propositions are being used to characterize, via truth, the standard worlds in a nonstandard way.

But there are possible views, we shall see, that accept Thesis 1 and reject Thesis 3. That is, there are views that are perfectly orthodox with respect to the structure of the propositions, but that hold that the propositions are not truth-functionally standard at every world. Indeed, one of the reasons for positing the propositional structure without reference to truth, or to worlds, is that it allows for the formulation of such nonstandard conceptions of worlds. What grounds could one have for rejecting the nonstandard conceptions?

It would be a mistake to expect an argument here. An argument, if it were not just to run in a circle, would have to be based upon principles somehow even more secure, or more obvious, than principles (3.1) through (3.8) themselves. No such principles can be found. A view that rejects some or all of (3.1) through (3.8) is not thereby guilty of rejecting some even more obvious principle, or of committing some logical fallacy. Nor need it even be guilty of using words in an uncommon way. With respect to the word 'world', neither philosophical nor ordinary usage is sufficiently determinate to settle the matter in favor of Thesis 3. No, I think we must agree that a view that rejects Thesis 3 might simply be presenting an alternative conception of worlds, or of truth-at-a-world. Such alternative conceptions of worlds cast no more doubt upon Thesis 3 than the alternative conceptions of propositions discussed above cast doubt upon Theses 1 and 2; for Thesis 3, like Theses 1 and 2, is analytic in the sense that it determines in part the conception of worlds that is here under discussion.

On the conception of worlds that I favor, Thesis 3 follows as a matter of course, and does not stand in need of argument. On this conception, worlds are _concrete particulars_, where a defining feature of a concrete particular is that, for any property that can be meaningfully applied to it, either that property or its complement is true of it, but not both.[9] Since propositions are properties of worlds, Thesis 3 holds

[9] I use the term 'concrete particular' roughly in the sense of Goodman, _The Structure of Appearance_, 2nd ed. (Indianapolis: Bobbs-Merrill, 1966), pp. 248-250, but without the phenomenalist trappings. My use also agrees with Armstrong, _Nominalism and Realism_ (Cambridge: Cambridge University Press, 1978), pp. 120-121. _Pace_ Stalnaker, "Possible Worlds", _Nous_, 10 (1976), pp. 65-75, the concrete-abstract

on this conception given the intended interpretation of the propositional structure. But there are conceptions according to which worlds are abstract entities of one sort or another, and on some such conceptions Thesis 3 might plausibly be denied. Let me briefly canvass the possible alternative conceptions, and evaluate them for compatibility with Thesis 1.

A view that rejects Thesis 3 must also reject at least one of the truth-functional principles (3.1) through (3.8). But not all of these principles are equally open to debate. Let us look more closely to see with respect to which principles different conceptions of worlds might differ. Each of the principles places some constraint upon the way in which truth can be distributed throughout the algebra of propositions. Could there be a conception of worlds so liberal as to reject all of these constraints, thus allowing any set of propositions whatsoever to be the set of propositions true at a world? I think not. That would completely undermine the propositions' role as world-characterizers. In particular, sets of propositions would be cut off from their consequences with respect to the transference of truth. Since for a proposition to be true at a world is for it to characterize that world, this would allow that one might add something further to the characterization of a world by asserting the consequences of what had already been asserted. But this would be simply to misunderstand the

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distinction is not, in my opinion, the crux of the distinction between extreme and moderate realism about possible worlds. All realists, properly so-called, hold that (at least some) merely possible worlds are as concrete as the actual world; realists differ as to whether or not merely possible worlds have an irreducibly modal status.

intended interpretation of 'implication' as given in a previous section: a proposition says nothing over and above the propositions that imply it.

The point was neatly summarized by Wittgenstein:

> If a god creates a world in which certain propositions are true, then by that very act he also creates a world in which all of their consequences are true.[10]

Since, on the Boolean framework, the consequences of a set of propositions are just those propositions implied by some conjunction of members of that set, it follows immediately that the principles (3.1), (3.3), (3.4), and (3.7) will hold on any conception of worlds. Algebraically speaking, the set of propositions true at a world, for any world, is a *principal filter* over the Boolean algebra of propositions.

If, in addition, principle (3.6) holds on a conception of worlds, then the remaining principles (3.2), (3.5), and (3.8) will hold as well. So let us henceforth focus our attention on the two halves of (3.6). One half states that the law of contradiction holds at every world. Is there a legitimate conception of worlds on which the law of contradiction fails at some worlds, the so-called *impossible worlds*? I think that there is, at least for some sense of the word 'world', although I do not know how the details of such a conception could best be worked out. One way of motivating impossible worlds is in connection with some of the drawings of M. C. Escher. For example, consider his *"Klimmen en Dalen"*: the drawing depicts fourteen figures ascending a

[10] "Wenn ein Gott eine Welt erschafft, worin gewisse Sätze wahr sind, so schafft er damit auch schon eine Welt, in welcher alle ihre Folgesätze stimmen." *Tractatus Logico-Philosophicus*, 5.123.

staircase on top of a square tower with each figure a few steps in front of another figure, all together forming a closed loop. The situation depicted is impossible, and occurs in no possible world. But why not say that, in addition to possible worlds, there are impossible worlds in which such impossible situations occur? Indeed, such a world would be impossible, for among the propositions true at it are: (1) Space, throughout the region depicted, is (approximately) Euclidean; and (2) Within this region, one can travel a closed path through space while moving always in an upward direction. Since the negation of (2) follows from (1), such a world would be an impossible world, a world at which the law of contradiction is violated. Nonetheless, it is not trivially impossible in the sense that any proposition whatever is true at the world. For example, it is false in the situation depicted in the drawing that there are thirteen figures ascending the stairs: there are fourteen. It is this feature that gives impossible worlds their point: we seem to be able to think about them, and reason about them, in a nontrivial way. Of course, there is no room for impossible worlds on a conception of worlds as concrete particulars. On such a conception, impossible worlds, if they are wanted, will have to be introduced as things of an entirely different sort than possible worlds. But on a conception of worlds as abstract entities, both possible worlds and impossible worlds may be things of one kind, and so seem equally admissible -- for example, both might be identified with sets of propositions.

But a nontrivial conception of impossible worlds, whatever its merits, is beyond the reach of the present approach. For, given what was said above, if a proposition and its negation are both true at a world, then so must be all of their consequences. But on the Boolean framework given us by Thesis 1, every proposition is a consequence of a contradictory pair of propositions. So all propositions would be true at any impossible world. What is needed in order to capture a nontrivial notion of impossible worlds is a different, weaker notion of consequence, and thus a non-Boolean approach to the propositions under implication.[11] But since there is little point to admitting impossible worlds if the only impossible world has all propositions true at it, any conception of worlds based upon the present Boolean framework will presumably endorse the law of contradiction. Thus, principle (3.2), in addition to those listed above, will be satisfied by any reasonable conception of worlds; in other words, the set of propositions true at a world, for any world, is a proper, principal filter over the Boolean algebra of propositions.

Finally, we come to the other half of principle (3.6), the law of excluded middle. Is there a legitimate conception of worlds on which the law of excluded middle fails at some worlds, the indeterminate worlds? Again, I think that there is. It arises quite naturally if one

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[11] Some have attempted to use the consequence relation of relevance logic for this purpose. See, for example, Robert K. Meyer and Richard Routley, "Dialectical Logic, Classical Logic, and the Consistency of the World", Studies in Soviet Thought, 16 (1976), pp. 1-25. But I know of no reason to think that relevance logic provides a consequence relation appropriate to reasoning with respect to impossible situations such as that depicted above.

takes the view that talk of possible worlds is best understood as an extension of talk of works of fiction.[12] Indeed, we do sometimes speak of "the world" of a work of fiction; and we speak of propositions being true in the fiction, or in the world of the fiction. If one takes these locutions to be paradigmatic of our metaphysical talk about worlds, and truth-at-a-world, then one will be lead to reject the law of excluded middle. For consider any work of fiction, say, Hamlet. Innumerable details of particular fact are left out of the story. Did Hamlet bite his nails? The play does not explicitly tell us. Nor can it be inferred from what the play explicitly tells us together with whatever background assumptions the story implicitly carries with it. So it is not true in the play that he did, nor that he did not, bite his nails. If it makes sense, as ordinary language suggests, to speak of the world of Shakespeare's Hamlet, then this world must be an indeterminate world, a world at which the law of excluded middle is violated. Of course, the usual notion of a work of fiction will have to be generalized so as to remove any restrictions on the length or complexity or coherence of the story. This will allow that there be fully determinate worlds as well, worlds at which the law of excluded middle holds. Presumably, the actual world is, or corresponds with, one of the fully determinate worlds.

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[12] Such a view is implicit in Robert Adams's theory of possible worlds, the world-story theory. See his "Theories of Actuality", Nous, 8 (1974), pp. 211-231. But Adams's theory does not admit indeterminate worlds. It will be discussed below in section 6.

But does the notion of an indeterminate world make sense? The answer depends upon the conception of worlds that one holds. If one holds a conception of worlds according to which the worlds are concrete particulars, then there are no indeterminate worlds: concrete entities are fully determinate. On this conception, the project of explaining possible worlds in terms of fictional "worlds" has things exactly reversed: fictional "worlds" are to be analyzed in terms of possible worlds.[13] On any such analysis, an indeterminate "world" of fiction will turn out to be something of quite a different sort than a possible world: perhaps the set of all those determinate worlds whose true propositions include the true propositions of the indeterminate "world". But on those conceptions according to which worlds are abstract entities, there might be no objection to taking indeterminate worlds to be on a par with determinate worlds. The most clearcut case would be a conception of worlds according to which worlds were identified with certain sets of propositions.[14] I have given reasons above why, on any such conception, worlds should not be identified with sets of propositions that are not proper, principal filters over the algebra of propositions. But if the proponent of such a conception chooses to take our talk of _the_ world of a work of fiction seriously, I know of no argument that could persuade him that worlds should only be identified with principal _ultra_filters over the algebra of propositions, that is, that worlds must be fully determinate. He can either admit

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[13] As, for example, is done in David Lewis's "Truth in Fiction", _American Philosophical Quarterly_, 15 (1978), pp. 37-46.

[14] See section 6 below.

indeterminate worlds or not without contravening his conception of worlds as sets of propositions.

Thesis 3, then, might plausibly be denied on certain conceptions of worlds. But that casts no doubt upon Thesis 3 as here intended. I have briefly mentioned such conceptions so as to illustrate what I do not mean by the word 'world'. This being done, I hereby adopt Thesis 3 into the present theory.

SECTION 4

SEPARATING WORLDS BY PROPOSITIONS

Fourth Thesis.

Take any world w. Each of the propositions true at w characterizes w: propositions are properties of worlds. In the previous section, we saw that the proposition that is the conjunction of all propositions true at w is itself true at w, and that, being an atomic proposition, it gives the complete characterization (or complete description) of w: nothing more can be said about w without falling into contradiction. But does the proposition that completely characterizes w also uniquely characterize w? That is, is this proposition false at every world distinct from w? Clearly, the answer had better be 'yes' if the propositions are not to be deficient in their role as the characterizers of worlds. Indeed, it follows from the present broad, purely metaphysical conception of the propositions that the answer is 'yes', and trivially so. It will be the purpose of Thesis 4 to guarantee that the present theory embrace a sufficiently broad conception of the propositions. In this section, I first discuss how Thesis 4 constrains the structure of logical space; then I try to show that the broad conception of the propositions elucidated by Thesis 4 is a natural one, and I defend it against possible objections.

If two worlds are such that there is a proposition true at one but not at the other, then I will say that the worlds are <u>separated by a proposition</u>. The fourth thesis to be considered asserts that every two worlds are separated by a proposition. In other words:

THESIS 4. For distinct worlds w and v, the set of propositions true at w is distinct from the set of propositions true at v; that is, if $w \neq v$, then $Pw \neq Pv$.[1]

It follows from Theses 1 through 4 that the atomic proposition that completely characterizes a world also uniquely characterizes that world. Indeed, in light of Theses 1 through 3, Thesis 4 is equivalent to:

(4.1) For distinct worlds w and v, the conjunction of all propositions true at w is not true at v; that is, if $w \neq v$, then it is not the case that $\Lambda Pw \varepsilon Pv$.

<u>Proof</u>. If (4.1) holds, then distinct worlds w and v are separated by a proposition: either ΛPw or ΛPv will do. Conversely, assume that Thesis 4 holds. Then there is a proposition p such that either: (1) $p \varepsilon Pw$ and not $p \varepsilon Pv$; or (2) $p \varepsilon Pv$ and not $p \varepsilon Pw$, in which case, by (3.6), $-p \varepsilon Pw$ and not $-p \varepsilon Pv$. Now, assume for purposes of a <u>reductio</u> that $\Lambda Pw \varepsilon Pv$. In case (1), we have, by (3.7), that $p \varepsilon Pv$. Contradiction. In case (2), we have, again by (3.7), that $-p \varepsilon Pv$, which contradicts the consistency of Pv. Therefore, it is not the case that $\Lambda Pw \varepsilon Pv$, and (4.1) holds.

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[1] Notation used throughout this work: for any world w, Pw is the set of propositions true at w; for any proposition p, Wp is the set of worlds at which p is true.

Since, given Theses 1 through 3, Thesis 4 and (4.1) are equivalent, I could choose to focus upon either one of them in the discussion that follows; for the most part, I will focus upon Thesis 4.

Thesis 4 guarantees that the propositions are sufficiently numerous to characterize the worlds not only in the weak sense given by (4.1), but also, with the aid of Thesis 2, in the following stronger sense:

(4.2) For any set of worlds, there is a proposition true at all and only the worlds in that set.

Proof. Consider any nonempty set of worlds W (the empty set, of course, has the null proposition true at all and only its members). For each world $w\varepsilon W$, there is an atom $aw=\Lambda Pw$ (by Theorem (3.13)). Consider the proposition $p=V\{aw\}w\varepsilon W$. Such a proposition exists by Thesis 2. Let $Wp=\{w: p\varepsilon Pw\}$. I claim that $W=Wp$. For let $w\varepsilon W$. Clearly, $aw\rightarrow p$, and so $p\varepsilon Pw$. Thus $w\varepsilon Wp$. Conversely, let $w\varepsilon Wp$. Otherwise put, $p\varepsilon Pw$. Now, since Pw is maximal consistent (by Thesis 3), it must contain at least one disjunct of every disjunction that it contains. Thus, for some $v\varepsilon W$, $av\varepsilon Pw$. But also $aw\varepsilon Pw$, and so it follows from Theorem (3.13) that $aw=av$, and that $w=v$. Whence $w\varepsilon W$. Therefore, $W=Wp$, as was to be proved.

(4.2) will be a crucial lemma in any attempt to establish a correspondence between propositions and sets of worlds. It is interesting to note that the above proof of (4.2) is the only place that Thesis 2 is used towards establishing that result: its only role is to assist Thesis 4 in making sure that the propositions are out in full force.[2]

The effects of Thesis 4 can also be seen by way of the constraints it puts on the structure of logical space. Recall that logical space is the unique field of sets (if any such exists) that is isomorphic to the Boolean algebra of propositions and has the set of worlds as the points of its space. According to a standard terminology, a field of sets is reduced if and only if distinct points of its space are always separated by a set in the field; that is, for any two points of the space, there is a set in the field that contains one of the points but not the other. Thesis 4 asserts that logical space is reduced. For if distinct worlds w and v are separated by a proposition p, then, in the field of sets isomorphic to the Boolean algebra of propositions, w and v are separated by a set in the field -- namely, the image of p under the isomorphism in question.[3]

Indiscernibility Principles.

Let us turn to evaluating the status of Thesis 4. Is it a controversial metaphysical thesis about the nature of possible worlds? Although I shall look at some attempts to generate controversy below, Thesis 4 is not intended to be controversial in any way. It follows quite trivially from the broad conception of the propositions here being

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[2] For further insight into the role played by Thesis 2, see the independence proof in the second appendix.

[3] What does Thesis 4 tell us about topological logical space? Thesis 4 asserts that topological logical space is totally disconnected, that is, that distinct points of the space are always separated by a set that is both open and closed in the topology.

used. Whoever views Thesis 4 as controversial has presumably confused it with some stronger thesis that invokes some narrower conception of the propositions. For example, all of the following stronger versions of Thesis 4 are controversial, and make substantive claims about the nature of possible worlds:

(4.3) Distinct worlds are separated by an expressible proposition.

(4.4) Distinct worlds are separated by an empirical proposition.

(4.5) Distinct worlds are separated by a purely qualitative proposition.

Exactly which substantive claims are made by the above theses will depend upon how one interprets the underscored terms; I shall have more to say about (4.3) through (4.5) in sections to follow. But for now it is enough to note that each of the above theses, as is seen by taking their contrapositives, represents a nontrivial special case of the Identity of Indiscernibles: they claim, respectively, the identity of empirically indiscernible worlds, of linguistically indiscernible worlds, and of qualitatively indiscernible worlds. Whereas, on the other hand, Thesis 4 represents the trivial version of the Identity of Indiscernibles as applied to worlds. Its triviality will be manifest from the triviality of the supporting arguments given below.

As a first attempt at an argument for Thesis 4, I offer the following instructive failure. Consider any two distinct worlds; call them w and v. Now, as long as one is broad-minded about what to count as a proposition, there is no problem finding a proposition that separates w and v. Consider the proposition expressed by: "The world is identical with w". Taking 'the world' to be an ordinary definite description that nonrigidly designates, at each world, that world itself, the above proposition is true at w, but false at v. So, Thesis 4 is established.

The above argument tries to do too much, and that is its downfall. It claims to exhibit in language a proposition that separates the two worlds in question. But to what language does the expression "The world is identical with w" belong? Not English! Nor does it belong to an extension of English that contains a new proper name 'w' introduced solely for the purpose of the argument. As used above, 'w' is not a genuine proper name at all. It was introduced into the argument, not to name some particular world, but to stand indifferently for any world; let us call it an arbitrary name. It is a common practice to introduce arbitrary names as a means of facilitating the presentation of quantificational arguments; indeed, I have done this throughout the present work. The practice is perfectly legitimate: rules of logic can be drawn up that govern the proper treatment of arbitrary names within the context of an argument.[4] In fact, once an arbitrary name has been introduced into an argument, it can be treated just as if it were a genuine proper name, so there is usually little harm done in confusing a proper name with an arbitrary name. Harm is done, however, if some part of the argument makes use of the fact that a name is a proper name. And that is exactly what the above argument does when it claims to exhibit in language a proposition that separates the two worlds. Since this

[4] For a standard source, see E. J. Lemmon, Beginning Logic (Indianapolis: Hackett, 1979).

claim is false, the argument, as it stands, must be rejected.

But an argument for Thesis 4 need not make any claims at all about the expressibility of propositions. The argument given above tried to argue for some version of (4.3), that distinct worlds are separated by an expressible proposition, not directly for Thesis 4. An argument for Thesis 4 need only show that certain propositions exist, not that they can be expressed in some actual or possible language. This allows us to reformulate the argument as follows. Consider any two distinct worlds; call them w and v. To show that there exists a proposition that separates w and v, it suffices to find something that is true of one of the worlds but not the other. What could be easier: only w is identical with w; v is not. This establishes Thesis 4. The argument is completely trivial in that its conclusion is but a thinly disguised restatement of its premise.

Objections to the Fourth Thesis.

How might this argument be challenged? A character in a Max Black dialogue named B seems to think that even the above argument involves an illegitimate use of the symbols 'w' and 'v' -- at least in the case where the two worlds in question are qualitatively indiscernible.[5] Since I do not want to presuppose (4.5), the identity of qualitatively indiscernible worlds, in the argument for Thesis 4, this would be a serious objection indeed.

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[5] "The Identity of Indiscernibles", Problems of Analysis (Ithaca: Cornell University Press, 1954).

Let me adapt B's comments, which were made in connection with the case of two qualitatively indiscernible globes within a single world, to the present case of two, perhaps qualitatively indiscernible, entire worlds. At the point where the names 'w' and 'v' were introduced, B would retort: "But which of the worlds is to be named 'w' and which of the worlds is to be named 'v'? Unless you provide some means by which the name 'w' is attached to one of the two worlds but not the other, you are not entitled to use this name in your argument. You have not provided any such means; indeed, that would be impossible in the case where the worlds in question are qualitatively indiscernible." Now, it should be clear in light of the above discussion that B is confused about the argument's use of the names 'w' and 'v': he mistakenly takes them to have been introduced as genuine proper names. But given that, on the contrary, they were introduced as arbitrary names, his objection makes no sense at all.

Although B's initial retort is based upon this confusion, it soon becomes apparent that his objection to the argument for Thesis 4 can be traced to a different source. For even after I had pointed out to him that the names 'w' and 'v' were not intended as genuine proper names, he would continue to object as follows: "But the argument depends upon 'w' being a genuine proper name. If the argument does not use 'w' as a genuine proper name, then it cannot succeed in showing that the proposition in question exists."

But why would B claim that an argument for Thesis 4 must make a detour through proper names? Presumably, because he takes it to be a defining characteristic of the propositions that they be, in some sense, expressible. That is, he requires that an argument for Thesis 4 be at the same time an argument for (4.3). But that is the confusion with which we began this discussion, and now we have come full circle. B's objection simply disregards the stipulation that the propositions, on the present conception, have no essential ties to language. Perhaps B's objection is motivated by the belief that this purely metaphysical conception of the propositions is somehow incoherent. If so, he owes us an argument; none has been forthcoming.

I suspect that what bothers some people about the argument for Thesis 4 is just its complete triviality. If they misunderstand the nature of that triviality, they might take it to be an objecion to the broad conception of the propositions. The argument claims that distinct worlds w and v can be separated by the proposition that the world is identical with w. Moreover, according to the present theory, this proposition completely and uniquely characterizes w. But the objector finds this absurd: how, he asks, can the trivial and uninformative proposition given above be said to characterize a world? Thus, he rejects the argument's claim to have provided a proposition that separates w from v by completely and uniquely characterizing w. But the objector is confused. There is nothing trivial or uninformative about the proposition! It is as full-blooded and informative as can be, implying everything that can be said about the world in question. It is only the way in which the argument picks out the proposition that is trivial and uninformative, not the proposition itself. Once one realizes that the triviality is not in the proposition but in the way it was picked out, there is no longer a temptation to reject the proposition as unfit to perform the task of separating the two worlds in question, or of completely and uniquely characterizing one of them.

SECTION 5

CONSISTENCY AND REALIZABILITY

Fifth Thesis.

Thesis 4, we saw, ensures that there are enough propositions around to perform the task of characterizing the worlds: for any set of worlds, there is at least one proposition true at exactly the members of that set. But might there be more than one such proposition? Or are there only as many propositions as are needed to fully characterize the worlds? Nothing in the idea of propositions as world-characterizers limits the number of propositions: it is one thing to characterize efficiently, quite another just to get the job done. Moreover, for all that has been said to this point, additional propositions might have an additional role to play, perhaps as characterizers of some mathematical or metaphysical reality existing independently of the worlds. The fifth and final thesis to be included in the present theory, however, requires that the propositions form the minimal structure (closed under the Boolean operations) capable of fully characterizing the worlds, and thus that distinct propositions always play distinct world-characterizing roles. The thesis is controversial in ways that the previous four theses were not, although it seems to me that the controversy has generally been misplaced. As we shall see, the thesis has significant implications for realism about possible worlds, and for the customary possible-worlds analysis of the alethic modalities.

Before turning to the controversy, let me present Thesis 5 and develop some of its consequences. In order to ensure that the propositions be minimal characterizers, it suffices to posit either of the following two statements:

(5.1) Every proposition distinct from the null proposition is true at some world.

(5.2) Every proposition distinct from the universal proposition is false at some world.

A restriction on the number of propositions at either of the two extremes, it turns out, reverberates throughout the entire propositional structure. Since the converse of each of these statements is already given by Thesis 3, an acceptance of (5.1) and (5.2) would entail accepting that the null proposition is the one and only proposition false at every world, and that the universal proposition is the one and only proposition that is universally true. It is not necessary to explicitly incorporate both (5.1) and (5.2) into the theory; they are equivalent in light of Theses 1 and 3. Proof: Assume (5.1), and let p be any proposition distinct from the universal proposition. It follows from Boolean algebra that -p is then distinct from the null proposition, and so, by (5.1), -p is true at some world. But then, by Thesis 3, p is false at some world, as was to be shown. A similar argument shows that (5.1) follows from (5.2). Since (5.1) will be the more convenient formulation for what follows, I will choose to focus upon it as the fifth thesis:

THESIS 5. Every proposition distinct from the null proposition is true at some world.

In other words: every consistent proposition is realizable.

This relation between consistency and realizability easily generalizes to sets of propositions:

(5.3) Every consistent set of propositions is realizable.

Proof. Let P be a consistent set of propositions; that is, $\Lambda P \neq \emptyset$. By Thesis 5, ΛP is true at some world w. By Thesis 3 (in particular, (3.7), the truth-functional principle for set-conjunction), every member of P is true at w, and so P is realizable.

Since the converse of (5.3) is provided by Thesis 3, Thesis 5, if accepted, requires that a set of propositions is consistent if and only if it is realizable. This coincidence between two propositional notions, one defined in terms of the Boolean structure, the other defined in terms of truth and of worlds, is a special case of a more general result to be proved in section 7: all propositional notions definable in terms of the Boolean structure have counterparts definable in terms of worlds, and truth-at-a-world.[1] This result is reminiscent of completeness results for formal logical systems, whereby it is shown that two sets of sentential concepts, one syntactic and one semantic,

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[1] See Theorem (7.2) in section 7.

exactly coincide. The difference is that here the two sets of concepts apply to propositions rather than sentences, and thus the concepts in question are neither syntactic nor semantic, but metaphysical in nature.

The general result to be proved in section 7 will also serve to guarantee that the notion of logical space defined in section 2 is not vacuous: it follows from Theses 1 through 5 that there exists an appropriate field of sets composed entirely of worlds and isomorphic to the Boolean algebra of propositions. It is easy to see the role that Thesis 5 plays here. For suppose that there were two propositions false at every world. Both of them would have to be mapped into the empty set, thereby collapsing the Boolean structure. That's no way to make an isomorphism.

In addition to providing for the existence of logical space, Thesis 5 further constrains its structure, thereby indirectly constraining the structure of the Boolean algebra of propositions. We saw in section 3 that if a proposition is true at some world w, then there is an atomic proposition that implies it -- ΛPw will do. Since Thesis 5 asserts that every nonnull proposition is true at some world, it follows immediately that every nonnull proposition is implied by an atomic proposition. In other words:

(5.4) The propositions with respect to implication form an atomic Boolean algebra.

The effects of Thesis 5 upon the structure of logical space can now be summarized in the following way. Thesis 5 guarantees, on the one hand, that every consistent proposition is implied by an atomic proposition, or, what is equivalent according to Theorem (3.13), that every consistent proposition is contained within a maximal consistent set; and, on the other hand, that every atomic proposition, or, equivalently, maximal consistent set of propositions, is realizable. Without Thesis 5, a consistent proposition might fail to be realizable either because it is not implied by any atomic proposition, or because none of the atomic propositions that imply it are realizable.[2] In an extreme case, the algebra of propositions might have no atomic propositions or maximal consistent sets of propositions at all; although given Thesis 3, this could occur only if there were no worlds.

The Compact Theory.

Whether or not the algebra has any maximal consistent sets of propositions, it will always have maximal finitely consistent sets. These sets appear to play a role in the semantics for modal logic, for example, in completeness proofs involving canonical models.[3] Actually, it is sets of sentences that directly play this role; the sets of propositions come into it, if it all, indirectly by way of the interpretation of the language in question. Call a sentence or set of sentences Consistent (with a capital 'C') just in case it is

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[2] Thesis 5 can thus be broken down into two independent subtheses, what will be called Thesis 5' and Thesis 5^+ in section 6. That they are independent is shown in the second appendix.

[3] See Brian Chellas, Modal Logic (Cambridge: Cambridge University Press, 1980), pp. 60-62. Cf. also Hintikka's use of "model sets" in, for example, "The Modes of Modality", Models for Modalities (Dordrecht: D. Reidel, 1969), pp. 71-87.

syntactically consistent with respect to the logic in question, that is, just in case no explicit contradiction can be derived from it by using the axioms and rules of the logic. Since the rules of the logic are assumed to be finitary, a set of sentences is finitely Consistent if and only if it is Consistent. Now, the maximal (fintely) Consistent sets of sentences of the logic are taken to be the "worlds" of the canonical model, and this suggests with respect to the corresponding sets of propositions that a stronger version of Thesis 5 might be wanted:

THESIS 5*. Every finitely consistent set of propositions is realizable.

Unfortunately, Thesis 5* is inconsistent with Theses 1 through 4 unless logical space is finite. To show this, it suffices to show that if logical space is infinite, there will be a finitely realizable set of propositions that is not realizable, that is, that logical space is not compact. In section 3, I presented an intuitive counterexample to compactness; but now we are in a position to see more generally why compactness fails.

(5.5) Logical space is not compact unless it is finite.

Proof. Assume that logical space is infinite; that is, assume that there are infinitely many sets of worlds in the field, and thus infinitely many worlds in the space. Call a set of worlds cofinite if its complement in the field is a finite set of worlds. By (4.2), for each cofinite set of worlds there is a proposition true at just the worlds in that set. Let Pcf be the set of all such propositions true at cofinitely many worlds. I claim that Pcf is finitely realizable but not realizable. Indeed, every finite subset of Pcf is realizable, since the intersection of finitely many cofinite sets is again cofinite and, *a fortiori*, nonempty. But Pcf is not realizable, since the intersection of all cofinite sets of worlds is the empty set. Since there is a set of propositions that is finitely realizable but not realizable, logical space is not compact.

Note that, by Thesis 3, the set Pcf is finitely consistent; it thus also serves as a counterexample to Thesis 5* under the assumption that logical space is infinite.

An examination of the above proof reveals that the failure of compactness can ultimately be attributed to Thesis 3, and, in particular, to the truth-functional principle for set-conjunction, (3.7). Whoever wants his logical space compact had better accept only the weaker version of Thesis 3 introduced in section 3, the version that does not entail (3.7):

THESIS 3*. For any world, the set of propositions true at that world is a maximal finitely consistent set of propositions.

The compactness of logical space is consistent with a theory that replaces Thesis 3 by Thesis 3*, even if logical space is infinite. Moreover, if the theory also replaces Thesis 5 by Thesis 5*, then logical space is required to be compact; for every finitely realizable set of propositions will be finitely consistent by Thesis 3, and thus realizable by Thesis 5*. Let us call the theory that accepts Theses 1 through 5 the standard theory, and the theory that accepts Theses 1, 2, 3*, 4, and 5* the compact theory. In effect, the compact theory results from the standard theory by having finitely consistent sets of propositions play the role, vis-à-vis worlds, previously played by the consistent sets of propositions. The compact theory entails that logical space is a perfect, reduced field of sets, the sort of field that the Stone Representation Theorem for Boolean algebras tells us can always be found.[4] Worlds are now in one-to-one correspondence with maximal finitely consistent sets of propositions, but not with maximal consistent sets of propositions (unless logical space is finite). Moreover, it is no longer the case that every set of worlds has a proposition true at just the worlds in that set, lest the set of propositions Pcf still be around to undermine compactness.

There are two quite different strategies available to the theorist who wants his space compact. Starting from the standard theory, the compact theorist can either shrink the algebra of propositions or expand the space of worlds (or do a combination of both). The first sort of compact theorist holds a narrower conception of propositions, countenancing some but not all of the propositions countenanced by the standard theorist. What the compact theorist calls "propositions" will have to form a Boolean subalgebra of the full Boolean algebra of propositions so as not to run afoul of Thesis 1. But which propositions will be left out of the subalgebra? To allow for compactness, every set

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[4] According to the compact theory, topological logical space is the Stone Space of the Boolean algebra of propositions, with worlds as points -- a totally disconnected, compact topological space.

of propositions over the full algebra that was a counterexample to compactness, such as the set Pcf defined above, will have to be eliminated by eliminating at least one of its members. Beyond that, anything goes. Note that the resulting subalgebra (if infinite) will have sets of propositions that are both inconsistent and realizable; but, properly understood, this is no cause for alarm. These sets are inconsistent *relative to the subalgebra*, but consistent *in fact*, that is, consistent relative to the full algebra of propositions. It is just that the nonnull lower bounds of such sets were left out of the subalgebra. I have no objection to the compact theory so interpreted, as long as one is clear that a narrower conception of propositions is involved. The propositions expressible within some language, for example, might well be represented by a compact "logical space", although the representation is only a subfield of logical space, properly so-called.

There is a second way to ensure that logical space be compact: instead of subtracting propositions, one adds new "worlds", one new world for each set of propositions that is maximal finitely consistent, but not maximal consistent.[5] Each of these added worlds is impossible in a weak sense according to which the conjunction of all propositions true at it is inconsistent, but not in a stronger sense according to which there is a single inconsistent proposition true at the world; the inconsistency, so to speak, is hidden among the infinite, rather than

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[5] Topologically speaking, this results in the Stone-Czech compactification of topological logical space. See John Kelley, *General Topology* (New York: D. Van Nostrand, 1955), pp. 152-3.

out in the open. Is this strategy coherent from a metaphysical standpoint? It has the effect of imposing compactness upon logical space by divine decree: Let there be worlds to ground all finitely consistent sets of propositions! But, as Leibniz might have pointed out, it is far from clear that even God has the power to create an impossible world. Moreover, the impossible worlds here invoked do not even seem to be well-motivated by impossible-world theorists' standards. If there are impossible worlds, why must their impossibility be subtly hidden among infinite sets of propositions? Why not allow finite inconsistent sets of propositions to have all their members true at an impossible world? Or why not require that the impossibility be hidden among sets of propositions of some higher infinite cardinality, say, the cardinality of the continuum? I can think of no answers to these questions that would not rest irrelevantly upon some semantic or epistemological point. Whatever role such impossible worlds might play in semantics or epistemology, then, they are best left out of metaphysical discussions of worlds.[6]

We are now in a position to assess the significance of canonical model constructions in modal semantics. But first it should be pointed out that, on a purely formal level, such constructions do not stand in need of justification. If the only goal is to prove a mathematical theorem establishing the coextensiveness of two notions -- provability

[6] For example, such impossible worlds could be used to (slightly) simplify the truth conditions for counterfactuals, though, I would say, at a disproportionate cost. See David Lewis, "Counterfactuals and Comparative Possibility", Contemporary Research in Philosophical Logic and Linguistic Semantics (Dordrecht: D. Reidel, 1975), Hockney et. al., eds., pp. 13-14.

within some modal logic and validity within some class of structures -- then it matters not at all what the elements of the structures are taken to be, as long as there are enough of them. But completeness proofs can be given for interpreted languages as well as for uninterpreted languages, and in such cases it is not irrelevant to ask about the connection between the language's true interpretation and the interpretation provided by the canonical model.

From the perspective of the present theory, completeness proofs involving canonical models in general employ a combination of the two strategies described above. First, since only the propositions expressible within some countable language are under consideration, attention is focused upon a subalgebra of the algebra of propositions. Second, every set of propositions that is maximal finitely consistent in the subalgebra is supplied with (at least) one "world" at which all of its members are true: the maximal Consistent set(s) of sentences that express it.[7] Thus, the algebra of expressible propositions is represented by a field of sets whose space is composed, not of worlds, but of maximal Consistent sets of sentences. Of course, there is no guarantee that, under the true interpretation, every maximal Consistent set of sentences expresses a set of propositions that is maximal consistent in the subalgebra. We can distinguish three cases:

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[7] Of course, the <u>set</u> of propositions expressed by a <u>set</u> of sentences contains all and only those propositions expressed <u>individually</u> by some sentence in the set.

(1) The maximal Consistent set of sentences contains a sentence that is Consistent, but not consistent: the sentence expresses the null proposition. If such a set of sentences is taken to represent a world, it will have to be an impossible world in the strong sense given above: an inconsistent proposition is true at it.

(2) The maximal Consistent set of sentences expresses a consistent set of propositions. It can then be taken to represent, at least in part, a possible world.

(3) The maximal Consistent set of sentences expresses a set of propositions that is finitely consistent, but not consistent (in the subalgebra). There are two cases:

(a) The set of propositions is <u>in fact</u> consistent, and only inconsistent with respect to the subalgebra of expressible propositions. In this case, it can still be taken to represent, at least in part, a possible world, since it is genuinely realizable.

(b) The set of propositions is also inconsistent in the full algebra of propositions. In this case, if the set of sentences is taken to represent a "world", it will be an impossible world in the weaker sense given above: the <u>set</u> of propositions true at the world is inconsistent, even though no inconsistent proposition is true at the world.

If all maximal Consistent sets of sentences in the canonical model fall under case (2) or case (3a), then the interpretation provided by the canonical model matches the true interpretation and the the maximal Consistent sets of sentences can be taken to represent worlds in more than just name. Except for one problem. A maximal Consistent set of sentences of a countable language will generally be only a partial description of a world; it represents, not a single world, but a set of worlds indiscernible with respect to the expressive powers of the language in question.[8] Nonetheless, the canonical model for such an interpreted language has a clearcut metaphysical meaning, and the completeness proof can be given more than merely formal significance.

On the other hand, if even one maximal Consistent set of sentences of the language falls under case (1) or case (3b), then the interpretation provided by the canonical model does not adequately match the true interpretation, and the completeness result should be relegated to a purely formal status. That is all to the good. For, although one may still wish to call the uninterpreted language in question complete, the interpreted language, properly speaking, is not: the semantic notions, such as consistency, and the syntactic notions, such as Consistency, do not in this case coincide.

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[8] This issue is discussed extensively in section 8.

<u>The Mathematical Objection</u>.

The compact theory accepted Thesis 5, and more besides. The more serious objections to the standard theory, however, come from the other direction. In this subsection and the next, I will consider in turn two arguments for rejecting Thesis 5. The first is concerned with the status of mathematical propositions, and is undoubtedly familiar; the second is concerned with the status of certain modal propositions.

The standard theory, we saw, requires that there be only one proposition that is true at every world. This confronts the standard theorist with the following dilemma: either he must hold that there is only one true mathematical proposition, or he must hold that some mathematical propositions are contingently true, that is, true at some worlds but not at others.[9] I find both of these options utterly implausible. The standard theorist, it seems to me, has no choice but to sidestep the dilemma by modifying the rules of the game in one of two ways: he can allow logical space to contain points that are not, properly speaking, worlds; or he can factor out and discard the mathematical propositions from the full algebra of propositions, thus restricting the scope of his theory. Because the present work is not chiefly concerned with the mathematical propositions (or the other species of necessary truth), I will here opt for the latter, more modest approach. Either way, one must relinquish any hope of analyzing <u>all</u>

[9] A similar dilemma arises with respect to metaphysical propositions (such as the theses of the standard theory!), and all other species of necessary truth. I will focus upon the mathematical case in what follows, though my treatment of the other cases would be essentially the same.

propositions (of a metaphysical sort) in terms of possible worlds.

Let us briefly consider the two horns of the dilemma. What is wrong with the view that there is only one true mathematical proposition? Not the usual complaint. The problem is not that one can believe one and not the other of two true mathematical statements. That is so, but irrelevant. Belief and the other propositional attitudes are relevant to epistemological notions of proposition, but not directly relevant to the metaphysical notion of proposition here under discussion. Indeed, if the argument from belief were relevant to the individuation of the metaphysical propositions, it would argue just as well for a plurality of logical truths as for a plurality of mathematical truths. But a single logically true proposition suffices. The metaphysical propositions are individuated by (an admittedly elusive notion of) content, and all logical truths are alike in this respect; they all say the same thing -- namely, nothing.[10]

Similarly, all logical falsehoods are alike in content and say the same thing -- namely, everything. Logically false propositions thus imply all propositions, and, by Thesis 1, are equivalent to the null proposition. Since, again by Thesis 1, equivalent propositions are identified, it follows that the null proposition is the only logically false proposition. This "argument", if it deserves the name, is almost but not quite question-begging: it rests upon Thesis 1, whereas it is

[10] Although the notions of logical truth and logical falsehood are primarily semantic (or syntactic) notions applying to sentences, they can safely be transferred to the propositions: a proposition is logically true (false) if and only if some logically true (false) sentence expresses it.

here Thesis 5 that is in dispute.

Now, one might think (and I have heard it suggested) that a parallel argument can be used to defend the view that there is only one false mathematical proposition. For, in mathematics as well as in logic, it often seems that from any one false statement, any other false statement can be derived. This is conveniently illustrated with respect to a subset of the statements of arithmetic: all numerical equations involving addition, subtraction, multiplication, or division of whole numbers. It is a familiar fact that any false numerical equation can be transformed into any other false numerical equation by uniformly performing the operations of addition, subtraction, multiplication, and division upon both sides of the equation. For example, "3+2=7" can be transformed into "1=2" by, first, subtracting 3 from each side, then, dividing each side by 2. Since all false numerical equations are in this way interderivable, it follows, the argument runs, that they all express the same proposition. Moreover, the argument continues, it is plausible that this interderivability of falsehoods can be extended to all of mathematics, once all of mathematics is couched within a single theory, presumably, set theory. If this is so, the argument concludes, then the view that there is only one false mathematical proposition is exactly on a par with the view already endorsed that there is only one logical falsehood.

But the analogy between the mathematical case and the logical case is spurious. Indeed, the above argument does not even succeed in showing that all false numerical equations express the same proposition. In the course of deriving one false numerical equation from another, various assumptions are needed, such as the associative laws, the commutative laws, and other general laws of arithmetic. When these assumptions go unmentioned, as in the example above, they may also go unnoticed, thus giving the impression that the derivation depends only upon trivial logical truths such as that performing the same operation upon equals always yields equals. But, in fact, a substantial portion of the theory of arithmetic must already come into play.[11] Call the conjunction of all mathematical propositions that need to be used in such derivations z. All that the above argument shows is that, where p and q are two propositions expressed by false numerical equations, p&z and q&z are equivalent, and so identical. It does not follow from the argument that p and q are identical unless both p and q are the null proposition, which is the very point at issue, or unless z is the universal proposition, which is at least as controversial as the point at issue.

One way to see clearly why the argument fails is to note that an exactly similar argument could be used to show that there is only one proposition false at the actual world. Let p@ be the proposition that gives the complete description of the actual world. All false propositions are interderivable -- that is, equivalent -- <u>relative to</u> the (true) assumption p@. But, of course, Thesis 1 cannot then be invoked to identify propositions that are equivalent only in this relative sense; the assumption in question has to be not only true, but logically true, and so not really needed in the first place. Equivalent

[11] This echoes Frege's famous criticism of Leibniz's derivation of '2+2=4'. See <u>Die Grundlagen der Arithmetik</u>, section 6.

propositions are to be identified only if they are absolutely equivalent: each implies the other with respect to the relation of implication posited by Thesis 1.

In summary, the above argument fails unless it can be shown that, at the very least, the mathematical proposition z is identical with the one logically true proposition, the universal proposition T. More generally, it appears that any argument for the view that there is but one true mathematical proposition must rest upon some version of the logicist thesis that mathematics is reducible to logic, and indeed, to a logic empty of content. A discussion of logicism, of course, is beyond the scope of this work, but perhaps it will suffice to point out that it is generally agreed that mathematics is reducible to logic only if "logic" is taken to include, overtly or covertly, a significant amount of set theory, say, Zermelo-Fraenkel set theory plus the axiom of choice (ZFC). If this is so, then the argument at best reduces the question whether all true mathematical statements express the universal proposition to the question whether the axioms of ZFC all express the universal proposition.[12] But is it plausible that the axioms of ZFC all express the universal proposition and are devoid of content? That is not an account of mathematics, but a discounting of it, a failure to take mathematics seriously. The statements of set theory stand in material relations such as implication, consistency, and independence just as surely as do the contingent propositions; and such relations

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[12] I say "at best" because even this ignores the problem of mathematical truths that cannot be proven from the axioms, as must exist given Gödel's Theorem and the law of excluded middle (part of Thesis 3).

bear witness, it seems to me, to a rich and varied content inexplicable on the view that there is only one true mathematical proposition. If propositions are differentiated by differences in content, then there seems to be no choice but to admit a plurality of true mathematical propositions.[13]

That takes us to the other horn of the dilemma. It is consistent with the standard theory to admit a plurality of true mathematical propositions, but only if all but one of them are contingently true, true at the actual world but false at some nonactual possible world. It will not be possible for me to discuss here the various positions that have been taken according to which mathematics is, in some sense, contingent. There is, for example, the super-empiricism of Mill, the pragmatism of Quine, and the intuitionism of Brouwer. It is doubtful, in any case, whether any of these positions allow for the sort of thoroughgoing contingency required by the standard theory. I would, however, like to briefly consider some suggestions made by Robert Stalnaker in his paper "Propositions".[14] Towards the end of this paper, Stalnaker attempts to defend a version of the standard theory against the mathematical objection. He seems to be attempting to straddle the

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[13] This argument, of course, is not conclusive; it rests upon some form of mathematical Platonism rather than supporting it. It is always open to the non-Platonist to try to explain away what he takes to be merely an appearance of genuine content.

[14] In Issues in the Philosophy of Language (New Haven: Yale University Press, 1976), Alfred MacKay and Daniel Merrill, eds. Stalnaker defines the propositions in terms of their epistemological role; but since he appears not to distinguish between epistemological and metaphysical notions of proposition, I believe it is not inappropriate to interpret his views within the present context.

two horns of the dilemma by playing one bad option off against the other; I suspect that he has been twice impaled.

Stalnaker holds that, strictly speaking, the statements of mathematics express the universal proposition if true, or the null proposition if false: "There are only two mathematical propositions". Realizing, however, that this does not do justice to the complexity of mathematical investigation, he proposes that the mathematicians' "objects of study" are not (just) the mathematical propositions themselves, but rather propositions asserting that mathematical statements express the universal proposition. Thus, for any mathematical statement, say, the Goldbach Conjecture, there are two propositions involved. First, there is the proposition that the statement expresses, either the universal or the null proposition; this Stalnaker calls the mathematical proposition. Second, there is the proposition asserting that the Goldbach Conjecture expresses the universal proposition; this is what a mathematician would come to know if he proved the Goldbach Conjecture, what Stalnaker calls the object of study.

Why this puzzling separation of "mathematical propositions" from "the objects of study in mathematics"? Granted, by taking the objects of study to be sufficiently numerous and complex, Stalnaker avoids the objection that his theory fails to take <u>mathematicians</u> seriously. But if there are only two mathematical propositions, is he not still open to the objection that his theory fails to take <u>mathematics</u> seriously? In any case, it seems perverse not to call the objects of study in mathematics, assuming they are propositions at all, "mathematical propositions". Let us amend Stalnaker's theory in this way, thereby avoiding the first horn of the dilemma without affecting the situation with respect to the second horn.

Stalnaker's account splits into two versions depending upon how the word "statement" is taken in the above description of his view. If we take mathematical statements to be sentences of some particular language (as Stalnaker does at first), then the resulting view takes mathematical propositions to be propositions about language. That makes mathematical propositions contingent all right, as required by the standard theory, but it makes them contingent for the wrong reason. Consider any true mathematical statement, say, Euclid's Theorem on the infinity of primes (expressed in English). Is there a world at which this statement does not express the universal proposition? Of course there is; 'prime' might have meant 'crime' or 'grime' or 'slime'. So mathematics might have been other than it is? Nonsense! All that follows is that language might have been other than it is, not mathematics. Since Stalnaker's mathematical propositions could be false without mathematics being different, these propositions do not correctly capture the content of mathematics. Therefore, the fact that they are contingent does not provide a reply to the mathematical objection.

Presumably in order to avoid this and other difficulties that arise if mathematical propositions are taken to be about language, Stalnaker proposes a second version of his theory; it results from taking the mathematical statements referred to in the above account to be nonlinguistic "structures" intermediate between sentences and the propositions that they express. On this account, the mathematical propositions assert that these structures express the universal proposition. Stalnaker does not say whether these structures are semantic, like the semantically structured propositions mentioned in the introduction, or mathematical, perhaps set-theoretic constructions similar to "models" (in the logician's sense) or sets of such models. It does not matter. In either case, the structures are purely abstract entities, and so presumably do not vary from world to world. Moreover, the universal proposition certainly does not vary from world to world. So, any relation that a given structure bears to the universal proposition in one world, it bears to the universal proposition in all worlds. That is, any mathematical proposition true at the actual world is true at all possible worlds. No contingency here.

The only way out for the standard theorist is to allow the abstract structures to vary from world to world. That at least preserves the form of the standard theory, but at the cost of greatly weakening its metaphysical content. It makes mathematics "contingent" in an extended sense, with respect to an extended notion of world. I, for one, cannot locate these "worlds" at which mathematics is different among the entities I recognize as possible worlds; but to say that mathematics is different at various impossible worlds merely sidesteps, rather than confronts, the dilemma raised by the mathematical objection. For if limitations are not placed in advance upon what is to count as a world, then Thesis 5 is vacuous and the mathematical objection can be ignored.

Impossible worlds are too easy to come by if they do not have to be the same sort of entity as possible worlds. For example, one can think of the impossible worlds here in question as ordered pairs consisting of an ordinary possible world and a mathematical world, where a mathematical world can be taken to be the set (or proper class?) of all those structures that express the universal proposition at the impossible world in question.[15]

I do not know whether Stalnaker would accept that (the second version of) his view requires the introduction of impossible worlds, or whether instead he would somehow argue that these "worlds" are possible after all. Possible worlds differing only with respect to the abstract entities that exist at them do not seem to be consistent with Stalnaker's gloss of "possible world" as "alternative possible state of affairs or course of events".[16] One difficulty with understanding Stalnaker's view is that he wavers between two incompatible approaches to the standard theory. On the one hand, he claims that the standard theory is "neutral with respect to the form of individual possible

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[15] This works as long as the algebra of mathematical propositions and the algebra of contingent propositions (defined below) are independent Boolean algebras, overlapping only at the two extremes. Then, the algebra of mathematical-cum-contingent propositions is the Boolean product of these two algebras. This product algebra can be represented by a field of sets that is the field product of the fields that separately represent the two algebras, the one whose points are ordinary possible worlds, the other whose points are the mathematical worlds. It is easily shown that the points of the field product can be taken to be ordered pairs of the points of the separate fields. For the relevant definitions of product algebras and field products, see Sikorski, Boolean Algebras (New York: Academic Press, 1964).

[16] Stalnaker, op. cit., pp. 84-5.

worlds" allowing, for example, that possible worlds might be "structures of Platonic universals participating together in alternative ways".[17] But then, as already noted, the mathematical objection would not provide a serious challenge to the standard theory; the worlds constructed above could simply be included among the possible worlds. On the other hand, Stalnaker does take the mathematical objection seriously, which only makes sense if some specific notion of possible world has been fixed upon in advance. But then the standard theory is certainly not "neutral with respect to the form of individual possible worlds". Stalnaker wants the standard theory to have a substantial metaphysical content, but then his "solution" to the mathematical objection robs the theory of the desired content. He can't have it both ways.

I have argued that the standard theory can be taken to apply to all propositions only if impossible worlds are introduced, and the project of analyzing propositions in terms of possible worlds is abandoned. But since in this work I am not chiefly concerned with distinctions among necessary propositions, a more modest interpretation of the standard theory will suffice. Rather than extend the notion of world in Theses 3 through 5 so as to encompass impossible worlds, let us restrict the notions of proposition and implication in Theses 1 through 5 so as to apply only to what I will call the algebra of contingent propositions (although it contains two noncontingent propositions to round out the structure).

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[17] Ibid., p. 85.

The algebra of contingent propositions is a (Boolean) subalgebra of the full algebra of propositions. But it won't do to simply discard from the full algebra of propositions all necessary propositions except the universal proposition and all impossible propositions except the null proposition. That still leaves various mixed propositions such as "Grass is green and 2+2=4", which, though contingent, are not what I will call thoroughly contingent: they also serve in part to characterize matters of noncontingent fact. Worse, the resulting set of propositions is not closed under conjunction or disjunction, and so does not even form a subalgebra of the full algebra of propositions. We need a way to factor all noncontingent content out of the full algebra of propositions; we need, that is, to form an appropriate quotient algebra.

The way to ignore content related to matters of noncontingent fact is to identify propositions that agree in content with respect to matters of contingent fact. Propositions agree in content with respect to matters of contingent fact just in case they are necessarily equivalent, that is, just in case their material biconditional is a necessary proposition. This suggests that propositions in the full algebra that are necessarily equivalent be identified. More formally, the elements of the quotient algebra are the equivalence classes of propositions with respect to the equivalence relation necessary equivalence. One equivalence class implies another in the quotient algebra just in case some member of the one implies some member of the other in the full algebra. It is easily shown that the equivalence classes together with the defined notion of implication satisfy the ten postulates of section 1, that is, they form a Boolean algebra. Indeed, this follows from the fact that the set of necessary propositions is a filter over the full algebra: (1) The conjunction of two necessary propositions is again necessary; and (2) Any proposition implied by a necessary proposition is again necessary.[18] The resulting quotient algebra is, up to isomorphism, the algebra of contingent propositions.

To transform the quotient algebra into the algebra of contingent propositions it suffices to choose an appropriate representative from each equivalence class: the universal proposition or the null proposition for the equivalence classes at the two extremes; otherwise the one and only proposition that is thoroughly contingent, and thus says nothing about matters of noncontingent fact. By what criterion are the thoroughly contingent propositions to be recognized? Let me answer this with respect to the impossible worlds introduced above, the ordered pairs of possible worlds and mathematical worlds.[19] When a proposition is true at such an impossible world, let us say that it is _true at_ the possible world _relative to_ the mathematical world; a proposition is simply _true at_ a possible world, of course, just in case it is true at that possible world relative to the actual mathematical world, that is, relative to the way mathematics actually is. Now, a proposition is _thoroughly contingent_ just in case whenever it is true at a possible world relative to _one_ mathematical world, it is true at that possible

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[18] See Sikorski, op. cit., on the relation between filters and quotient algebras.

[19] More generally, further "worlds" can be introduced to account for matters of nonmathematical noncontingent fact.

world relative to <u>all</u> mathematical worlds. That captures the idea that a thoroughly contingent proposition says nothing about mathematics; its truth or falsity at a possible world is independent of the truth or falsity of any mathematical proposition. Note that this definition gives a way of characterizing the algebra of contingent propositions without making a detour through the quotient algebra: the algebra of contingent propositions is that subalgebra of the full algebra of propositions that contains the universal proposition, the null proposition, and the thoroughly contingent propositions. But the quotient algebra, it seems to me, is useful in providing an intuitive grasp of the structure in question.

Let us henceforth interpret Theses 1 through 5 so as to refer to the algebra of contingent propositions, rather than the full algebra of propositions. So interpreted, the various propositional notions defined in terms of implication are identified with familiar modal notions: a proposition is <u>possible</u> if and only if consistent, <u>impossible</u> if and only if the null proposition, <u>necessary</u> if and only if the universal proposition, <u>contingent</u> if and only if neither the null proposition nor the universal proposition, and so on. As a result of these identifications, the standard theory entails the standard analyses of the modal notions in terms of possible worlds, for example, that a proposition is possible if and only if there is a possible world at which it is true. But the standard theory no longer entails that, say, mathematical notions can be analyzed in terms of possible worlds. That is certainly a retreat for the gung-ho possible-worlds theorist, but by no means a total defeat. The standard theory still makes the nontrivial claim that a logical space composed entirely of possible worlds is adequate to represent the algebra of contingent propositions. That this claim is nontrivial will be shown, I believe, by the challenge to be posed in the next subsection.

The Modal Objection.

Thesis 5 has been made immune to counterexamples involving impossible propositions that are not realizable. It has not been made immune, however, to counterexamples involving possible propositions that are not realizable. According to the modal objection now to be considered, such counterexamples can be found. It will not be my purpose in these pages to attempt to refute the modal objection; I doubt that talk of refutation is even appropriate here, since we are dealing with competing metaphysical pictures that will ultimately have to be adjudged on largely pragmatic grounds. Instead, I will focus upon how the modal objection makes life difficult for those who accept the standard theory, or even just Thesis 5. These are not straw men. The great majority of contemporary philosophers who write on the topic of modality, no matter what their metaphysical persuasion, take as one of their starting points the (supposedly) Leibnizian idea embodied in Thesis 5 that whatever is possible is true at some possible world. The nature and ontological status of possible worlds has been a constant source of debate, but not this fundamental connection between possible worlds and possibility.[20]

[20] William Lycan writes: "[The idea that our modal idioms can be analyzed as quantifiers over possible worlds] is now so commonplace that we easily forget how stunning the idea is." "The Trouble with

But if I am correct, the Leibnizian idea is not nearly as innocent as its tautological ring might suggest. I will claim that standard theorists who are nonrealists have no satisfactory reply to the modal objection. Only realists, I believe, can satisfactorily reply to the modal objection without abandoning the standard theory. But their adherence to realism is severely tested in the process, for the reply requires measures that, I suspect, will make even some of the most confirmed realists blanch.

In order to get the modal objection rolling, I want to convince the reader that a certain proposition is possible. As is common in philosophical writing, I will use the English "might have" construction to express the broad sense of metaphysical possiblility _de dicto_ here under discussion (sometimes misleadingly called logical possibility). Rather than state the proposition outright, I will work my way to it down a slippery slope.

Certainly, everyone agrees that some of the entities that actually exist might have been different than they are. But it seems no less certain that some of the entities that actually exist might not have existed at all, and that nonactual entities might have taken their place.[21] This is true not only of gross material objects such as tables

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Possible Worlds", _The Possible and the Actual_ (Ithaca: Cornell University Press, 1979), Michael Loux, ed., p. 274. I would add: we seem also to forget how _controversial_ the idea is. No doubt the successes of possible-world semantics for modal logic had much to do with the idea's current widespread acceptance.

[21] Any history of the pernicious influence of semantics upon metaphysics would have to include a section on the way in which the early constant-domain semantics for quantified modal logic warped some

and rocks, but also of the more fundamental particles of which tables and rocks are composed such as neutrons and neutrinos. It would risk inconsistency to put no restrictions on the number and distribution of, say, tables and rocks, while at the same time requiring that they be entirely reconstituted out of the elementary matter that actually exists: for one thing, there might not be enough elementary matter to go around. How many of the entities that actually exist might not have existed? Seventeen? Seventeen billion? I see no way to justify drawing a line here. It might have been the case that all of the (concrete) entities that actually exist failed to exist, and that other entities have taken their place.

Not only might an individual table or an individual neutrino not have existed, but there might have been no tables or neutrinos at all. Some of the kinds of entity that actually exist, be they natural or artificial, might not have existed.[22] Of course, if a kind of entity is sufficiently fundamental, then a world at which it failed to exist would not be a physically possible world, but it would nonetheless be possible in the appropriate metaphysical sense. How many of the kinds of entity that actually exist might not have existed? Again, I see no way to justify drawing a line. It might have been the case that all of the (concrete) kinds of entity that actually exist failed to exist, and that

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philosophers' intuitions on the Barcan formula and its converse.

[22] I say that a kind exists at a world if and only if some member of the kind exists at the world; and that a kind exists, simpliciter, if and only if it exists at the actual world. There may be other, less restricted, senses according to which a kind can be said to exist even if it does not exist at the actual world, or at any world.

other kinds of entity have taken their place.[23]

The new kinds of entity might be, to varying degrees, within our powers of conception: unicorns, perpetual motion machines, phlogiston. Or they might be utterly inconceivable to us except in the broadest general terms such as 'entity' and 'thing'. Whoever denies this on the grounds that whatever is possible is conceivable risks coming into conflict with one of the most basic modal principles, that whatever is actual is possible. For who can say whether or not we are capable of conceiving the inner workings and fundamental entities of the actual world; I doubt that many who are familiar with modern physics would hold out much hope for that.

Putting this all together, we have that the following proposition is possible:

(M) All of the (concrete) kinds of entity that actually exist failed to exist, and other kinds of entity beyond our powers of conception have taken their place.

That is the proposition I want to focus upon in discussing the modal objection. I find it difficult to believe that anyone who accepts the notion of metaphysical possibility here in question could deny that this

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[23] What about spacetime points for the absolutist about spacetime who takes spacetime points to be concrete? Might it have been the case that this kind of entity failed to exist, that there were no spacetime points at all? I believe so; but the modal conservative can take spacetime points to be a kind of entity that necessarily exists, and only allow that the kinds of entity distributed over spacetime might not have existed, without affecting the discussion to follow.

proposition is possible; it leads to no contradiction, posits no round squares or other impossible objects. Of course, those who simply reject all talk of possibility and necessity that is not syntactically based will claim to have trouble understanding the last few paragraphs; but they presumably have no need for the modal objection, since they also claim not to understand the theory that is here being objected to.

Once the standard theorist accepts that the proposition (M) is possible, the problems come thick and fast. By Thesis 5, there must be a possible world at which (M) is true. Thus far this world has been characterized primarily in negative terms, by saying that the world's kinds of entity are <u>not</u> the kinds of entity that actually exist, and <u>not</u> the kinds of entity of which we can conceive. But if this is to be a full-fledged possible world, there will also have to be propositions that characterize the world in a positive way, indeed, <u>singular</u> propositions asserting that particular inconceivable entities have particular inconceivable properties.[24] But that, it may seem, goes too far. Whereas the general proposition (M) was perfectly comprehensible and even expressible within ordinary language, these singular propositions are utterly mysterious and forever beyond our comprehension. It is one thing to accept that (M) is possible, quite another to admit a full-fledged possible world at which it is true, and all the singular propositions that go with it. But according to the

[24] The distinction between general and singular propositions, when divorced from syntax and semantics, is admittedly problematic, but I think clear enough for present purposes. It is not definable in terms of the Boolean notions that we officially have available. See the discussion of the last alternative conception of logical space in section 2.

standard theory, admitting the one possibility requires admitting a host of others, even though these latter possibilities seem much more dubious than the first. The propounder of the modal objection seeks a middle ground according to which the general proposition (M) can be admitted as possible without thereby having to admit the host of singular propositions that instantiate it.

The modal objection should not be confused with a more familiar complaint about the existence of nonactual objects. On the one hand, a realist about possible worlds might take the modal objection seriously even though he had no qualms in general about positing nonactual objects. Indeed, all possible worlds but one are nonactual for any realist worth the name,[25] as are some or all of the (concrete) entities and kinds of entity that exist at the nonactual worlds. But even realists can admit that there is a point beyond which it is unnecessary, and perhaps unreasonable, to posit further nonactual entities. According to the modal objection, that point has been reached by the time one contemplates whether or not there is a possible world at which

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[25] What about realists, like Stalnaker and perhaps Plantinga, who take possible worlds to be actually existing irreducible abstract entities? These philosophers seem to hold that realism about possible worlds is made compatible with actualism -- the view that whatever exists is actual -- merely by taking possible worlds to be abstract entities. I have no idea how the view that possible worlds are actual is supposed to follow from the view that possible worlds are abstract entities, unless possible worlds are constructed out of abstract entities already believed to be actual. Why not say instead that since possible worlds are abstract, some abstract entities have turned out to be nonactual? For the views in question, see Stalnaker, "Possible Worlds", Nous, 10 (1976), pp. 65-75; and Plantinga, "Actualism and Possible Worlds", Theoria, 42 (1976), pp. 139-160. Both articles are reprinted in Michael Loux, ed., op. cit.

the proposition (M) is true. On the other hand, the modal objection does not disappear for the realist about propositions, even if he posits no nonactual objects because he takes worlds to be constructed out of propositions, and propositions to be actual entities.[26] The singular propositions required by the standard theory seem just as dubious as the nonactual entities they are purportedly about. So, the modal objection cuts across the objection to nonactual objects. It tries to convince the standard theorist, be he a realist about worlds or not, that his theory is weighed down with excess baggage; his theory requires entities that are not needed in providing a metaphysical framework for our conceptual scheme.

The options available to the standard theorist in replying to the modal objection are formally identical with the options he had available in replying to the mathematical objection. In this case, he faces the following dilemma: either he denies that (M) is possible, or he admits that there is a possible world at which (M) is true. Moreover, there are again two ways to sidestep the dilemma, one by expanding the space of worlds, the other by shrinking the algebra of propositions. But in this case, it seems to me, neither of these methods for sidestepping the dilemma are satisfactory.

Let me take the second first. The standard theory can be secured from objection forevermore by identifying propositions whenever they are true at the same set of worlds. Theses 1 through 5 will then refer, by

[26] This last choice is not inevitable; possible but false propositions could with equal justice be taken to be possible but nonactual entities.

definition, to the minimal algebra of propositions capable of fully characterizing the worlds. But such a stipulation would undermine the entire project.[27] Granted, it would no longer be a problem for the standard theorist if (M) were false at every world, since (M) would be identified with the null proposition. But then both possible and impossible propositions are identified with the null proposition, and logical space no longer adequately captures modal distinctions. Wherefore all this talk of possible worlds if not to provide a foundation for modality? That would seem to be a minimal condition for justifying the project that the standard theorist is engaged in. Thus, to retreat here as was done in the face of the mathematical objection would spell defeat for the possible-world theorist.

The second method for sidestepping the dilemma fares no better. This method attempts to circumvent the second horn of the dilemma by introducing a world at which the general proposition (M) is true without thereby also introducing the host of mysterious singular propositions. But it can only do this by introducing a host of mysterious worlds, worlds at which only general propositions are true, not any singular propositions. Let us call these general worlds. Note that these worlds need not be indeterminate in the sense discussed in section 3. There need not be any proposition such that neither it nor its negation is true at the world because, according to the view being considered, the singular propositions do not exist. General worlds are indeterminate in a way that cannot be captured in purely truth-functional terms. Call a

[27] Unless it is bolstered by an appropriate metatheory. See the discussion of the trivial world-based theory in section 7.

set of propositions witnessed if for every existential proposition in the set, there is at least one singular proposition in the set that instantiates it. Surely, the set of propositions true at a possible world must be a witnessed set of propositions; this merely extends the maximality requirement of Thesis 3 outside of the truth-functional domain. But then a general world at which (M) is true is not a possible world; for it is true at the world that some (concrete) entity exists, but there is no singular proposition true at the world to instantiate this existential proposition. Of course, the standard theorist is free to introduce general "worlds" into logical space, but then he has given up the project of analyzing modal notions such as possibility in terms of possible worlds. There is no way to sidestep the dilemma posed by the modal objection without undermining the project.

The standard theorist must meet the modal objection head-on, and insist that there is a possible world at which the general proposition, and the singular propositions needed to instantiate it, are true. But not all standard theorists are in a position to make this reply. Some standard theorists are conceptualists about possible worlds and propositions: possible worlds and propositions must be constructed out of what we do or can conceive.[28] Can such a conceptualist consistently posit the requisite possible world and singular propositions? We cannot of course directly conceive of the possible world or singular

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[28] Rescher defends such a conceptualism in "The Ontology of the Possible", Michael Loux, ed., op. cit., pp. 166-182. The 'we' refers, of course, to us humans. The view that possible worlds and propositions are ideas in the mind of God is, for present purposes, not conceptualism but realism with a crutch.

propositions in question (although perhaps we can conceive that there be such a world, and that there be such propositions); but that is not quite the end of the matter, since these entities only need to be constructed out of what is conceivable, and need not themselves be conceivable. However, I see no way that constructions out of what is conceivable can be made to represent an appropriate possible world, or the singular propositions true at it. On the one hand, whatever is used to represent one of the singular propositions true at the world will equally have to be used to represent any other, since the singular propositions are conceptually indiscernible to us. On the other hand, since there are numerous worlds at which the general proposition is true, all of which are conceptually indiscernible to us, no construction out of what is conceivable can be said to represent one such world without representing them all. But if the conceptualist cannot supply a unique surrogate for each individual possible world and proposition required by the standard theory, then the standard theory is incompatible with conceptualism about worlds and propositions.

Some standard theorists are nominalists about possible worlds and propositions: possible worlds and propositions must be constructed out of actual concrete entities.[29] One form of nominalism, known as combinatorialism, holds that possible worlds can be represented as alternative distributions of actual entities over space and time. But

[29] For a fervent rendition of the nominalist credo, see Nelson Goodman's "The Passing of the Possible" in Fact, Fiction, and Forecast, 3rd ed. (Indianapolis: Bobbs-Merrill, 1973), pp. 31-59. But I also include less stringent forms of nominalism than Goodman's that allow set-theoretic constructions out of actual concrete entities. See section 8.

this view cannot even account for the fact that there might have been some nonactual entity, let alone the more outré possibility here being contemplated. The most successful form of nominalism, surprisingly, is one that takes possible worlds and propositions to be constructed out of linguistic entities. This view will be extensively discussed in section 8, but the reason for its failure can be given in advance. This form of nominalism has available to it at best the resources of conceptualism and combinatorialism combined, and these resources will not in general be sufficient to represent worlds whose entities are both inconceivable and nonactual. Nominalism too is incompatible with the standard theory, and, in particular, with the Leibnizian Thesis 5.

Since the various forms of conceptualism and nominalism exhaust the nonrealist positions with which I am familiar, it appears that only the realist about possible worlds or propositions[30] can consistently maintain the standard theory. Because he takes possible worlds or propositions as primitive,[31] the realist is free to posit whatever possible worlds or propositions are dictated by the existential commitments of the standard theory. For a realist, to accept the standard theory is to posit the entities dictated by the theory. There are no reductionist constraints imposed from outside the theory that the entities have to meet in order to be accepted.

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[30] or similar entities -- like properties or states of affairs -- out of which possible worlds and propositions can be constructed.

[31] He need not take both as primitive if he is willing to reduce one to the other. See sections 6 and 7 below.

What we have here is a qualified argument for realism. The qualifications are two in number. First, the proposition (M) (or something similar) must be agreed to be possible; but not to do so, it seems to me, would be to show little respect for firmly entrenched modal intuitions. Second, the argument for realism applies only to those who accept the standard theory. But if the standard theory (or even just its Leibnizian part) is accepted by anywhere near as many philosophers as it appears to be, there must be a lot of realists out there waiting to come out of the closet.

The modal objection might cause some standard theorists to embrace realism; but it also might cause some realists to abandon the standard theory on account of its extravagant ontological demands. Metaphysical theories must be tested against their consequences; in this case, the virtues of the standard theory must be balanced against its ontological vices. There is no fixed scale for performing these measures. Different philosophers will undoubtedly attach different weights to the various pragmatic factors involved. In the next section, we will be in a better position to consider arguments as to whether that part of the standard theory that is open to the modal objection should be rejected. But the arguments, not surprisingly, will be inconclusive.

SECTION 6

PROPOSITION-BASED THEORIES

Tripartite Correspondence.

The standard theory makes use of four primitive notions: propositions, implication, worlds, and truth-at-a-world. In the next two sections, we will consider definitional extensions of the standard theory that reduce the number of primitives to one or two. Let us first consider theories that take propositions and implication as primitive, and that define worlds and truth-at-a-world in terms of them (plus set theory). These will be called *proposition-based theories*. Note that it is the binary relation truth-at-a-world that becomes defined; the absolute notion of truth, as well as the related notion of actuality, play no role in the theories of this work, either as primitive or defined.

One might wonder whether the theorist who wishes to take the algebra of propositions as basic need endorse all of the theses of the standard theory. It follows from Theses 1 and 3 alone that every possible world is associated with a maximal consistent set of propositions, namely, the set of propositions true at the world. It might seem, therefore, that only Theses 1 and 3 are needed in order to *reduce* possible worlds to propositions and implication; one can simply identify each possible world with its associated maximal consistent set of propositions. That would be a mistake, as will be explained more fully later in this section.[1] A reduction of possible worlds to maximal consistent sets of propositions requires, at the very least, a one-to-one correspondence between the two, and Theses 4 and 5 are needed in order to establish such a correspondence. Indeed, Theses 4 and 5 were framed with this correspondence in mind. Interestingly, Thesis 2 is not needed by the proposition-based theorist; but, owing to its uncontroversial nature, I think we can safely assume that all proposition-based theorists will accept it.

In order to establish the one-to-one correspondence between possible worlds and maximal consistent sets of propositions, we need only extract from Thesis 5 the following simple lemma:

LEMMA (6.1). For any maximal consistent set of propositions M, there exists a world w such that M=Pw.

Proof. Let M be any maximal consistent set of propositions. Since M is consistent, $\bigwedge M \neq \emptyset$. By Thesis 5, there is a world w at which $\bigwedge M$ is true; that is, $\bigwedge M \varepsilon Pw$. So $\bigwedge Pw \rightarrow \bigwedge M$. But $\bigwedge M$ is an atom by Lemma (3.10). So $\bigwedge Pw = \bigwedge M$, since $\bigwedge Pw \neq \emptyset$ by Thesis 3. Therefore, M=Pw by Lemma (3.11).

Let the set of possible worlds be symbolized by WOR, and the set of maximal consistent sets of propositions by MAX. Let Φ be the mapping that assigns to each possible world the set of propositions true at the world; that is, for all $w \varepsilon WOR$, $\Phi(w)=Pw$. We can now prove:

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[1] Stalnaker apparently makes this, or a related, mistake. See the discussion of his theory in section 7.

THEOREM (6.2). Φ maps the set of worlds one-to-one, onto the set of maximal consistent sets of propositions: Φ: WOR <--> MAX.

Proof. By Thesis 3, Φ assigns to each world a unique maximal consistent set of propositions. By Thesis 4, the mapping is one-to-one. By Lemma (6.1), the mapping is onto.

For those who want to do away with worlds, Theorem (6.2) suggests that maximal consistent sets of propositions might be able to take their place. But there is another possibility. Recall that, according to Theorem (3.13) in section 3, the maximal consistent sets of propositions are in one-to-one correspondence with the atomic propositions. It follows from the transitivity of one-to-one correspondences that worlds are in one-to-one correspondence with atomic propositions as well, each atomic proposition being, so to speak, a complete description of a world. We thus have a tripartite correspondence between worlds, maximal consistent sets of propositions, and atomic propositions.

One-to-one correspondences can be tickets to ontological reduction; in this case, possible worlds might be reduced either to maximal consistent sets of propositions or to atomic propositions. Of course, a one-to-one correspondence is not by itself a sufficient condition for reduction; there is no danger that some enterprising philosopher will succeed in ontologically reducing our left feet to our right. Some of the other conditions necessary for ontological reduction will be presented and discussed in sections 8 and 10.

For now it suffices to note that, if the further conditions are met, a reduction of possible worlds to propositions might recommend itself for either of two reasons.

(1) Economy. Ontological reduction saves on the kinds of entity that a theory needs to posit, and, other things being equal, a theory that posits fewer kinds of entity is to be preferred.

(2) Clarity. When some kind of entity that one finds obscure is eliminated in favor of another kind of entity that one finds less obscure, the entire theory gains in clarity as a result. Thus a theorist who finds possible worlds to be more obscure than propositions will, other things being equal, perform the reduction.

The only catch is that the notion of possible world must be obscure enough: nothing already known about possible worlds can be incompatible with possible worlds turning out to be propositions, or sets of propositions. That would be the case, for example, if everything that was known about possible worlds was given by the standard theory.[2] Unfortunately, evaluations of clarity tend to be highly subjective; often what one philosopher finds tolerably clear, another finds hopelessly obscure. This threatens to make the choice between philosophical theories subjective as well. I suspect that this must be accepted as a philosophical fact of life. The only consolation is that

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[2] But if one assumes with David Lewis that all possible worlds are of the same kind as the actual world, and that the actual world is not a proposition or set of propositions, then the reduction can be rejected out of hand. See Counterfactuals (Oxford: Basil Blackwell, 1973), sec. 4.1.

such subjectivity also seems to affect the choice of scientific theories, though perhaps to a lesser degree.

Let us turn now to a proposition-based theory that identifies possible worlds with maximal consistent sets of propositions. Since two notions are eliminated by the theory, it makes use of two definitions.[3]

DEFINITION (6.3). Something is a possible world if and only if it is a maximal consistent set of propositions.

Intuitively, a possible world is identified with the set of propositions true at it. Since, trivially, a proposition is true at a world if and only if it is a member of the set of propositions true at the world, we have:

DEFINITION (6.4). One thing is true at another if and only if the first is a member of the second, and the second is a maximal consistent set of propositions.

Note that these are so-called explicit definitions: they allow all predicates, names, and variables having to do with possible worlds and truth-at-a-world to be individually replaced by predicates, names, and variables having to do with maximal consistent sets of propositions and set-membership.

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[3] These are definitions in a weak sense because they require only extensional equivalence, not sameness of meaning. That is all right, for otherwise it is hard to see how these definitions could be informative, which they are. Moreover, if these were stipulative definitions, it would be pointless to compare the standard theory with the proposition-based theory: tokens of the word 'world' in the two theories would be mere homographs.

Given these definitions, it is not necessary to incorporate all five theses of the standard theory into the proposition-based theory. We need Thesis 1 of course,[4] and we have assumed that we want Thesis 2. But Theses 3 and 4 are not needed because they turn into the following tautologies when 'world' is replaced by 'maximal consistent set of propositions', and 'true at' is replaced by 'is a member of':

THESIS 3'. Any maximal consistent set of propositions is a maximal consistent set of propositions.

THESIS 4'. Distinct maximal consistent sets of propositions are distinct.

The translation of Thesis 5, on the other hand, places nontrivial constraints upon the structure of the algebra of propositions, and will be incorporated into the theory by a standard theorist:

THESIS 5'. Every nonnull proposition is a member of a maximal consistent set of propositions.

The first proposition-based theory, then, consists of Definitions (6.3) and (6.4) and Theses 1, 2, and 5'.

The proposition-based theory that identifies possible worlds with atomic propositions makes use of the following two definitions:

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[4] But see the discussion of Adams's theory below.

DEFINITION (6.5). Something is a _possible world_ if and only if it is an atomic proposition.

DEFINITION (6.6). One thing is _true at_ another if and only if the first is implied by the second, and the second is an atomic proposition.

Again, the translations of Theses 3 and 4 are not needed. Thesis 5 becomes:

THESIS 5''. Every nonnull proposition is implied by an atomic proposition; that is, the Boolean algebra of propositions is atomic.

Theses 5' and 5'', of course, are equivalent in light of Theorem (3.13). The second proposition-based theory, then, consists of Definitions (6.5) and (6.6) and Theses 1, 2, and 5''. In what follows, I will focus upon the first, more commonly proposed, proposition-based theory; what I have to say will apply with obvious modifications to the second theory as well.

Weak and Feeble Proposition-Based Theories.

The above proposition-based theories entail the five theses of the standard theory, and thus the two objections to Thesis 5 considered in the previous section apply to these proposition-based theories with equal force. Let us consider the options available to a proposition-based theorist who takes these objections seriously, and is willing to reject Thesis 5 because of them. Can he simply weaken the first proposition-based theory presented above by eliminating Thesis 5', the counterpart of Thesis 5? Let us call the resulting theory the _weak_ proposition-based theory, and the theory from which it came the strong proposition-based theory. It might seem that the weak theory has the following two advantages over the strong theory: it can apply to all the propositions, necessary and contingent; and it can avoid the strong realist commitments required by Thesis 5. But alas, it isn't so.

It isn't so because eliminating Thesis 5' amounts to eliminating only a part of Thesis 5, and, as it turns out, eliminating that part is not sufficient to avoid either the mathematical or the modal objection. To see this, one has to solve the following logical conundrum: Thesis 5 was needed to establish the one-to-one correspondence between possible worlds and maximal consistent sets of propositions; this correspondence was then incorporated in Definitions (6.3) and (6.4); these definitions were then used to transform Thesis 5 into Thesis 5'; and, finally, Thesis 5' was eliminated. What happened to Thesis 5?

The situation becomes clearer if one works backwards from the proposition-based theory to the standard theory. Since Thesis 5 can only be derived within the strong theory by using both Thesis 5' and the definitions, it follows that the content of Thesis 5 has been divided into two parts: one part, the part concerned with the relation between possible worlds and maximal consistent sets of propositions, is embodied in the definitions; the other part, the part concerned with the structure of the algebra of propositions, and, in particular, the existence of maximal consistent sets, is embodied in Thesis 5'. Thus, by eliminating only Thesis 5', the weak theory has not entirely eliminated Thesis 5. It still maintains:

THESIS 5^+. Every maximal consistent set of propositions is realizable.

Thesis 5 is the conjunction of Theses 5' and 5^+.[5]

Now, if the goal of the weak proposition-based theory is to avoid the mathematical and modal objections, then it has eliminated the wrong part; it has eliminated the part of Thesis 5 that is not responsible for the trouble pointed to by the objections. The mathematical objection did not claim that false mathematical propositions do not belong to maximal consistent sets of propositions; it is rather that the maximal consistent sets of propositions that they belong to are not associated with possible worlds. Similarly, the modal objection did not claim that the set of propositions "true at" a general "world" cannot be taken to be a maximal consistent set of propositions; it is rather that such a maximal consistent set, because unwitnessed, is not properly associated with a possible world. The weak proposition-based theory is an uninteresting hybrid: too strong to avoid the mathematical and modal objections; but not strong enough to do the work of the standard theory.

The weak proposition-based theory can be further weakened, however, so as to expunge all trace of Thesis 5. Definition (6.3) can be replaced by the weaker:

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[5] Moreover, Theses 5' and 5^+ are independent, and thus might with some advantage have been split up from the start. See the second appendix.

(F) Something is a possible world only if it is a maximal consistent set of propositions.

The resulting theory no longer entails that every maximal consistent set of propositions is, or even corresponds with, a possible world. It is thus free to take the full algebra of propositions as its subject matter without being open to the mathematical or modal objection. Let us call it the feeble proposition-based theory for reasons to be given presently.[6]

It might seem that the feeble proposition-based theory, like its two predecessors, provides a reduction of possible worlds to sets of propositions. For although (F) is weaker than Definition (6.3), it is still sufficient to ensure that there are no entities called worlds existing over and above the maximal consistent sets of propositions. If worlds are maximal consistent sets of propositions as asserted by (F), then worlds as ontologically independent entities have disappeared. Only propositions and sets of propositions need to be included in the domain of quantification of the feeble theory.

But if worlds have thus disappeared from the ontology of the theory, they leave their imprint as an unreduced part of its ideology[7]: the feeble theory provides no means for eliminating the predicate 'world' in favor of predicates definable in terms of propositions and implication;

[6] It does not much matter whether Thesis 5' is reintroduced into the theory, since that will not make it any less feeble.

[7] This use of the term 'ideology' comes from Quine, for example, "Ontological Reduction and the World of Numbers", The Ways of Paradox (Cambridge, Mass.: Harvard University Press, 1966), p. 215.

(F) does not provide a definition of 'world'. Indeed, the feeble theory still makes use of the notion of world to mark the distinction between two kinds of maximal consistent sets of propositions -- the worldly and the nonworldly. Since the distinction thus marked cannot be constructed out of the available Boolean notions, it must be taken as primitive by the theory. But then, even though worlds have disappeared as primitive entities, worldliness remains as an unreduced part of the theory. Since proposition-based theories were to have only the notions of proposition and implication as primitive, the feeble theory is not really a proposition-based theory after all.

Reductions would be too easy to come by if one were allowed to simply trade ontological commitments for ideological commitments.[8] For example, someone who accepts an ontology of sets can avoid accepting in addition an ontology of numbers by positing that numbers are sets. But unless he provides a method for constructing the numbers within set theory, he has no guarantee that he can do more than simply trade unreduced numbers for a new irreducible property of sets -- the property of being a number, or of being numerical. But surely, such a trade should not count as a reduction of numbers to sets. There is no obvious diminution of his total commitments, ontological and ideological combined. In a similar way, the feeble theory manages only to shift commitments, not decrease them. Without Thesis 5^{+}, the notion of world or worldliness remains an unreduced part of the theory.[9]

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[8] In section 8, such purported reductions will be characterized as violating a noncircularity condition.

[9] This is not to say, of course, that the notion of world or worldliness

The proposition-based theorist cannot eat his cake and have it. He wants to provide a reduction of talk about possible worlds to talk about sets of propositions. But then he cannot also interpret his theory so as to apply to the full algebra of propositions, and he cannot alleviate his realist commitment to the host of singular propositions needed as witnesses. At any rate, that is his predicament if what was said in the last section about the mathematical objection and the modal objection is essentially correct. A decision to identify possible worlds with some (but not necessarily all) of the maximal consistent sets of propositions does not by itself diminish the force of these objections.

The friend of propositions must choose between eating his cake and having it, between endorsing the strong theory and endorsing the feeble theory. The issues are many and complex, and I will here make only a few remarks about the question of economy.[10] Both theories are committed to the existence of propositions, but the strong theory, we have seen, is committed in particular to the existence of dubious singular propositions, a commitment that the feeble theory can disclaim. On the other hand, the feeble theory is committed to an unreduced notion of world, a notion that the strong theory takes as defined. Let us say that one theory is _qualitatively more parsimonious_ than another if it is committed to fewer basic concepts or kinds; and _quantitatively more parsimonious_ with respect to some kind of entity if it is committed to

might not be reducible within the context of some more inclusive metaphysical theory. This issue is discussed, tentatively, in section 10, subsection "Against the Proposition-Based Theory".

[10] See further section 10, especially the subsection "Against the World-Based Theory".

fewer entities of that kind.[11] The strong theory is qualitatively the more parsimonious because it posits one less kind. But the feeble theory is quantitatively the more parsimonious with respect to propositions because it need not, and presumably does not, posit the dubious singular propositions. Which sort of parsimony is to be preferred?

David Lewis writes: "I subscribe to the general view that qualitative parsimony is good in a philosophical or empirical hypothesis; but I recoginze no presumption whatever in favor of quantitative parsimony."[12] If Lewis's view is accepted, it would decide the matter in favor of the strong theory. But since it seems to me that the quantitative parsimony of the feeble theory does count for something, there must be something wrong with Lewis's view. I would agree with Lewis that once a theory has posited entities of some kind, it is of no consequence whether it posits further entities of that kind if the further entities are merely replicas of the entities already posited, or at least similar to them in all important respects; Lewis's example involving different numbers of electrons suggests that this might have been his paradigm case. But entities of the same kind might be different in ways that matter, even if those differences are not

[11] This terminology is taken, more or less, from Lewis, op. cit., p. 87. Note that each distinction within the ideology of a theory commits the theory to a distinct concept or kind, even if it is subsumed under others. Otherwise every theory could be said to be committed to only one kind of entity: the objects in its domain of quantification. But note that not all of the kinds taken as basic by a theory need count equally towards an evaluation of qualitative parsimony.

[12] Ibid.

captured by the theories in question. I see no reason why these differences might not be such as to make some entities of a kind ontologically more respectable than other entities of the same kind. In such cases, quantitative parsimony will be an important factor in the evaluation of the theories involved. The modal objection strongly suggests that we have before us just such a case.

Given that both qualitative and quantitative parsimony can play a role in the evaluation of theories, the merits of each will have to be weighed against the other in cases where the two conflict. The result will depend upon the scale used, and undoubtedly no one scale will be agreed upon by all. For example, the strong theory will most likely be favored by those with stronger realist tendencies. Disputes are likely to end in a standoff.

But there is a further consideration to be discussed in section 10 that would clearly favor the feeble theory, at least with respect to the question of economy. The strong theory might not satisfy all of the conditions necessary for reduction.[13] The strong theory would then have to give way to the standard theory, and would lose its edge in qualitative parsimony. But that, of course, would not be the end of the matter. Economy is only one of many criteria in theory choice.

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[13] I will claim that it fails to satisfy a methodological principle having to do with explanatory adequacy. See "Against the Proposition-Based Theory" in section 10.

Adams's World-Story Theory.

Robert Adams has proposed that a proposition-based theory is just what is needed to provide an actualist solution to the problem of actuality.[14] Adams does not attempt to give a precise formulation of the theory. Indeed, most of what he says is contained in the following brief passage:

> The analysis which I have in mind is a reduction of talk about possible worlds to talk about sets of propositions.
> Let us say that a world-story is a maximal consistent set of propositions. That is, it is a set which has as its members one member of every pair of mutually contradictory propositions, and which is such that it is possible that all of its members be true together. The notion of a possible world can be given a contextual analysis in terms of world-stories.[15]

Adams then gives three examples illustrating how sentences about possible worlds can be translated into sentences about world-stories. The examples clearly show that Adams accepts Definitions (6.3) and (6.4) of the strong proposition-based theory.[16]

What theses about worlds and propositions would Adams choose to include in his world-story theory? It is hard to tell. Adams says virtually nothing about the structure that his theory imposes upon the propositions. Since Theses 2 and 5' are not needed for the reduction he

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[14] In "Theories of Actuality", Nous, 8 (1974), pp. 211-231.

[15] Ibid., p. 225.

[16] I am somewhat puzzled as to why Adams speaks of a contextual analysis, since, as was noted above, Definitions (6.3) and (6.4) provide explicit, not contextual, definitions. Perhaps he has in mind that Definition (6.3) is not alone sufficient for translating a sentence about possible worlds into a sentence about world-stories, but must be jointly applied with Definition (6.4) (and perhaps other definitions in an extension of the theory).

proposes, it cannot be inferred from the above-quoted passage that he would or that he would not accept them. Earlier, however, Adams included a version of Thesis 5' as part of his actualist credo: "possibility is holistic rather than atomistic, in the sense that what is possible is possible only as part of a possible completely determinate world."[17] Does Adams mistakenly believe that this thesis follows from the reduction he proposes? In any case, it appears that Adams will want to add Thesis 5' to his world-story theory. On the other hand, nothing Adams says in his paper is relevant to Thesis 2.

The Boolean postulates of Thesis 1, it would seem, are needed by Adams to guarantee that the two notions he uses in defining a world-story -- the notion of a pair of propositions being contradictory, and the notion of a set of propositions being consistent or possible -- have their customary interpretations. Thus I assume that Adams would endorse Thesis 1, with perhaps the one exception to be noted below.

What about Theses 3, 4, and 5^+, the theses that serve to relate possible worlds and maximal consistent sets of propositions? As we saw in the last subsection, Adams must accept these three theses if, as he claims, he is providing a "reduction of talk about possible worlds to talk about propositions". But a problem arises with respect to Adams's interpretation of Thesis 5 (in particular, Thesis 5^+, but I will focus upon the more general Thesis 5). Does Adams's intend to restrict the scope of Thesis 5 to the algebra of contingent propositions? This is necessary, I have argued, if the mathematical objection is to be avoided

[17] Ibid., p. 225.

without enfeebling the theory. There is some evidence that this is Adams's intention. For example, in a quotation already given above he explicitly endorses Thesis 5 under its restricted interpretation: "what is possible is possible only as a part of a possible completely determinate world".[18] Moreover, the fact that he takes 'consistent' and 'possible' to be interchangeable provides further evidence for the same.

On the other hand, Adams includes among the propositions out of which worlds are constructed what he calls semantical propositions, under which heading he apparently includes propositions that make assertions about the truth or falsity of propositions. Furthermore, he gives an example involving giraffes that suggests that he takes a proposition p to be distinct from the proposition that asserts that p is true.[19] But that suggests that Adams does not hold the restricted interpretation of Thesis 5. For this interpretation requires that there be but one necessary proposition, whereas Adams would be committed to at least two: the universal proposition T, and the semantical proposition asserting that T is true. More generally, it seems likely that anyone who admits a multitude of semantical propositions would admit a multitude of mathematical propositions as well, although the mathematical propositions are not mentioned in Adams's article. Thus Adams may not intend to restrict the scope of his theory to the algebra of contingent propositions. In this case, he owes us an explanation as to how he would deal with the mathematical objection. For Thesis 5, or, at any

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[18] Ibid.

[19] Ibid., p. 229.

rate, Thesis 5^+, must in some way be made palatable if the reduction is to go through.

There is a crucial difference, however, between introducing semantical propositions and introducing mathematical propositions, a difference that might come into play if Adams intends only to introduce the former. It is reasonable to suppose that every semantical proposition is equivalent to a nonsemantical proposition. The problematic cases would involve self-referential propositions that are ungrounded or paradoxical.[20] But one need not admit such self-referential propositions on the metaphysical conception, since they are lacking in the appropriate sort of content. Indeed, whatever content they may seem to have, I believe, is semantical rather than metaphysical, being derived from the meanings of sentences that would be used in attempting to express them. Now, if Adams does hold that every semantical proposition is equivalent to a nonsemantical proposition, then the semantical propositions violate Thesis 5 only in an innocuous way that does not rule out the possibility of reduction; they violate Thesis 5 only because of a less significant violation of (1.3), the Boolean postulate that identifies equivalent propositions. Indeed, Adams does say with respect to the giraffe example that a proposition p implies the proposition that asserts that p is true; and I see no reason to think that he would not accept the reverse implication. So, if Adams accepts only the semantical propositions, and not other varieties of

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[20] For explications of these two notions as applied to sentences, see Kripke, "Outline of a Theory of Truth", Journal of Philosophy, 72 (1975), pp. 690-716.

noncontingent proposition, he may still be able to carry through the reduction of worlds to sets of propositions.

What would be the consequences for Adams's theory if, indeed, he does reject postulate (1.3)? Postulate (1.3) was needed to prove the uniqueness of greatest lower bounds, least upper bounds, complements, a maximal element, and a minimal element. Thus, the operations of conjunction, disjunction, and complementation cannot be defined in terms of implication without (1.3); nor can the universal and null propositions be defined. Adams's theory would thus have to have a less economical base than the proposition-based theories considered thus far -- assuming, of course, that Adams wants to include these Boolean operations and constants within his theory.

But (1.3) can be discarded by the standard theorist without serious repercussions. Its role is one of simplification: it serves only to identify elements indistinguishable with respect to the theory.[21] Even without (1.3), the notion of a maximal consistent set of propositions can be defined in terms of implication. Adams can define a consistent proposition to be any proposition that does not imply every proposition; and a consistent set of propositions to be any set that has a consistent proposition as one of its lower bounds. Then, a maximal consistent set of propositions can be defined as a consistent set of propositions that is not properly included in any other consistent set of propositions.[22]

[21] See the second appendix for more on the relation between (1.3) and the rest of the standard theory.

[22] The notion of a contradictory pair of propositions was thus not needed by Adams in his definition of the notion of a world-story. In

With these definitions, Thesis 5 holds in the now weakened sense: every consistent proposition is realizable. But this version of Thesis 5 is still strong enough to entail that every maximal consistent set of propositions corresponds with a world -- and that is just what is needed for the reduction to go through.

But does Adams view the semantical propositions in such a way that they violate only (1.3) without essentially violating Thesis 5? Some remarks towards the end of the article suggest otherwise. Adams thinks that semantical paradoxes might result if the semantical propositions are allowed to be members of world-stories. But I do not see how semantical paradoxes could result unless it is not the case that every semantical proposition is equivalent to a nonsemantical proposition. Thus, Adams's semantical propositions apparently include such things as paradoxical and ungrounded propositions, in which case they violate Thesis 5 after all, even under the weakened interpretation given in the last paragraph. Adams could only accept Thesis 5 by admitting paradoxical and ungrounded worlds! At any rate, it does not seem plausible to me to say that paradoxical or ungrounded propositions are equivalent to nonsemantical propositions. For example, if there is a proposition that asserts of itself that it is true, it does not say the same thing as any nonsemantical proposition, not even the universal proposition; for, unlike the universal proposition, it can with equal

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any case, it too can be defined in terms of implication. Call a proposition refutable if it is not the case that every proposition implies it. Two propositions are contradictory if no consistent proposition implies them both, and no refutable proposition is implied by them both. But note that, without (1.3), more than one proposition might be contradictory to a given proposition.

justice be taken to be either true or false at a world.

In conclusion, then, if Adams takes the semantical propositions to have an essentially semantical content, and if he allows them to be members of world-stories, then his reduction of worlds to sets of propositions fails. He is left with the feeble proposition-based theory. But if, for whatever reason, Adams excludes from the scope of his theory the semantical propositions, and all other sorts of noncontingent proposition, then he is free to accept the strong proposition-based theory presented at the beginning of this section. At any rate, he is free to accept this theory if he is willing also to undertake the strong realist commitment to propositions that goes with it.

SECTION 7

THE WORLD-BASED THEORY

Isomorphic Algebras.

In the previous section there was no mention of logical space. For the proposition-based theorist, logical space is a rather complicated structure with little independent interest. The points of logical space are maximal consistent sets of propositions; each proposition corresponds, via the relation truth-at-a-world, with the set of maximal consistent sets of propositions that contain it. Logical space still represents the algebra of propositions as a field of sets, but since the field is constructed out of the elements of the algebra itself, it points toward nothing new. For the proposition-based theorist, the algebra of propositions is the fundamental object of study, not logical space.

A representation of the algebra of propositions in terms of worlds has independent interest only if worlds are not in turn reduced to propositions. The world-based theory of this section goes even further. According to this theory, it is the algebra of propositions that lacks independent interest: the algebra of propositions is identified with logical space, each proposition being identified with the set of worlds at which it is true. Such identifications, we saw in the last section, do not automatically count as reductions. It must be shown that the propositions can be eliminated without merely stepping up the ideology of the theory. It turns out that Theses 1 through 5 are just what is needed to provide this guarantee. Call a field of sets the full subset algebra over a given space if every subset of the space is a member of the field. It follows from the five theses of the standard theory that logical space is the full subset algebra over the space of worlds, and thus that the algebra of propositions is isomorphic to this full subset algebra. To this result I now turn.

Recall that, for any proposition p, Wp is the set of worlds at which p is true; by definition, then, $w \in Wp$ if and only if p is true at w if and only if $p \in Pw$. Let Ψ be the function that maps a proposition into the set of worlds at which it is true: $\Psi(p)=Wp$. I need to show that Ψ is an isomorphism between the algebra of propositions and the full subset algebra of worlds. One of the two lemmas needed for this result has already been proven in section 4:

LEMMA (4.2). For any set of worlds, there is a proposition true at all and only the worlds in that set.

Lemma (4.2) does not rest upon the controversial Thesis 5. The second lemma, however, does rest upon Thesis 5, and so shares its controversial status. It asserts, in effect, that distinct propositions always play distinct world-characterizing roles:

LEMMA (7.1). For any propositions p and q, if $p \neq q$, then $Wp \neq Wq$.

Proof. Assume that $p \neq q$. We can also assume without loss of generality that p does not imply q (otherwise q does not imply p, by (1.3), and 'p' and 'q' can just be switched throughout the proof). By Boolean algebra, if p does not imply q, then $p\&-q \neq \emptyset$. By Thesis 5, there is a world w at which p&-q is true; that is, $p\&-q \varepsilon Pw$. By Thesis 3, $p \varepsilon Pw$ and not $q \varepsilon Pw$. Hence, by definition, $w \varepsilon Wp$ and not $w \varepsilon Wq$, as was to be shown.

It follows from Lemmas (4.2) and (7.1) that Ψ provides a one-to-one correspondence between propositions and sets of worlds. But the world-based theorist requires more; he needs to know that Ψ maps the Boolean relations and operations into set-theoretic relations and operations that he already has available. Since all the Boolean relations and operations are definable in terms of implication, it suffices to show that Ψ maps implication into some familiar set-theoretic relation. Indeed, let PROP be the set of propositions, let WOR be the set of worlds, let S(WOR) be the set of all subsets of worlds, and let $\subseteq$ be the set-inclusion relation. Then,

THEOREM (7.2). The algebra of propositions <PROP, →> is isomorphic to the full subset algebra of worlds <S(WOR), $\subseteq$> under the mapping: Ψ: PROP <--> S(WOR).

Proof. The mapping Ψ does take each proposition into a unique set of worlds. The mapping is one-to-one by Lemma (7.1), and onto by Lemma (4.2). It remains only to show that, for any propositions p and q, $p \rightarrow q$ if and only if $Wp \subseteq Wq$. Assume that $p \rightarrow q$, and let $w \varepsilon Wp$. Then $p \varepsilon Pw$. It follows from Thesis 3, in particular, principle (3.3), that $q \varepsilon Pw$ as well. Thus $w \in Wq$. Conversely, suppose that it is not the case that $p \rightarrow q$. Then $p\&-q \neq \emptyset$, and, by Thesis 5, there is a world w at which $p\&-q$ is true. By Thesis 3, p is true at w but q is not. Hence $w \in Wp$ and not $w \in Wq$, and it is not the case that $Wp \subseteq Wq$.

Implication between propositions is thus mapped into inclusion between sets of worlds. It is easy to check that all the other propositional notions are mapped into their familiar set-theoretic analogues: conjunction into intersection, disjunction into union, the null proposition into the null set, and so on.

Let me pause briefly to note a simple corollary to Theorem (7.2):

COROLLARY (7.3). Let α be the cardinality of the set of worlds and β the cardinality of the set of propositions. Then $\beta = 2^{\alpha}$.

Proof. By Theorem (7.2), the cardinality of PROP is equal to the cardinality of S(WOR). The rest is set theory.

At this point, the content of the standard theory has been pretty much exhausted. Indeed, not only does Theorem (7.2) follow from the five theses, but all five theses follow from Theorem (7.2). Theses 1 and 2 follow because whatever is isomorphic to a full subset algebra is a complete Boolean algebra. Thesis 3 follows because, for any world, the set of all sets of worlds containing that world is a principal ultrafilter over the subset algebra, and only maximal consistent sets of propositions are mapped by the isomorphism into principal ultrafilters over the subset algebra. Thesis 4 follows because, for distinct worlds, there is a set of worlds containing one and not the other, and thus, by

Theorem (7.2), a proposition true at one but not the other. Finally, Thesis 5 follows because every nonnull proposition corresponds to a nonempty set of worlds, and is thus realizable.

Since Theorem (7.2) is by itself equivalent to the five theses taken together, it provides a more succinct formulation of the standard theory, a formulation that would likely be preferred by the world-based theorist. But for those who have qualms about accepting Theorem (7.2), it is more advantageous to divide the standard theory into a number of independent components. It is then easier to see what the various alternative theories are, and at what point or points in the standard theory criticism might best be directed. As we have seen, Thesis 5 -- and, in particular, Thesis 5^+ -- provides the most likely target.

Once Theorem (7.2) is accepted, however, opportunities abound for the reductionist. The notions of proposition, implication, and truth-at-a-world can all be defined in terms of the notion of world (together with a little set theory). Theorem (7.2) suggests the following three definitions:[1]

DEFINITION (7.4). Something is a proposition if and only if it is a set of worlds.

DEFINITION (7.5). One thing implies another if and only if the first is included in the second, and both are sets of worlds.

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[1] By "set of worlds" I mean, of course, any subset of the set of all worlds, including the empty set.

DEFINITION (7.6). One thing is true at another if and only if the second is a member of the first, and the first is a set of worlds.

The world-based theory consists of Definitions (7.4), (7.5), and (7.6). No further assertions are needed, since all the theses of the standard theory follow from these definitions alone. Thus, neither the standard theory nor its world-based extension place any formal constraints upon the set of possible worlds. But obvious extensions of these theories might include, for example, an assertion that worlds exist, or that one and only one world is actual, and so on.

The world-based theory and the strong proposition-based theory are alike in accepting the standard theory as common neutral ground; each then strikes out on its own in search of reduction. The world-based theory is unlike the strong proposition-based theory, however, in that it cannot in any way be weakened if the reduction is to remain intact. In order to see this, it is crucial to keep in mind the distinction between an identification of the entities of one kind with the entities of another kind, and a reduction of one kind to another. As we saw in the last section, it is possible to have the former without the latter.

Not all five theses of the standard theory are required if the goal is simply to identify the algebra of propositions with logical space by identifying each proposition with its corresponding set of worlds. Such identification requires only those theses that were needed to ensure the existence of logical space: Thesis 1, Thesis 5, and that part of Thesis 3 contained in the principles (3.1) through (3.6) (or, equivalently, Thesis 3*). The remainder of the standard theory -- Thesis 2, Thesis 4, and the principles (3.7) and (3.8) -- were used not to establish the existence of logical space, but rather to determine its structure. What if a world-based theorist were to drop any or all of these structure-determining theses from his theory? Definition (7.4) would then have to be replaced by the weaker:

(G) Something is a proposition only if it is a set of worlds.

The resulting theory might be called a <u>feeble</u> world-based theory on analogy with the feeble proposition-based theories of the last section. (G) provides no assurance that every set of worlds is a proposition. It should thus be clear from what was said in the last section that this weakened theory merely trades the propositions as independently existing entities for an unreduced property applying to sets of worlds, the property of being a proposition. In this case, identification does not amount to reduction. Fortunately, there is little reason why a world-based theorist would want to reject any of the structure-determining theses; all of them, I have argued, are uncontroversial on the metaphysical conception of propositions here in question.

Although all world-based theorists must accept the five theses of the standard theory,[2] they need not agree upon their interpretation. In this sense, there is a plurality of world-based theories. For example, the ontological implications of the world-based theory will depend upon the ontological status attributed to worlds, and, in particular, upon

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[2] Stalnaker apparently disagrees. See the discussion of his view below.

whether worlds are in turn reduced to something else. Moreover, the world-based theory comes in stronger or weaker versions depending upon how broad or narrow a conception of propositions and of worlds is invoked. For example, we have seen that the strength of the world-based theory tends to vary directly with the extension of the term 'proposition' and to vary inversely with the extension of the term 'world'. Thus, a theory that allows 'proposition' to range over the full algebra of propositions is, other things equal, stronger than a theory that restricts the application of 'proposition' to the algebra of contingent propositions. Contrariwise, a theory that allows 'world' to range over impossible as well as possible worlds is, other things equal, weaker than a theory that restricts the application of 'world' to possible worlds alone. The weakest world-based theory takes Definitions (7.4) through (7.6) to be stipulative definitions. Since the theory then consists entirely of stipulative definitions, it is devoid of content. Nothing about propositions, as that term has been used prior to the stipulations, can be inferred from the theory. The theory does not provide an analysis of propositions, or of anything else for that matter. I will call it the trivial world-based theory.

A number of philosophers have explicitly endorsed the world-based theory, although not always under the same interpretation.[3] Prominent

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[3] I include among the world-based theorists those who identify propositions with functions from worlds to truth values, instead of with sets of worlds. The difference is trivial unless one has ontological qualms about truth values.

[4] In Meaning and Necessity, 2nd ed. (Chicago: University of Chicago

among them are Carnap,[4] Montague,[5] Stalnaker,[6] Lewis,[7] and Cresswell.[8] I will say just a few words about these philosophers' views. I listed Carnap's name as a courtesy for having fathered the whole idea. His conception of worlds and propositions was thoroughly linguistic in nature, and thus far removed from the metaphysical conception here in question. Just how far removed will be discussed in the next section. Cresswell does not take worlds as primitive, but reduces them to what he calls "basic particular situations", or what might as well be called basic propositions. His theory, then, can be viewed as a reduction of both propositions and worlds to basic propositions, a version of logical atomism. Stalnaker and Lewis agree in taking worlds as primitive, but differ sharply as to their ontological status. Lewis does, whereas Stalnaker does not, take all possible worlds to have the same ontological status as the actual world, where the actual world is characterized as: David Lewis and all his surroundings.[9] As for the remaining member on the list, Montague, I know of no place where he has

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Press, 1956), p. 181.

[5] In, for example, "On the Nature of Certain Philosophical Entities", Formal Philosophy (New Haven: Yale University Press, 1974), p. 153.

[6] In, for example, "Pragmatics", Semantics of Natural Language (Dordrecht: D. Reidel, 1972), D. Davidson and G. Harman, eds., p. 381.

[7] In, for example, Counterfactuals (Oxford: Basil Blackwell, 1973), p. 46.

[8] In "The World is Everything that is the Case", The Possible and the Actual (Ithaca: Cornell University Press, 1979), Michael Loux, ed., p. 133.

[9] For Stalnaker's summary of this debate, see "Possible Worlds", Nous, 10 (1976), pp. 65-75.

attempted to clarify his notion of world.

The cavalier way in which Montague, Lewis, and Cresswell identify propositions with sets of worlds suggests that it might be the trivial world-based theory that they are putting forward: they merely stipulate that by 'proposition' they mean 'set of worlds'. That might seem to be a good way to avoid controversy, but it also seems rather pointless since the definiendum has the same number of letters as the definiens. Actually, it does not much matter whether or not one attributes to these philosophers the trivial theory; they can still claim to provide an analysis of proposition, as long as they accept the trivial theory in conjunction with something else. What else? Endorsing the trivial theory merely shifts the nontrivial content of the analysis of propositions into the metatheory, at which level the relation between the newly defined term 'proposition' and traditional uses of the term 'proposition' will have to be discussed; such a discussion will be needed to justify the intended applications of the theory to, for example, the analysis of our modal idioms. Indeed, all the theses of the standard theory will then make their appearance in the formal mode, in which guise they will be just as controversial or uncontroversial as they were before. So if, as I assume, these philosophers intend to provide an analysis of some notion of proposition, then they cannot avoid tangling with the theses of the standard theory, later if not sooner.

It is no accident that those who have explicitly endorsed the world-based theory are all proponents of possible-worlds semantics. But it would be a mistake to suppose that, conversely, whoever accepts possible-worlds semantics (and accords it more than purely formal significance) implicitly endorses the world-based theory.[10] For although the semantics assigns sets of worlds to sentences as their interpretation, it does not require that one identify the sets of worlds with the propositions those sentences express; the semantics need not be taken to imply anything about propositions in the sense here under discussion. Moreover, the possible-worlds semanticist need not even accept the standard theory; it is consistent with his semantical project, for example, to endorse the compact theory, in which case the reduction of propositions to sets of worlds is out of the question. The possible-worlds semanticist will presumably endorse the Leibnizian version of Thesis 5 so as to get our patterns of modal inference right; but much of the rest of the standard theory could be rejected without adversely affecting his semantical project. Thus, the acceptance of possible-worlds semantics need not lead to an acceptance of the world-based theory. That is a separate decision to be made, the pros and cons of which will be discussed in section 10.

Stalnaker on Adams.

The most extensive treatment of the proposition-based and world-based theories occurs in Stalnaker's article "Possible Worlds", and in a more

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[10] As might be suggested by some of Kaplan's remarks in "Transworld Heir Lines", or by Plantinga's discussion of what he calls "the canonical conception" in "Actualism and Possible Worlds". Both articles are in Michael Loux, ed., The Possible and the Actual (Ithaca: Cornell University Press, 1979).

recent, but as yet unpublished, expansion of this article.[11] Even so, Stalnaker provides but the briefest sketch. His goal is to compare Adams's world-story theory, which I discussed in the last section, with his own world-based theory, what he calls the possible-worlds analysis of proposition. He focuses upon two theses which, he claims, are just what need to be added to Adams's theory in order to make it equivalent to the possible-worlds analysis of proposition with respect to the structure imposed upon the algebra of propositions. Unfortunately, Stalnaker is only slightly more explicit in his presentation of Adams's world-story theory than Adams was himself. Since it is unclear just what theory Stalnaker is contemplating adding the two theses to, it is difficult to evaluate much of what he says. In this subsection, I will consider some possible interpretations of Stalnaker's remarks.

Stalnaker attributes what he calls the "minimal world-story theory" to Adams. If by this he means the minimal proposition-based theory that permits a <u>reduction</u> of possible worlds to maximal consistent sets of propositions, then we know exactly what theses are included: Thesis 1 (except perhaps (1.3)), Thesis 3, Thesis 4, and Thesis 5^+. But it soon becomes apparent that Stalnaker attributes an additional thesis to Adams. He writes of Adams's use of the notion of possibility:

> There is no assurance from anything Adams says that the notion [of possibility] will even coincide with the corresponding possible worlds concept of possibility: simultaneous truth in some possible world, or being a subset of some world-story.

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[11] Section IV of a manuscript entitled "Propositions", which is to become the first part of a forthcoming book. Because this unpublished manuscript contains the more complete and up-to-date presentation of Stalnaker's view, I will refer to it in what follows. It is quoted by permission of the author.

> Presumably, an explicit formulation of the world-story theory would contain postulates sufficient to ensure this. One that would do the job is this: every subset of a possible set of propositions is itself possible.[12]

This passage is extemely puzzling. For one thing, the postulate that Stalnaker proposes would not in fact do the job: it is a simple consequence of Thesis 1 (even without (1.3)), whereas we have seen that it is Thesis 5 that is needed to give the notion of possibility its customary Leibnizian interpretation. In particular, the postulate that Adams would want to add is Thesis 5', which ensures that every possible set of propositions is included in a maximal consistent set of propositions, and so has all of its members simultaneously true at a world. We saw in the last section that Adams clearly endorses Thesis 5'. Thus, I will assume for now that Stalnaker merely slipped here, and intended to attribute Thesis 5' to Adams. When Thesis 5' is added to the other theses needed for the reduction of possible worlds to propositions, the resulting theory entails Thesis 1 (except perhaps (1.3)), Thesis 3, Thesis 4, and Thesis 5.

Attributing this theory to Adams, however, creates a problem for interpreting the rest of Stalnaker's remarks. One of the two theses that Stalnaker claims follows from the possible-worlds analysis but not from Adams's world-story theory is what he labels:

(I) Necessarily equivalent propositions are identical.[13]

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12 "Propositions", typescript, pp. 70-71. Recall that for Adams 'possibility' and 'consistency' are interchangeable.

13 Ibid., p. 74. I assume that by "necessarily equivalent" Stalnaker means "true at the same set of worlds", although this is controversial according to the modal objection.

If Adams is committed to Theses 1, 3, 4, and 5, then he is committed to (I) as well. Stalnaker's (I) is just the contrapositive of Lemma (7.1), and a quick check of this lemma's proof shows that Thesis 2, the only missing thesis, is not used in it. So if Stalnaker holds that (I) is not a consequence of Adams's world-story theory, and he has made no logical error, then he must not be attributing all four of the above-mentioned theses to Adams. Since he explicitly attributes Theses 3 and 4 to Adams, the problem must come with either Thesis 1 or Thesis 5 (or both).

Perhaps Stalnaker does not attribute Thesis 1 to Adams because he does not take Adams to endorse (1.3), the Boolean postulate asserting that propositions mutually implying one another are identical. I have claimed that (1.3) holds on any metaphysical conception of the propositions: propositions that mutually imply one another have the same content and, on the metaphysical conception, propositions having the same content are identified. But perhaps Stalnaker takes Adams to hold a quasi-metaphysical conception of the propositions according to which (1.3) does not hold. Indeed, we saw in the last section that the "semantical propositions" tentatively endorsed by Adams probably violate (1.3). Now, postulate (1.3) is needed to derive (I) within any of the proposition-based theories; without (1.3) there can be distinct propositions that are true at exactly the same set of worlds. So, one way to understand Stalnaker's claim that (I) does not follow from Adams's theory is to take Stalnaker to be attributing to Adams Thesis 1 without (1.3), Thesis 3, Thesis 4, and Thesis 5.[14]

There is another way to understand Stalnaker's claim that (I) does not follow from Adams's theory. I suspect that Stalnaker does not intend to attribute Thesis 5^+ to Adams, and that it is the absence of Thesis 5^+ rather than the absence of (1.3) that is held responsible for the absence of (I). For one thing, we saw that Adams's "semantical propositions" probably violate Thesis 5^+ as well as (1.3), and that Adams thus may well be putting forth some version of the feeble proposition-based theory. Moreover, Stalnaker takes the chief challenge to (I) to come from the possibility of a plurality of true mathematical propositions. But I have argued that the mathematical propositions are most naturally seen as a counterexample to Thesis 5^+, not as a counterexample to (1.3).[15] It seems plausible, then, that Stalnaker does not take Adams to endorse Thesis 5^+, whether or not he takes Adams to endorse postulate (1.3).

If this interpretation of Stalnaker is correct, then Stalnaker takes Adams to hold (some version of) the feeble proposition-based theory. That creates another problem. I have claimed that the feeble theory is

[14] Recall that without (1.3) 'consistent' in Theses 3 and 5 must be redefined: a set of propositions is consistent iff it has some lower bound that does not imply everything; a proposition is consistent iff it does not imply everything.

[15] See section 5, subsection "The Mathematical Objection". Of course, if one simply defines implication between propositions in terms of inclusion between the sets of worlds at which they are true, then the mathematical propositions will violate at most (1.3), not Thesis 5^+ (assuming that mathematics is not contingent). But that is certainly not the ordinary sense of implication between mathematical propositions; a mathematician would not normally assent to the claim that 2+2=4 implies the axiom of choice.

not sufficient for the reduction that Adams proposes: the feeble theory does not provide a reduction of worlds to propositions and implication because it needs to make use of an unreduced property of sets of propositions, the property of being a world. Stalnaker, however, never questions that Adams's theory provides a reduction of worlds to propositions. Unless there is evidence that Stalnaker disagrees with my assessment of the feeble theory, it makes no sense to claim that he is attributing the feeble theory to Adams.

There is such evidence. Although Stalnaker never discusses the feeble proposition-based theory, he does say something about the feeble world-based theory that helps to clarify his general position on reduction. It emerges from his discussion of the other thesis that he considers adding to Adams's theory, what he labels:

(C) For every set of propositions, there is a proposition which is true if and only if every member of the set is true.[16]

Stalnaker's (C) can be derived in the standard theory from Thesis 2 and principle (3.7). Since Thesis 2 is not needed by Adams for his reduction of worlds to propositions, Stalnaker is correct to point out that (C) would not be part of a minimal proposition-based theory. On the other hand, we have seen that all five theses of the standard theory, and so also (C), are required for the reverse reduction of

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[16] Ibid., p. 74. As stated, this is too weak, since it only refers to truth at the actual world. It needs to be prefixed with a universal quantifier ranging over worlds. (The version of (C) in "Possible Worlds" has the word 'necessarily' appropriately inserted, which gives the same effect.)

propositions to possible worlds; without (C) one has to replace Definition (7.4) by the weaker (G), and this results in the feeble world-based theory. Yet Stalnaker writes: "Nothing important hangs on condition (C). One who rejects it can still identify propositions with sets of possible worlds, but not every such set will be a proposition."[17] Stalnaker is right that the identification in question does not require condition (C), but wrong that nothing important hangs on it. The possibility of reduction hangs on (C), and since Stalnaker claims to provide a "reductive analysis of proposition", that would seem to be important in the present context. Perhaps Stalnaker simply means by 'reduction' what I mean by 'identification', and believes that identification alone is sufficient for providing a "reductive analysis of proposition".[18] I have given my reasons in the last section for holding that identification by itself is too weak to be of much metaphysical interest. The feeble proposition-based theory, for example, does not provide a method for eliminating the term 'world' from all contexts in which it occurs.[19]

Given that Stalnaker takes the feeble world-based theory to provide a reductive analysis of proposition, it is not implausible that he similarly takes the feeble proposition-based theory to provide a reductive analysis of world. This could explain why he might attribute

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[17] Ibid., fn. 53.

[18] Indeed, others have used the term 'reduction' in this weak sense. See Montague on events, op. cit., pp. 148-152.

[19] Thus, identification by itself is too weak to satisfy Adams's stated purpose. See further sec. 8, "Three Necessary Conditions on Reduction".

the feeble proposition-based theory to Adams, and why he might think that (I) does not follow from Adams's world-story theory. On my view, however, the proposition-based theory thus attributed to Adams is not, as Stalnaker claims, minimal, but <u>sub</u>minimal.

In summary, Stalnaker's discussion of Adams's theory can be interpreted in a number of ways depending upon whether Stalnaker is taken to be attributing to Adams some, none, or all of the assertions: Thesis 5', Thesis 5^+, and (1.3). Since Stalnaker does not distinguish between these three assertions, I can do no more than guess at his intentions. Much of this confusion might have been avoided if Stalnaker had focused upon theses that were independent of Adams's theory, whatever he takes it to be, instead of focusing upon conditions (I) and (C). From the standpoint of a proposition-based theory, (I) is a rather complicated assertion, resting as it does upon Thesis 1, Thesis 3, Thesis 5', and Thesis 5^+. Adams's theory endorses some but not all of the content of (I), and, similarly, some but not all of the content of (C). If one subtracts that part of the standard theory endorsed by Adams's theory from that part of the standard theory endorsed by the world-based theory, it is Thesis 2 and some selection from Thesis 5', Thesis 5^+, and (1.3) that remain, not conditions (C) and (I). For this reason, Thesis 2 and some selection from Thesis 5', Thesis 5^+, and (1.3) would have been the more appropriate candidates to consider adding to Adams's theory.

SECTION 8

REDUCING POSSIBLE WORLDS TO LANGUAGE

Introduction.

The standard theorist with reductionist tendencies might look upon the results of the last two sections as an embarrassment of riches. Given both the possibility of reducing possible worlds to propositions and the possibility of reducing propositions to possible worlds, which if either reduction should the standard theorist endorse? The answer to this question would not much matter, however, should it turn out that propositions or possible worlds can be reduced to some further kind of entity, provided that this further kind of entity is taken to be ontologically more respectable than either propositions or possible worlds. For then, whether the world-based or proposition-based theory is endorsed, both propositions and possible worlds turn out to be set-theoretic constructions out of entities of this further kind. Let us then postpone the confrontation between world-based and proposition-based theories, and focus in the present section upon nominalist[1] proposals for reduction. Since, given the standard theory, any proposal for reducing possible worlds can be turned into an equivalent proposal for reducing propositions, it will suffice in what follows to restrict

[1] On my use of 'nominalist' see section 5, subsection "The Modal Objection", and also the comments below.

our attention to proposals for reducing possible worlds.

The most promising nominalist approach to modality, I will argue below, would involve a reduction of possible worlds to language. The question to be posed in this section is then: can talk about possible worlds be reconstrued as talk about respectable linguistic entities? I do not think that any such reduction can succeed; indeed, my reasons for this have already been foreshadowed in discussing the modal objection in section 5. But if my argument is to be conclusive, it will have to be directed against the strongest possible case for reduction, not against the proposals most commonly heard. The commonly heard proposals succumb to a simple cardinality argument: on quite modest assumptions, it can be shown that there are more possible worlds than there are linguistic entities provided by the proposal; it follows straightway that the linguistic entities cannot <u>be</u> the possible worlds. One might be tempted to think that some version of the cardinality argument could be used quite generally to show tht any attempt at reducing possible worlds to language must fail. This, however, is simply not the case. In this section, I will show how the standard proposals for reduction can be generalized in a natural way ao as to make better use of the resources available to them, and thereby circumvent the cardinality argument. Once we see just what the limitations are on these more general proposals for reduction, we will be able to see more clearly where the real difficulty lies with any attempt to reduce possible worlds to language.

In order to set the stage for what will follow, let me briefly recapitulate some of the ontological positions that might be held with respect to possible worlds. The realist with respect to possible worlds takes possible worlds to be primitive to his ontology. He means his talk about possible worlds to be taken literally, and not as disguised talk about some other kind of entity. Nonrealists with respect to possible worlds include at least the following two sorts: those who simply reject possible worlds outright, and those who try to rehabilitate possible worlds by reducing them to something else. The first sort of nonrealist, if he is consistent, must also reject the philosophical analyses that have been based upon the notion of a possible world, or, at any rate, take them to have at most heuristic value and to lack any serious philosophical import. On the other hand, the reductionist with respect to possible worlds often finds such analyses to be fruitful, illuminating, and philosophically important. The problem is not that he finds talk of possible worlds meaningless or incomprehensible, but that his philosophical conscience will not permit him to take such talk at face value. Possible worlds are nowhere to be found in his ontology! Let us focus upon a reductionist who is a nominalist (of sorts): his ontology consists only of actual, concrete entities together with what can be constructed out of such entities by means of set theory.[2] Some philosophical consciences, it is true, have

[2] This is the ontology accepted by Quine throughout most of his career. See, for example, W. V. Quine, Word and Object (Cambridge, Mass.: The MIT Press, 1960), pp. 266-270. Possible exceptions are an early paper in which he rejected the abstract entities (Nelson Goodman and W. V. Quine, "Steps Towards a Constructive Nominalism", Journal of Symbolic Logic 12 (1947), pp. 105-122); and a late paper in which he seems to reject the concrete ones (W. V. Quine, "Whither Physical Objects",

even balked at an ontology of sets; but, as will become clear, any attempt at reduction would be crippled at the outset if the reductionist did not at least have set theory at his disposal. Now, what the reductionist must do if he is to succeed in appeasing his philosophical conscience is to show that all talk about possible worlds that he wants to preserve can be interpreted as disguised talk about some kind of entity that he does accept.

It is natural at this point for the reductionist to turn to language. For among the entities that he does accept are linguistic entities: finite sequences of types of concrete marks or sounds (where sequences and types are given their standard interpretations in terms of sets), and set-theoretic constructions out of such sequences. Moreover, we do commonly gain access to the notion of a possible world through language, through what purport to be descriptions of possible worlds. So the reductionist might hope that talk about possible worlds could be made respectable by identifying possible worlds with their purported linguistic descriptions, or, at any rate, constructions out of such descriptions.[3]

To what language shall we assume that these descriptions belong? If we allow the reductionist to take the notion of a language too broadly, the resulting reduction is likely to be circular. For example, if he

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Essays in Memory of Imre Lakatos, R. S. Cohen et al., eds. (Dordrecht: Reidel, 1976), pp. 497-504).

[3] Of course, when I speak of language, it will always be an interpreted language that I have in mind; uninterpreted linguistic entities do not purport to describe possible worlds, or anything else.

can merely stipulate that the language contains a name for every possible world, then the existence of the language (with this interpretation) is at least as dubious as the existence of the possible worlds themselves. The surest safeguard against such circularity is to require that possible worlds be reduced to a natural language, perhaps enriched by the languages of science, mathematics, and formal logic. Unfortunately, many sentences of a natural language are unsuitable for use in constructing the possible worlds. I will thus assume that the reductionist has extracted from the enriched natural language a sublanguage satisfying all of the following conditions:

(1) All sentences are declarative sentences.

(2) Truth conditions for sentences are independent of contexts of inscription or utterance.

(3) All sentences are unambiguous.

(4) There is no vagueness in the truth conditions for sentences, let alone indeterminacy of a more radical sort; a sentence is either true or false at a possible world.

(5) Sentences can be uniquely parsed so as to exhibit their truth-functional and quantificational form.

Any sublanguage of a natural language satisfying these conditions will be called a reasonable language -- reasonable in the sense that it is appropriate for the project of reducing the possible worlds.[4] One

[4] Some would say that no natural language has a significant sublanguage satisfying all of the above conditions, even if the natural language is enriched by the languages of science, mathematics, and formal logic. This claim is certainly controversial; but there is no space to discuss it in the present work.

characteristic of the original natural language will be shared by all of its sublanguages: the expressions of the language are finite sequences over a finite alphabet, and thus the language has at most a countably infinite number of distinct expressions. One of the chief contentions of this section, however, is that this limitation is not as severe as might first appear.

Three Necessary Conditions for Reduction.

What conditions must be met by a successful reduction of possible worlds to language? Let us suppose that the reductionist has fixed upon a particular theory of possible worlds, that is, the set of those sentences about possible worlds whose truth he wishes to preserve. Let us call this the possible-worlds theory. Just which statements about possible worlds will be included in this theory is largely up to the reductionist. But I assume that at the very least he will want the theory to be strong enough to support the standard analysis of the alethic modalities as quantifiers over possible worlds; that is, he will want to include Thesis 5. Since the other theses of the standard theory are relatively uncontroversial, let us assume that all five theses of the standard theory are included within the possible-worlds theory.

Of course, the reductionist cannot hope to preserve the truth of everything that a realist would assert about possible worlds. For example, a realist would assert that possible worlds are not linguistic entities, whereas this cannot be preserved by any reduction of possible worlds to language. Similarly, other statements directly or indirectly about the ontological status of possible worlds must remain a source of disagreement between the realist and the reductionist. What the reductionist must do is to fix upon a theory that is strong enough to support the possible-worlds analyses that he wants to accept, but not so strong that the very possibility of reduction is excluded by the theory itself. Now, a minimal condition that the reduction must satisfy is that it provide a translation of sentences about possible worlds into sentences about linguistic entities that maps truths of the possible-worlds theory into truths, and falsehoods into falsehoods. (Sentences that are not assigned a determinate truth value by the theory can be mapped into either truths or falsehoods, as convenience dictates.)

But the reductionist not only wants the translation to preserve the truth of the possible-worlds theory, he wants it to preserve the structure of the theory as well. For example, sentences that make existential claims about possible worlds are to be translated into sentences that make existential claims about linguistic entities. The reduction is not intended to provide a restructuring of logical form, but only a switch of the underlying ontology. If a reduction provides a truth- and structure-preserving translation of the possible-worlds theory, then I will call the translation _faithful_, and say that the reduction satisfies the _faithfulness condition_.

There is a routine method for providing such a faithful translation, at least if we assume that the possible-worlds theory is couched within an extensional language. Before presenting the method, it will be useful to introduce some terminology. Two worlds are _discernible with_ respect to the predicates of the possible-worlds theory if there exists some binary relation expressible within the language of the theory, and some object from the domain of the theory, such that one but not the other world bears the relation to the object, or the object bears the relation to one but not the other world.[5] Note that if the possible-worlds theory makes use of a primitive identity predicate applying to worlds, then any two worlds are discernible with respect to the predicates of the theory. On the other hand, if there is no such primitive identity predicate, but the theory makes use of one or more binary predicates expressing metaphysical relations of discernibility between worlds, then discernibility with respect to the predicates of the theory will correspond to some metaphysical notion of discernibility.[6]

Now, the routine method for providing a faithful translation is as follows. To each possible world one assigns a linguistic entity to serve as that world's representative -- in effect, to be the possible world. Distinct possible worlds are assigned distinct linguistic entities if they are discernible with respect to the predicates of the possible-worlds theory; but in general the assignment need not be one-to-one. Any such assignment uniquely determines a faithful translation

[5] Note that I use the term 'predicate' to cover all the nonlogical vocabulary of the theory. Quine would call the notion defined above relative discernibility with respect to the predicates in question. On the distinction between relative and absolute discernibility, see W. V. Quine, Word and Object (Cambridge, Mass.: The MIT Press,1960), p. 230.

[6] But, as we shall see, it will not correspond to a notion of qualitative discernibility if the possible-worlds theory is Haecceitist.

of the possible-worlds theory into a theory of the linguistic entities. For example, given any unary predicate expressing some property of possible worlds, a corresponding predicate expressing some property of the linguistic entities is introduced as follows: the new predicate is true of just those linguistic entities that were assigned to some possible world of which the old predicate was true. The old predicate, as it were, casts a shadow upon the linguistic entities, and a new predicate is introduced to represent the shadow. More generally, the entire possible-worlds ideology (to use Quine's term), as represented by the predicates and function symbols of the language of the possible-worlds theory, is transferred by way of the assignment into a corresponding ideology of the linguistic entities, as represented by corresponding new predicates and function symbols. Every sentence about possible worlds can then be translated into a sentence about linguistic entities simply by replacing old predicates and function symbols by corresponding new ones. The translation clearly preserves logical structure. Moreover, by wending one's way up Tarski's inductive definition of truth, it can easily be shown that the translation is truth-preserving as well. So, any such assignment of linguistic entities to possible worlds can be used to ensure that the faithfulness condition be satisfied.

But this whole procedure has an air of circularity about it. The reductionist wishes to show that he need not admit a primitive notion of possible world to his ontology. According to the above procedure, he does this by showing that, for each possible world, a corresponding linguistic entity can be found. But if he thus invokes the possible worlds in selecting the linguistic entities, how can he claim to have eliminated the possible worlds? I do not think that he can unless the reduction meets the following noncircularity condition, which then shows that the circular way of describing the reduction can be avoided: the possible worlds, and all the possible-worlds ideology, must be constructed out of the actualist ontology and ideology that is already available to the reductionist. In other words, all the primitive predicates and function symbols of the possible-worlds theory must be translated by predicates and function symbols that are definable within the reductionist's theory of the language in question. To put the point circularly in terms of the shadowcasting image introduced above, the shadows that the possible-worlds ideology casts upon the linguistic entities had better only mark off distinctions that are available to the reductionist. Otherwise, the reduction simply exchanges the possible worlds for their shadows; and, surely, the shadows continue to attest to the existence of the possible worlds from which they came. Thus, the reduction will be circular if the possible worlds are eliminated by stepping up the ideology of the reductionist's theory of language. As we shall see, the noncircularity condition is difficult to apply in practice if it is unclear just what the reductionist should be allowed to include within his ideology. My general policy will be to allow the reductionist whatever property of linguistic entities he thinks he can make sense of; he will need all the help he can get.[7]

[7] The two conditions required thus far are in rough agreement with the conditions given by Quine in, for example, "Ontological Relativity", Ontological Relativity and Other Essays (New York: Columbia

There is a further condition that, I believe, must be satisfied by any successful reduction of possible worlds to linguistic entities. As yet I have said nothing to require that the linguistic entities correspond in a natural way with the possible worlds that they are to replace. It might be that the entire syntactic structure of the possible-worlds theory was duplicated, sentence for sentence, by the syntactic structure of some wholly unrelated actualist theory of linguistic entities. But the existence of such a duplication of structure cannot by itself provide grounds for concluding that the linguistic entities are fit to play the role of the possible worlds. We need to require that the linguistic entities be naturally linked to the possible worlds that they are to replace. Now, the obvious place to look for such a natural correlation is to the semantics of the language: the linguistic entities represent possible worlds in virtue of what they mean. More exactly, I will say that the reduction satisfies the *naturalness condition* as long as each possible world is replaced by a linguistic construction that can serve as a complete description of that possible world, where a linguistic construction *completely describes* a possible world if it is true at that world, and perhaps at worlds indiscernible from that world with respect to the predicates of the possible-worlds theory, but at no others.

Again I have taken the realist's perspective, and spoken as if there were possible worlds existing over and above the linguistic entities. Only the realist can speak of a correlation between possible worlds and

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University Press, 1969), pp. 26-29. Not so for the third condition now to be discussed.

linguistic entities as being more or less natural. As far as the reductionist is concerned, the linguistic entities just <u>are</u> the possible worlds, and so the correlation cannot but be the most natural one of all: identity. But I think that the naturalness condition can be restated in a way that the reductionist can accept. For although it makes no sense to the reductionist to speak of a linguistic construction as providing a complete description of a possible world, it does make sense to speak of a linguistic construction as <u>purporting</u> to provide such a description, even if he doesn't believe that anything exists for the description to describe. So the reductionist too can recognize which linguistic entities are naturally suited to play the role of the possible worlds, although, as we shall see, modal notions will be needed for this purpose.

It will be useful to illustrate an aspect of the naturalness condition that will be crucial to the arguments given below against the possibility of reducing possible worlds to a reasonable language. Consider the following possible-worlds theory. The language of the theory contains a primitive identity predicate, and an indiscernibility predicate that holds of two worlds just in case they are indiscernible with respect to the predicates of the theory <u>not including identity</u>. Moreover, suppose that whenever two worlds are thus indiscernible, they are also indiscernible with respect to the predicates of the language to which the possible worlds are to be reduced, as will always be the case when the possible-worlds theory includes a possible-worlds semantics for the language. Finally, suppose that the possible-worlds theory contains the assertion: there exist distinct but indiscernible worlds. That is, the theory contains a denial of the identity of indiscernible worlds, where the notion of indiscernibility depends upon the language of the theory itself.[8] Can such a possible-worlds theory be reduced to an actualist theory of linguistic entities?

If the reduction is to satisfy the faithfulness condition, there must be mutually indiscernible worlds that are not represented by the same linguistic entity. For if this were not the case, the above denial of the identity of indiscernible worlds would be translated into the falsehood that there exist distinct but identical linguistic entities. But can the reductionist provide distinct linguistic entities for mutually indiscernible worlds?

He cannot if the reduction is to satisfy the naturalness condition. For it is not enough to simply assign to every set of mutually indiscernible worlds an equinumerous set of linguistic entities, even if each linguistic entity can serve as a description of each of the indiscernible worlds. The linguistic entities cannot possibly provide the required <u>complete</u> descriptions. Since the possible-worlds theory contains a primitive identity predicate, a complete description of a world would have to hold true of that world and no other. But there is no semantical property in virtue of which a linguistic entity constructed out of the language in question can be taken to represent one but not the others of a set of mutually indiscernible worlds; by

[8] Terminological warning: for this example, indiscernibility with respect to the predicates of the possible-worlds theory coincides with identity, not with indiscernibility.

assumption, the worlds are indiscernible with respect to the predicates of the language. Thus, for no set of linguistic entities is there a natural one-to-one correspondence between it and a set of mutually indiscernible worlds.

How will the reductionist recognize whether or not he has violated the naturalness condition? Whenever he introduces as replacements for the worlds distinct linguistic entities that do not differ with respect to their modal properties, that is, their properties relevant to the description of worlds, that is proof that the naturalness condition has been violated. The naturalness condition prohibits using artificial devices, such as indexing or subscripting, as a means of multiplying linguistic entities; and only such artificial devices are available to the reductionist in the case at hand.

But the reductionist need not yet despair. The moral to be drawn from the above example is simply that the possible-worlds theory must not contain the above denial of the identity of indiscernible worlds. Let us henceforth assume that the possible-worlds theory has the indiscernibility predicate described above, and that identity between worlds is defined in terms of indiscernibility. In what follows I will speak simply of worlds being discernible or indiscernible, rather than being discernible or indiscernible with respect to the predicates of the possible-worlds theory; but it must be remembered that discernibility and indiscernibility continue to be relative to the possible-worlds theory chosen by the reductionist.

First Proposal.

With these three conditions in hand -- faithfulness, noncircularity, and naturalness -- let us turn to the evaluation of specific proposals for reducing possible worlds to language. Suppose that the reductionist has fixed upon some particular language L satisfying the five requirements listed in the introduction to this section. If two worlds are such that there is a sentence of L true at one of the worlds but not the other, then I will say that the worlds are *linguistically discernible with respect to L*, or, *L-discernible*, for short.[9] The first proposal to be considered is based upon the following idea: a class of possible worlds (in particular, the class of all possible worlds) is reducible to L as long as any two discernible worlds from that class are also L-discernible. For if discernible worlds are always L-discernible, then the reductionist can succeed in assigning distinct linguistic entities to discernible worlds by assigning to each possible world the set of those sentences of L that are true at the world.

Which sets of sentences of L will be identified with possible worlds under this proposal? Such a set of sentences must be *consistent*, that is, there must be a possible world at which all of the sentences of the set are true. Moreover, since the possible-worlds theory contains Thesis 3 of the standard theory, it follows from the possible-worlds theory that possible worlds are fully determinate, and thus that for any sentence of L and for any possible world, either that sentence or its

[9] Note that, to use Quine's terminology, L-discernibility is, in effect, *absolute* discernibility with respect to the predicates of L.

negation is true at that world. So the set of sentences true at a world will be a maximal consistent set, containing for any sentence of L either that sentence or its negation. Thus the proposal for reduction that we are considering can be formulated as follows:

PROPOSAL 1. Possible worlds are maximal consistent sets of sentences of L.

Note that the notion of consistency used in Proposal 1 is a modal notion; it cannot be taken to be narrowly logical consistency, where this is defined, for example, as truth under some reinterpretation of the nonlogical vocabulary, lest there turn out to be possible worlds in which bachelors are married.

Proposal 1 is the proposal made by Richard Jeffrey, who called such maximal consistent sets of sentences novels.[10] It is related to Carnap's well-known proposal to identify possible worlds with state descriptions, where a state description is defined as a set of sentences (of some given language) which contains for every atomic sentence either that sentence or its negation, but not both, and no other sentences.[11] One of the ways in which Carnap's proposal differs from Proposal 1 is that it places additional restrictions upon the language in question.[12] Thus,

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[10] In Richard C. Jeffrey, The Logic of Decision (New York: McGraw-Hill, 1965), pp. 196-197. But Jeffrey may not have had the modal notion of consistency in mind.

[11] In Rudolf Carnap, Meaning and Necessity (Chicago: University of Chicago Press, 1947), p. 9.

[12] Cf. Carnap's discussion in Logical Foundations of Probability (Chicago: University of Chicago Press, 1950), pp. 70-76.

Carnap must assume that the language contains individual constants (or there would be no atomic sentences at all) and that distinct individual constants denote distinct individuals. More significantly, Carnap must assume that the primitive predicates of the language have been chosen (and so can be chosen) in such a way as to guarantee that all the state descriptions are consistent (in the modal sense). This second assumption, which is quite strong, can be avoided by changing the proposal to read that possible worlds are to be identified, not with state descriptions, but with maximal consistent sets of basic sentences, where a basic sentence is either an atomic sentence or the negation of an atomic sentence. Then the proposal can be put on an equal footing with Proposal 1 by assuming that the language contains an individual constant for every individual that can be uniquely picked out by some formula of the language. But this Carnapian proposal is not equivalent to Proposal 1 (barring an exceptionally narrow use of the term 'consistency'). The Carnapian proposal will come in for discussion towards the end of this section.

The Circularity Objection.

It might seem that Proposal 1 violates the noncircularity condition because it makes use of the notion of a consistent set of sentences, and the notion of consistency has simply been characterized in terms of the notion of truth at a possible world. It is as if the reductionist were allowed to assume that L has already been given a possible-worlds semantics, so that he can then use that semantics in order to eliminate the possible worlds. But clearly, if the reductionist does not believe that L can be given a purely actualist semantics, that is, a semantics that makes reference only to actual, concrete entities and set-theoretic constructions out of such entities, then he has no business trying to reduce possible worlds to sets of sentences of L. For, as has been said, it is only in virtue of what the sentences of L mean that sets of such sentences are fit to serve as replacements for the possible worlds.

But surely L can be given an actualist semantics if it can be given any semantics at all. For recall that L is assumed to be a sublanguage of an actual language used by actual people. If there are to be any facts of the matter at all as to what the sentences of L mean, then these facts will have to be grounded in actual usage. Possible-worlds semantics may provide a compendious and illuminating way of presenting the semantics of a natural language, but it must be based upon, and so can never replace, some form of empirical semantics for the language. Now, empirical data of actual usage is available for the reductionist to use in constructing the possible worlds. Moreover, unless the reductionist shares Quine's radical skepticism about analyticity in natural language (in which case he should never have begun the project of reducing possible worlds to language), he can expect to be able to infer from the data of actual usage various facts about relations among the meanings of expressions of L; in particular, facts about such broadly modal notions as consistency itself. Presumably, there will not be enough data to determinately answer all questions about the consistency of sets of sentences. But all that matters for the reduction is that there is enough data from which a notion of possible world can be constructed that faithfully reinterprets the possible-worlds theory.

These brief remarks are meant only to suggest how I think Proposal 1 could be vindicated with respect to the charge of circularity. Much more would have to be said as to just what methods the reductionist should be permitted to use in developing an actualist semantics, and, for that matter, as to just what an actualist semantics would look like. But I want to bracket the problem of circularity for the rest of this discussion and turn instead to the problem that results from considerations of the cardinality of the class of possible worlds. This will provide a decisive refutation of Proposal 1.

The Cardinality Objection.

We have numerous beliefs about what is possible, and about which possibilities exclude which other possibilities -- that is, about *compossibility*, or what I have called consistency. If we accept the thesis that whatever is possible is true at some (fully determinate) possible world, then our beliefs about possibility and compossibility will lead us to beliefs about the number of possible worlds: we should believe that there are *at least* as many possible worlds as are needed to support our beliefs about possibility and compossibility. In this subsection, I shall use this method to set a lower bound on the cardinality of the set of possible worlds.

Here are some modal beliefs that I take to be fairly uncontroversial. I believe that the world *might* have been such that space and time were

Euclidean, although I don't believe this actually to be the case. I believe that the world <u>might</u> have consisted of nothing but (a single kind of) uniformly dense matter distributed throughout space and time, although again I don't believe that the world is actually this way. Moreover, I believe that the world might have contained nothing but a single solid cube of such matter, persisting without change throughout all eternity.

Let us fix our attention upon one such world containing one such cube. I believe that that cube might have been otherwise than it was; in particular, that it might have had less matter than it had. For example, it might have been missing one of its corners, or it might have had holes through it like a swiss cheese. Moreover, it might have had all its matter missing except for that of a single point; perhaps such an isolated point of matter would be impossible to detect, but physicists have not been contradicting themselves when they have spoken of such things. Finally, for any collection of points of matter of the original cube, the cube might have had the matter of just those points, spatially arranged in just that way.

If these beliefs about what is possible are correct, how many possible worlds must there be? Let the worlds that result from the elimination of some of the matter of the original cube be called the <u>cube worlds</u>. Different metaphysical positions with respect to transworld identity will lead one to count the cube worlds in different ways, although all methods of counting ultimately give the same result. Let us first consider how a Haecceitist with respect to matter would count the cube worlds.[13] A *Haecceitist* is someone who believes that there are primitive facts as to whether individuals inhabiting different worlds are numerically identical or not, and that therefore two worlds might be just alike with respect to all their qualitative properties, but nevertheless differ with respect to which individuals inhabit them. A Haecceitist *with respect to matter*, then, believes that there are primitive facts as to whether individuals inhabiting different worlds are composed of the numerically same matter. Now, the Haecceitist with respect to matter calculates the number of cube worlds as follows. Sets of points of the original cube are in one-to-one correspondence with cube worlds, each set of points corresponding with the unique cube world in which the cube has retained just the matter occupying the points of that set, in just that spatial configuration. Since space is assumed to be Euclidean, there are continuum many, or $\beth_1$, points of the original cube, and thus there are power-set-of-the-continuum many, or $\beth_2$, sets of such points. It follows that there are $\beth_2$ cube worlds. Since every cube world is a possible world, there are then *at least* $\beth_2$ possible worlds. So calculates the Haecceitist.

According to the *anti-Haecceitist*, however, distinct worlds must differ in some qualitative feature. In particular, distinct cube worlds must differ in the size or shape of their respective aggregates of matter, for there simply are no other qualitative properties that could distinguish them. (Note that qualitative does not exclude quantitative

[13] I use the terms 'Haecceitism' and 'anti-Haeceitism' roughly in accordance with the usage of David Kaplan, "How to Russell a Frege-Church", *Journal of Philosophy* 72 (1975), pp. 716-729. But there is no general agreement as to exactly how to use these terms.

on this usage.) But then the anti-Haecceitist will find the above calculation guilty of double counting: where the Haecceitist sees many worlds, the anti-Haecceitist often sees only one. For example, imagine the original cube to be divided into two equal halves, b and c; and call the cube world in which just the matter in b is retained wb, and the cube world in which just the matter in c is retained wc. The Haecceitist claims that wb and wc are distinct worlds because they differ as to which half of the original cube, and thus as to which matter, they contain. But the anti-Haecceitist claims that wb and wc are one and the same world, namely, that cube world that can be completely described in qualitative terms as the Euclidean world consisting of nothing but a solid, rectangular block of a certain kind of matter, of a certain shape and size, persisting unchanged throughout all time. So according to the anti-Haecceitist, the calculation done above counted the same world twice.[14]

More generally, wherever the Haecceitist sees distinct cube worlds whose respective aggregates of matter have the same shape and size, the anti-Haecceitist sees but a single world. This can be made precise as follows. Two aggregates of matter contained within the original cube, x and y, have the <u>same shape and size</u> just in case one can be superimposed upon the other by some combination of translations, rotations, and reflections; that is, just in case one can be superimposed upon the

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[14] Of course, I am speaking of an anti-Haecceitist not only with respect to matter, but also with respect to points of space. Otherwise, he counts the cube worlds like the Haecceitist with respect to matter: wb and wc are distinct cube worlds because their respective blocks of matter occupy different points of space.

other by a Euclidean transformation. (Note that this definition also covers wildly scattered and discontinuous aggregates of points of matter.) Let wx and wy be the cube worlds that result from removing all the points of matter not contained within x and y respectively. Then whereas the Haecceitist holds that wx is identical with wy if and only if x is identical with y, the anti-Haecceitist holds that wx is identical with wy if and only if x and y have the same shape and size.[15] Thus, we must distinguish between the Haecceitist cube worlds on the one hand, and the anti-Haecceitist cube worlds on the other. Since the relation having-the-same-shape-and-size-as is an equivalence relation over the set of all aggregates of matter contained within the original cube, it induces an equivalence relation over the set of Haecceitist cube worlds, and thus partitions this set of worlds into equivalence classes. The anti-Haecceitist cube worlds, then, are in one-to-one correspondence with these equivalence classes.

It is now a simple matter to count the number of anti-Haecceitist cube worlds by counting the number of equivalence classes. Recall that there are $\beth_2$ Haecceitist cube worlds in all. Each equivalence class contains at most $\beth_1$ members because there are only $\beth_1$ Euclidean transformations of a Euclidean space onto itself. But it follows from the arithmetic of infinite cardinals that if a class of $\beth_2$ members is

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[15] It should perhaps be noted that nothing compels the anti-Haecceitist to use same shape and size as his criterion for individuating the cube worlds. He could use same shape alone; or he could use something even weaker such as same topological structure. It depends upon whether or not he believes that there are absolute notions of shape and size that can support transworld comparisons. A discussion of this issue is beyond the scope of the present work.

partitioned into subclasses each containing at most $\aleph_1$ members, then there are $\aleph_2$ such subclasses. Therefore, there are $\aleph_2$ anti-Haecceitist cube worlds. Although the Haecceitist and the anti-Haecceitist may disagree as to how to interpret the various modal beliefs listed at the beginning of this section, they can both agree that those modal beliefs commit them to believing that there are at least $\aleph_2$ possible worlds.

How does all this bear upon Proposal 1? Let us make the modest assumption that the possible-worlds theory will be sufficiently strong to guarantee that there are cube worlds as described above, and that every cube world is a possible world. Then, whether the theory is Haecceitist or not, it will follow from the theory that there are at least $\aleph_2$ possible worlds. If proposal 1 is to provide a faithful translation of the possible-worlds theory, it must provide at least $\aleph_2$ linguistic entities to serve as substitutes for the possible worlds. But it can't. Since there are only a countable number of sentences of L, there are at most $\aleph_1$ maximal consistent sets of sentences of L. That will not be enough linguistic entities to provide a faithful translation of the possible-worlds theory.[16]

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[16] The argument of this subsection is derived from a similar argument in David Lewis, Counterfactuals (Oxford: Basil Blackwell, 1973), p. 90. I have presented the argument in such a way as to make it more neutral with respect to controversial metaphysical issues about identity over time, and identity across possible worlds.

Second Proposal.

The reductionist needs a proposal for reduction that can provide more linguistic entities than is provided by Proposal 1. The problem is not that there is a lack of linguistic entities in his ontology: by taking sets of sets of expressions of L, $\beth_2$ linguistic entities can be made available. The problem is that, if the proposal is to satisfy the naturalness condition, these entities will have to be able to serve as complete descriptions of possible worlds. But I will show that Proposal 1 can be generalized in a natural way so as to provide $\beth_2$ linguistic entities, and thus circumvent the cardinality argument of the previous subsection. In order to motivate such a generalization, it will be helpful to look more closely at how Proposal 1 satisfies the naturalness condition.

The naturalness condition requires that possible worlds be identified with constructions out of L that can serve as their complete descriptions, but it does not require that any single sentence of L be a complete description of a possible world. For example, Proposal 1 may identify a possible world with a set of sentences none of whose members is a complete description of that world. It is rather the set of sentences as a whole that is to be taken as the complete description of a world; the set of sentences is to be thought of as describing a world at which all of the sentences of the set are true. Thus, Proposal 1 satisfies the naturalness condition because we can think of a maximal consistent set of sentences as playing the role, semantically speaking, of the infinite conjunction of all its members. This shows that

Proposal 1, in effect, allows the reductionist to make use of the expressive power of a certain *infinitary expansion* of L: an infinitary language that adds to the logical apparatus of L an infinitary connective of *sentential* conjunction.

This suggests a way of generalizing Proposal 1. An infinitary language that only permits infinite conjunctions of *complete sentences* is quite weak as far as infinitary languages go. Why not allow the reductionist to make use of an infinitary logical expansion of L that permits infinite conjunctions of *open formulas* as well? Let L* be the infinitary logical expansion of L that permits infinite conjunctions over sets of less than $\aleph_2$ open (or closed) formulas; infinitely long sentences of L* are formed by attaching finite strings of quantifiers to such infinitely long formulas.[17] The semantics for the language L* is developed in the obvious way. In particular, the appropriate clause in the definition of truth and satisfaction reads: an infinite conjunction of formulas is satisfied by an assignment of objects to its free variables if and only if every conjunct is satisfied by that assignment. In general, the language L* will be richer in expressive power than the original language L. For example, let L contain a predicate for the greater-than relation between real numbers and a name for every (standard) natural number, but no other nonlogical constants. Although the Archimedian property of the reals, that every real number is exceeded by some (standard) natural number, cannot be expressed in L, it can be expressed in L* by: for all reals r, it is not the case that r

[17] Thus, L* is the infinitary language $L_{\aleph_2\aleph_0}$, to use a standard nomenclature.

is greater than 0 and greater than 1 and greater than 2 and ... . Such examples as this suggest -- rightly, as we shall see -- that a stronger proposal for reduction will result if L is replaced by L* in Proposal 1:

PROPOSAL 2. Possible worlds are maximal consistent sets of sentences of L*.

How does Proposal 2 fare with respect to the three conditions for reduction laid down earlier in this section? It fares at least as well as Proposal 1 with respect to the noncircularity condition. Both proposals are faced with the problem of defining consistency. True, questions about the consistency of sets of sentences of L* go beyond questions about the consistency of sets of sentences of L in that they require, in effect, judgments about the compossibility of infinite sets of open formulas for their answers. But such judgments seem no more problematical than the judgments about the compossibility of finite sets of open formulas already required by Proposal 1.

Moreover, the constructions out of linguistic entities that are to replace the possible worlds, the maximal consistent sets of sentences of L*, are all noncircularly available to the reductionist. For the expressions of L are all assumed to belong to the reductionist's ontology, and the sentences of L* can be defined as set-theoretic constructions out of the expressions of L (perhaps together with some new symbols for the infinitary connectives). Indeed, the work of constructing the sentences of L* has already been done for the reductionist by the logicians who developed the syntax for infinitary languages.[18]

Proposal 2 also satisfies that part of the naturalness condition that requires that possible worlds be replaced by linguistic entities that can serve as their descriptions.[19] The sentences of L* all have a definite meaning as long as the sentences of L do. Indeed, truth conditions for sentences of L* are completely determined by the semantics for L together with the semantical rules for the infinitary connectives. These truth conditions naturally correlate the maximal consistent sets of sentences of L* with the possible worlds that they purport to describe.

With respect to the faithfulness condition, Proposal 2 is a definite improvement over Proposal 1; for Proposal 2 can provide enough linguistic entities to faithfully translate the sentence: 'There are at least $\beth_2$ possible worlds'. This will be shown with respect to the cube worlds introduced above. It will suffice to show that, for some appropriate choice of L, distinct cube worlds are L*-discernible; for then the set of sentences of L* true at one of the worlds and the set of sentences of L* true at the other will be distinct maximal consistent sets of sentences of L*. Since there are $\beth_2$ cube worlds, it follows that Proposal 2 provides at least $\beth_2$ maximal consistent sets of sentences.

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[18] For example, see C. R. Karp, *Languages with Expressions of Infinite Length* (Amsterdam: North-Holland Publishing Corp., 1964).

[19] Whether or not the linguistic entities provided by Proposal 2 can serve as *complete* descriptions of possible worlds (for a reasonable language L) will be discussed below.

First I will show that, if L contains some rudimentary mathematical language, then L* will contain, in effect, a name for every real number.[20] Thus, let us assume that L contains at least a predicate for the greater-than relation between real numbers, function symbols for the arithmetical operations, and individual constants for the numbers 0 and 1. Then every rational number is designated by some term of L. We can use the fact that every real number is the least upper bound of some set of rational numbers to show that every real number uniquely satisfies some open formula of L*. For consider any real number r, and let $\{q_i\}$ be a set of rational numbers (indexed by ω) that has r as its least upper bound (the set of all rationals less than r will do). There is a formula of L* that is satisfied by a real number just in case it is the least upper bound of the q_i; that is, just in case it is greater than q_1 and greater than q_2 and ... and such that any other real number that is greater than q_1 and greater than q_2 and ... is greater than it. Such a formula uniquely picks out the real number r.

The names of real numbers contained in L* can be used in describing the cube worlds if we assume that L has the means to speak of a Euclidean assignment of spatial coordinates to points of matter. For example, L might contain an 8-place predicate that holds between four points of matter and four triples of real numbers just in case the assignment of those four triples to those four points determines a Euclidean coordinatization of space, that is, an assignment of triples

[20] That is, L* will have the expressive power of a language that contains a name for every real number. Such names can be provided by introducing into L, and thus into L*, a description operator that is contextually defined à la Russell.

of real numbers to all points of matter that is in conformity with the Euclidean structure of space. And L might contain a 9-argument function symbol representing the assignment of coordinates (triples of real numbers) to points of matter relative to an initial assignment of coordinates to four points of matter.

Consider now the Haecceitist version of the cube worlds. Here we must further assume that L contains names for four points of matter of the original cube, and that these points have been assigned coordinates in such a way as to determine a Euclidean coordinatization of the entire cube. On these assumptions, it is a simple matter to formulate, for any two cube worlds, a sentence of L* that is true at one of the worlds but not at the other, that is, a sentence of L* that discerns the two worlds. This is because, since L* contains a name for every real number, and so (with a modicum of set theory) a name for every triple of real numbers, L* also contains a name for every point of matter of the original cube: every such point can be picked out by reference to its spatial coordinates. Now, consider any two Haecceitist cube worlds. Since the worlds are distinct, there must be some point of matter of the original cube that exists at one of the worlds but not at the other. But then any sentence of L* that asserts that that point of matter exists is true at one of the worlds but not at the other. Since any two Haecceitist cube worlds are thus L*-discernible, and since there are $\beth_2$ cube worlds in all, it follows that there must be at least $\beth_2$ maximal consistent sets of sentences of L*.

Let us now turn to the anti-Haecceitist version of the cube worlds. Here it does no good to use the names for real numbers to introduce names for the individual points of matter. Rather, the anti-Haecceitist uses the names for real numbers to formulate qualitative descriptions of the cube worlds, descriptions of the overall shape and size of the worlds' aggregates of matter. Thus, let X be the set of triples of real numbers that are assigned to points of matter of the original cube under some arbitrary Euclidean coordinatization; and let Y be an arbitrary subset of X, and wY the corresponding (anti-Haecceitist) cube world. The world wY can be described by a sentence of L* that asserts the following: On some Euclidean coordinatization of space, all and only the coordinates in Y are assigned to points of matter existing in the world. For each (anti-Haecceitist) cube world, there will be such a sentence of L* that is true at that world but at none of the others. So here again we see that Proposal 2 can provide $\beth_2$ maximal consistent sets of sentences, and thus enough linguistic entities to undermine the cardinality argument.

Proposal 2 can succeed where Proposal 1 failed because Proposal 2 is based upon a weaker requirement for reducibility to L. Recall that Proposal 1 was based upon the idea that a class of possible worlds is reducible to L if any two discernible worlds from that class are L-discernible. But this condition for reducibility to L, although sufficient, is not necessary. Proposal 2 makes use of a weaker, but still sufficient, condition for reducibility: a class of possible worlds is reducible to L if any two discernible worlds from that class are L*-discernible. Since, as we have seen, there may be worlds that are discerned by a sentence of L* but not by a sentence of L, Proposal 2 is essentially more powerful than Proposal 1, and can reduce wider classes of possible worlds.

Further Generalizations.

Proposal 2 can itself be generalized, in at least two directions. First, recall that L* went infinitary with respect to conjunction, but not with respect to quantification. L* can be further expanded by introducing a stock of $\aleph_2$ individual variables to be used in the construction of formulas, and permitting universal quantification with respect to sets of such variables of cardinality less than $\aleph_2$.[21] Secondly, the bound on the number of formulas that can be conjoined, and the number of variables that can be quantified over, can be extended to arbitrarily high cardinality. For each infinite cardinal α, there is an infinitary expansion of L, $L\alpha$, which permits conjunctions over any set of less than α formulas, and universal quantification with respect to any set of less than α individual variables. These infinitary languages provide, for each infinite cardinal α, a distinct proposal for reduction:

PROPOSAL 3α. Possible worlds are maximal consistent sets of sentences of $L\alpha$.

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[21] This gives the infinitary language $L_{\aleph_2}$, to use a standard nomenclature.

Each of the Proposals 3α is based upon the idea that L_α-discernibility provides a sufficient condition for reducibility to L. Taken together, the series of Proposals 3α suggests the following *necessary* and sufficient condition for reducibility to L: a class of possible worlds is reducible to L if *and only if* any two discernible worlds from that class are L_α-discernible, for some infinite cardinal α. Corresponding to this necessary and sufficient condition, there is a maximally general proposal for reduction. It can be most easily formulated by introducing the infinitary language L_∞, which is defined as the union of the languages L_α, for all infinite cardinals α:

PROPOSAL 4. Possible worlds are maximal consistent classes of sentences of L_∞.[22]

Note, however, that the maximal consistent classes of sentences of L_∞ are *proper* classes -- they are "too large" to be sets. Thus, only a reductionist who admits proper classes as well as sets into his ontology can feel free to make use of Proposal 4.

Whether or not the reductionist will need to make use of the full power of Proposal 4 will depend upon which theses about possible worlds he has included in the possible-worlds theory. For example, if the reductionist believes that any possible world can be described by giving the distribution of no more than $\aleph_1$ qualitative properties over a space-time of no more than $\aleph_1$ points, then the cardinality of the set of all

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[22] The languages L_α and the language L_∞ are discussed, for example, in M. A. Dickmann, *Large Infinitary Languages: Model Theory* (Amsterdam: North-Holland Publishing Corp., 1975).

possible worlds will be no more than $\beth_2$, and Proposal 2 will succeed if any proposal will. But if the reductionist believes that, for any ordinal number, there is a possible world in which time is composed of a succession of instants having the order type of that ordinal, then there will be no *set* of all possible worlds, and the full generality of Proposal 4 will be needed in any attempt to reduce the *class* of possible worlds to language. But the question now arises: what are the limitations on even this most general proposal for reducing possible worlds?

The Objection from Descriptive Impoverishment.

Initially, one might have thought that there were two sorts of limitation that must be overcome by any proposal for reducing possible worlds to a language L. Let us assume, as is customary, that the vocabulary of L has been divided into two parts: a logical part and a nonlogical, or descriptive, part. Then, one might have thought that the prospects for a successful reduction would have been limited, on the one hand, by any impoverishment of the logical apparatus of L, and, on the other hand, by any impoverishment of the descriptive apparatus. What I hope that the generalized proposals of the preceding subsection have shown is that an impoverishment of the logical apparatus does not in fact limit the prospects for a reduction at all. As long as the reductionist has set theory (and perhaps class theory) at his disposal, he can always cook up set-theoretic constructions out of the expressions of L that can do all the work that the expressions of any infinitary expansion of L can do. Indeed, the standard proposals for reduction such as Proposal 1 already make use of this idea in an implicit way. I have suggested making the idea explicit by simply using sentences of an infinitary expansion of L in forming the required set-theoretic constructions. The move is perfectly legitimate; it merely allows the reductionist to make full use of the descriptive resources of the original language L.

The problem of an impovershment of the descriptive vocabulary, however, cannot be dealt with as easily. I claim that any language L that is appropriate to the task of reducing the possible worlds will have its descriptive vocabulary impoverished in such a way as to present insurmountable difficulties for the reductionist. In brief, the problem is this. If the language L is to be able to provide a noncircular reduction of possible worlds to an actualist ontology, then the descriptive resources of L will have to be, in a sense to be illustrated, imprisoned within the actual world. But then only possible worlds that are, in some broad sense, rearrangements of the actual world can be constructed (in a natural way) out of the linguistic entities of L. That will not be all of the possible worlds.

Indeed, not even the cube worlds, it seems to me, can be taken to be rearrangements of the actual world, and thus constructible out of the expressions of a reasonable language. So if the cube worlds are possible worlds as has been claimed, then no attempt at reducing possible worlds to language can succeed. Let us first see where the problem lies with respect to the Haecceitist version of the cube worlds. When I argued above that any two Haecceitist cube worlds were

L*-discernible, and thus that the class of such worlds was reducible to L, I had to assume that L contained names for points of matter of the original cube. But if L is required to be a reasonable language, in particular, an actual language having what I called an actualist semantics, then that assumption is unacceptable. Presumably, there does not exist a perfect cube of uniform matter anywhere in the actual world; nor do there exist dimensionless points of matter out of which such a cube might be composed. But providing names for nonactual points of matter of a nonactual cube is beyond the reach of the descriptive apparatus of an actual language. Such points cannot be named by ostension; nor can they be distinguished one from the other by their qualitative properties, or by their qualitative relations to actual existents. It follows that there will be cube worlds that are discernible (according to the Haecceitist), but not linguistically discernible with respect to any infinitary logical expansion of L, for any reasonable language L: just take two cube worlds whose aggregates of matter have the same shape and size. But then no linguistic construction out of L can be naturally correlated with one of the two worlds but not the other; no linguistic construction can provide a complete description of one of the two worlds. In short: the Haecceitist cube worlds are not reducible to L. The generalized proposals for reduction of the preceding subsection cannot help the reductionist here because the problem arises from an impoverishment of the descriptive vocabulary, not from an impoverishment of the logical vocabulary.

The reductionist with anti-Haecceitist leanings runs into difficulties of a somewhat different sort in attempting to reduce the cube worlds. I argued above that, for any anti-Haecceitist system of cube worlds, distinct worlds of that system are L*-discernible. Moreover, no assumptions on L were needed that would make the reduction circular, as was the case with the Haecceitist cube worlds. Nevertheless, showing that any two worlds from some one system of cube worlds are L*-discernible shows only that the class consisting of worlds from that one system is reducible to L; it does not show that the class consisting of worlds from all the different systems of cube worlds is reducible to L. There are numerous different systems of cube worlds, differing with respect to the kind of matter that their worlds contain. For, surely, there are worlds with more than one kind of uniformly dense matter, and each of these kinds of matter has associated with it a different system of cube worlds. Unless L has the descriptive resources to single out one particular such kind of matter, there will be distinct worlds from different systems of cube worlds that are not L*-discernible, and so that cannot be assigned distinct linguistic constructions out of L: just take two worlds from different systems whose aggregates of matter have the same shape and size. But it seems implausible to suppose that an actual language could single out one particular kind of uniformly dense matter. According to modern science, nothing that exists in the actual world is the stuff of which a cube world is made; so we cannot fix upon the kind of matter of any system of cube worlds by means of direct reference. Moreover, for any attempt to provide a purely qualitative description of matter, it seems to me, there could always be different kinds of matter that satisfy that description. So, for the anti-Haecceitist as well as the Haecceitist, the cube worlds resist reduction. The Haecceitist's problem with respect to distinguishing individual points of matter rearises for the anti-Haecceitist with respect to distinguishing kinds of matter.

By arguing along lines similar to these, I think it can be shown that much of our talk about possible worlds cannot be successfully reinterpreted as disguised talk about linguistic entities. But might the reductionist have done better to have chosen other entities from his actualist ontology to be the possible worlds, rather than the linguistic entities? That would have been of no avail. The proposals presented in this section will succeed in reducing possible worlds to an actualist ontology if any proposal will. For assume that some proposal for reduction makes use of a nonlinguistic, actual, concrete entity in constructing the possible worlds. That entity can be uniquely described by some open formula of some infinite expansion of some reasonable language L. At any rate, this is true if, as I suppose, every actual, concrete entity can be singled out by means of its spatio-temporal and causal relations to entities with which we are familiar.[23] So the reductionist can just as well use the open formula in constructing the possible worlds as use the concrete entity that the open formula describes. In general, whatever can be reduced to actual, concrete entities can equally be reduced to their descriptions.

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[23] The most plausible exception would be a concrete part of the actual world that was spatio-temporally and causally disconnected from the part we inhabit.

I have supposed that the reductionist has a nominalist ontology and ideology. Would it help to provide him in addition with a conceptualist ontology and ideology? It would not. The proposals of this section have already, in effect, made use of such conceptualist resources. Whatever is conceivable by actual people is describable within a reasonable language, a actual language having an actualist semantics.[24] So whatever is conceivable can be represented by some linguistic construction. Moreover, since the reductionist's ideology has already been allowed to include modal notions such as consistency, there seems to be nothing left for a conceptualist ideology to offer. So, adding a conceptualist ontology or ideology would not improve the reductionist's chances of success. The proposals of this section already make full use of the combined resources of the nominalist and the conceptualist. The failure of these proposals, then, marks the failure of any nominalist or conceptualist proposal for reducing possible worlds.

The Fallback Position: Carnap's Proposal.

If the reductionist is to have any hope of escaping this conclusion, he must go back and reject one of the premises. Now, the problem faced by the reductionist is of a piece with the problem raised by the modal objection in section 5, and comes from the same source. For I assumed throughout that the reductionist accepts Thesis 5 of the standard theory, that whatever is possible is true at some possible world. It was only with the aid of Thesis 5 that I was able to argue from the

[24] Although, on some views, the public natural language would have to be supplemented with private languages, one for each creature capable of conception.

modal beliefs about possible distributions of matter to the beliefs about the number of possible worlds. Without Thesis 5 -- in particular, Thesis 5^+, the thesis that every maximal consistent set of propositions is realizable -- the reductionist need not admit that the intractable cube worlds exist. It thus might appear that the reductionist program can be saved simply by rejecting Thesis 5^+. Let us briefly examine this reductionist position.

If the reductionist chooses to reject Thesis 5^+ in order that it will no longer follow from the possible-worlds theory that the cube worlds exist, then he will have to alter the proposals for reduction considered in this section. If Thesis 5^+ is false, not all maximal consistent sets of sentences need represent possible worlds, at least not if 'consistent' continues to mean 'compossible'. If 'consistent' means instead 'realizable' -- that is, 'true at some possible world' -- then the circularity objection looms larger than ever. Let us continue to equate 'consistency' with 'compossibility', and focus upon an independent notion of realizability. The reductionist wants to identify possible worlds with maximal realizable sets of sentences; but the reduction will be circular unless the notion of realizability can be defined within his theory of the language in question.

It turns out that something like Carnap's original proposal to identify possible worlds with state descriptions is just what the reductionist needs. For this proposal provides a way of selecting from the consistent sets of sentences just those sets of sentences that the reductionist wants to be realizable; on this proposal, only possible worlds that are, in an appropriate sense, rearrangements of the actual world can be constructed. The proposal can be developed as follows. Suppose that the reductionist has introduced into the original language L an individual constant for every entity within his actualist ontology that can be uniquely picked out by some open formula of the language. Suppose also that distinct individual constants denote distinct entities. Call the resulting language L^+. Recall that a basic sentence is either an atomic sentence or the negation of an atomic sentence. A new proposal for reducing possible worlds can be given in terms of the language L^+:

PROPOSAL 5. Possible worlds are maximal consistent sets of basic sentences of L^+.

As noted earlier, Proposal 5 differs from Carnap's original proposal in that the requirement of consistency is made explicit, rather than being packed into the choice of primitive predicates. Moreover, it is required that the individual constants do not increase the expressive power of the original language L so that the noncircularity condition will not be violated, whereas Carnap does not explicitly place any restrictions upon the interpretation of the individual constants.

Proposal 5, of course, can be generalized along the same lines as Proposal 1. The language $L\alpha^+$ can be formed from $L\alpha$ by adding an individual constant for every entity that can be uniquely picked out by some open formula of $L\alpha$. Then, possible worlds can be identified with maximal consistent sets of basic sentences of $L\alpha^+$. I assume that the reductionist would want to make use of these more general proposals, but it will be enough to focus upon Proposal 5 for what I have to say.

Why is it that the possible worlds allowed by Proposal 5 are, in general, a proper subset of the possible worlds allowed by Proposal 1? To answer this, we need to be able to say when a maximal consistent set of basic sentences of L^+ and a maximal consistent set of sentences of L purport to represent the same possible world. The connection is most easily made via models, or interpretations, for the language L^+. Each maximal consistent set of basic sentences of L^+ uniquely determines a model for the language: the universe of the model consists of all and only those entities denoted by some individual constant of L^+; the extension of a predicate is completely given by the list of all those basic sentences that contain the predicate and are members of the maximal consistent set. This model, in turn, uniquely determines a maximal consistent set of sentences of L: the set of sentences of L that are true in the model. Thus, for every maximal consistent set of basic sentences of L^+, we can speak of _the corresponding_ maximal consistent set of sentences of L. Now, a maximal consistent set of basic sentences of L^+ and a maximal consistent set of sentences of L purport to represent the same possible world just in case the latter is the set of sentences corresponding to the former. This clearly captures the intention of Carnap's proposal, although, strictly speaking, it is not a part of the proposal itself: a maximal consistent set of basic sentences of L^+ does not imply, either logically or necessarily, every sentence of its corresponding maximal consistent set of sentences of L.

Proposal 1 and Proposal 5 are distinct proposals for reduction because not all maximal consistent sets of sentences of L will correspond to a maximal consistent set of basic sentences of L^+. This can be illustrated by considering a sentence of L that expresses the proposition (M) used to generate the modal objection in section 5. Since this sentence, I have claimed, is consistent, it belongs to a maximal consistent set of sentences of L (assuming the unobjectionable Thesis 5'). But such a maximal consistent set of sentences does not correspond to any maximal consistent set of basic sentences of L^+, because every such set of basic sentences represents a world in which all the entities denoted by individual constants of L^+ exist, whereas (M) is only true at a world in which none of these entities exist. Indeed, any maximal consistent set of sentences of L that implies that there exist entities other than those denoted by the individual constants of L^+ will not correspond to any maximal consistent set of basic sentences, and so will not represent a possible world according to Proposal 5.[25]

We now see how to reformulate Proposal 5 for easy comparison with Proposal 1. Define a set of sentences of L to be realizable just in case there is a maximal consistent set of sentences of L that includes it, and that corresponds to a maximal consistent set of basic sentences of L^+. Proposal 5 is equivalent in effect to:

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[25] Also, any maximal consistent set of sentences of L that implies that some of the entities denoted by individual constants of L^+ do not exist will not represent a possible world, although Proposal 5 could be changed so as to allow that there are possible worlds in which some of the actual entities fail to exist.

PROPOSAL 5'. Possible worlds are maximal realizable sets of sentences of L.

Since not all maximal consistent sets of sentences of L are realizable on this account, Proposal 5' provides fewer possible worlds than Proposal 1. Indeed, Proposal 5', and its generalizations that make use of the infinitary expansions of L, leave out all the "worlds" that made Proposals 1 through 4 vulnerable to the objection from descriptive impoverishment. Can the reductionist, therefore, succeed in reducing possible worlds to language by rejecting Thesis 5 and embracing Proposal 5 or 5'?

That is not a feasible alternative for the reductionist. Recall that the reductionist was motivated by a desire to preserve the various analyses that make use of the notion of possible world. But most of these analyses depend in one way or another upon the connection between alethic modalities and possible worlds given by Thesis 5; without Thesis 5, few if any of these analyses can be maintained. Thus, to reject Thesis 5 is to weaken the possible-worlds theory to such an extent that it will no longer be worth preserving. The reductionist might as well join the hard-nosed nonrealist in taking such analyses to have at most heuristic value, and to lack any serious philosophical import; for the possible worlds he can supply by means of Proposal 5 or 5' will not be sufficient to make those analyses work. The reductionist program occupies an untenable middle ground between a realist position with respect to possible worlds and a hard-nosed nonrealist position.

Section 9

THE DISAPPEARANCE THEORY

Consider again the reductionist who wants to endorse the standard theory to keep up appearances, but does not want to be ontologically committed to the worlds and propositions that the theory is purportedly about. His attempts to do way with worlds and propositions by reducing them to sets of some further kind of entity, we saw in the last section, end in failure. But there is another, albeit unorthodox, path still open to the reductionist. He can forgo the further kind of entity, and reduce worlds and propositions to sets of one another, thus apparently eliminating both worlds and propositions in one fell swoop. I will call this *the disappearance theory* of worlds and propositions, although, as with most conjuring tricks, the missing entities will not be far to seek. The disappearance theory threatens to make a joke of honest ontological toil. In part for this very reason, I dare not dismiss the theory itself as a joke. Fortunately, there is ample reason to reject the theory, and thus to close off this last path available to the reductionist.

The disappearance theory can be introduced as follows. Recall that the standard theory is strong enough to support two alternative reductions: the reduction of worlds to maximal consistent sets of propositions embodied in the (strong) proposition-based theory, and the reduction of propositions to sets of worlds embodied in the world-based theory. Since the standard theorist has two theories from which to choose, he has, by random combination, four options:

(1) If he finds the notion of world less clear, or in some way less to his liking, than the notion of proposition, he can endorse the proposition-based theory but not the world-based theory. This is apparently the option taken by Adams.

(2) If he finds the notion of proposition less clear, or in some way less to his liking, than the notion of world, he can endorse the world-based theory but not the proposition-based theory. This is the option taken by Stalnaker, Lewis, and others.

(3) If he finds both notions tolerably clear, he can endorse _neither_ the proposition-based _nor_ the world-based theory, perhaps because neither theory satisfies the conditions necessary for a successful reduction. This is the position I will argue for in section 10.

(4) If he finds neither notion ontologically acceptable, he can endorse _both_ the proposition-based _and_ the world-based theory, thus identifying both worlds and propositions with sets of a sort. Indeed, on this option it appears that the standard theorist is committed only to an ontology of sets: not only worlds and propositions, but their members, and their members' members, and so on down the line, are all sets. Let us take _the disappearance theory_ of worlds and propositions to be the

conjunction of the (strong) proposition-based and the world-based theory.

If this fourth option could hold up to criticism, it would be a reductionist's dream, a realist's nightmare. But can't the disappearance theory simply be ruled out of court? Isn't it outright contradictory to define worlds as sets of propositions and propositions as sets of worlds? That depends upon one's notion of set. It is only contradictory if one chooses to make it so by restricting one's notion of set, and I assume that the disappearance theorist will not so choose. But aren't the analyses provided by the disappearance theory blatantly circular? Indeed they are, and that, according to the disappearance theorist, is the theory's chief merit. The worlds devour the propositions, the propositions devour the worlds, and, as if by magic, the ontological slate is wiped clean except for ontologically respectable sets.

But these "sets" deserve a closer look. They are not your ordinary, everyday sets; indeed, they are not sets at all according to the iterative conception of set, the conception that, I suspect, is tacitly assumed in most philosophical discussions involving set-theoretic constructions.[1] The sets that worlds and propositions would be identified with by the disappearance theory violate the so-called *Axiom of Foundation*, or *Fundierungsaxiom*, which prohibits endless descents with respect to set membership.[2] Such endless descents are generated by

[1] On the iterative conception of set, see Boolos, "The Iterative Conception of Set", *Journal of Philosophy*, 68 (1971), pp. 215-231.

the disappearance theory because it accepts both the definition of truth-at-a-world given by the proposition-based theory, Definition (6.4), and the definition of truth-at-a-world given by the world-based theory, Definition (7.6): a proposition is true at a world if and only if the proposition is an element of the world if and only if the world is an element of the proposition. But if the proposition is an element of the world and the world is an element of the proposition, then one can continue taking members of members without ever reaching bottom. Every world and every proposition except the null proposition is a member of at least one of its members. These worlds and propositions will be called unfounded sets.

It will be useful to illustrate the disappearance theory with respect to the simplest case, the case of only one world, say w, and thus only two propositions, T and $\emptyset$. Since propositions are sets of worlds, $T = \{w\}$ and $\emptyset = \{\}$. Since worlds are maximal consistent sets of propositions, and the only maximal consistent set of propositions is $\{T\}$, $w = \{T\}$. Thus, by substitution of equals for equals:

$$w = \{\{w\}\} = \{\{\{\{w\}\}\}\} = \dots \text{ ; and } T = \{w\} = \{\{\{w\}\}\} = \dots .$$

We have a chain of sets, infinite both to the left and to the right, such that each set in the chain is a member of the set to its right, and identical to the set to the right of the set to its right:

$$\dots w \in T \in w \in T \in w \dots .$$

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[2] For a discussion of various equivalent versions of the Axiom of Foundation, see Quine, Set Theory and its Logic, rev. ed. (Cambridge, Mass.: The Belknap Press, 1963), pp. 285-286.

Or, what amounts to the same since there are but two distinct sets, we have a two-element chain in the form of a circle, with set membership moving clockwise:

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    ε
   / \
  w   T
   \ /
    ε
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Although both w and T have the same abstract structure -- namely, {{{... ...}}} -- it cannot be proven from the axioms of set theory that they are identical. When there are unfounded sets, the axiom of extensionality no longer serves as a *criterion* of identity for sets; for this axiom simply reduces the question of the identity of sets to the question of the identity of their members, and, in the case at hand, these are the same question. The identity of unfounded sets, then, is not determined by any relation of same structure. This contrasts sharply with the *pure sets*, the well-founded sets that are built up from the empty set by arbitrary iterations of the operation set-of. The unfounded sets are like pure sets in not requiring nonsets as their members, or members' members, and so on down the line; but they are unlike pure sets in that they are not individuated by structure alone. There seems to be some sort of hidden content that can serve to individuate structurally identical unfounded sets, a source of impurity that will play a role in the discussion that follows.

One thing suggested by the disappearance theory is the holistic nature of the ontological enterprise. Questions of modal ontology cannot be separated from questions of mathematical ontology, not as long as the ontologist is permitted to use mathematical entities such as sets in his ontological constructions. We already saw in the last section that a decision as to whether or not to countenance proper classes may affect the possibility of reducing possible worlds. In the present case, a decision as to whether or not to countenance unfounded sets may affect the possibility of reduction. Proper classes may be more familiar than unfounded sets, but if anything unfounded sets are the better understood entities.[3] There are consistent axiomatizations of set theory that contain the denial of the Axiom of Foundation, and so posit the existence of unfounded sets. If such sets turn out to be a useful tool for the reductionist, and if such sets are at least as acceptable to him as the entities he is trying to dispose of, then I know of no grounds for prohibiting their use. The iterative conception of set has no monopoly on constructive methods; it is one tool among others.

But if unfounded sets will ever play a role in modal ontology, this is not the place. The disappearance theory purports to be a reduction of worlds and propositions to sets constructed out of neither worlds nor propositions. As such, it must satisfy the noncircularity condition set down in the last section: all properties and relations expressible within the possible-worlds theory must be definable within the reductionist's theory of sets. But that will not be possible, as can be seen by considering the entities that are to replace the worlds. The

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[3] I say this because the most familiar class theories, such as von Neumann's, do not adequately capture the notion of proper class, since, for one thing, proper classes are not permitted themselves to be members of classes. For objections to familiar class theories, see Penelope Maddy, "Proper Classes", unpublished manuscript.

unfounded sets that replace the worlds all have the same abstract structure because their structure depends only upon the cardinality of the set of worlds. But then there is no property definable within set theory that holds of one of these unfounded sets without holding of them all. Not a single nontrivial property of worlds can be constructed within the reductionist's ontology and ideology. If the reductionist claims to have captured distinctions given by the possible-worlds theory, it can only be because he has imported them straight from the possible-worlds theory itself. Such importation steps up the ideology of his theory and makes the reduction circular.[4]

Distinct but structurally identical unfounded sets, I said, are individuated by some sort of hidden content. With respect to the unfounsets that are supposed to replace the worlds, we now see what that hidden content might be: just the worlds themselves, though in a clever disguise. Noone would think that the worlds had been eliminated if they had been replaced by sets of worlds, or by sets of sets of worlds, and so on for any finite iteration. What the disappearance theory does, in effect, is to replace the worlds by sets of sets of ... of worlds. The worlds must still be there, as it were, beyond infinity, for there is nothing else that can serve to individuate one of these unfounded sets from another. The worlds may be hiding, but they have not disappeared. Their presence dashes the last hope of the

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[4] Does the reduction also fail to satisfy an appropriate naturalness condition? Given that the reduction is circular, the relation between the worlds and the unfounded sets is as natural as one could want -- like the relation between an object and its shadow.

reductionist who seeks to eliminate both propositions and possible worlds.

SECTION 10

PROPOSITION-BASED VS. WORLD-BASED THEORIES

Introduction.

I see no way for the standard theorist to eliminate both worlds and propositions from his ontology, but perhaps he can eliminate one or the other. Which, if either, should he try to eliminate? Should he try to eliminate worlds by endorsing the (strong) proposition-based theory, or propositions by endorsing the world-based theory? The three conditions for reduction set down in section 8 will not help in answering this question. Both the proposition-based and the world-based theory provide faithful reinterpretations of the standard theory, as follows from the one-to-one correspondences established in Theorems (6.2) and (7.2).[1] Both theories provide reductions that are noncircular, at least if the theorist is sufficiently realist with respect to the unreduced kind of entity. And both theories provide reductions that satisfy an appropriate naturalness condition, since both reductions are mediated by the natural relation truth-at-a-world. We might say that the satisfaction of these three conditions shows that worlds and

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[1] Moreover, the identifications of the proposition-based and world-based theories can be used to provide faithful reinterpretations of any extension of the standard theory -- say, the total theory of worlds and propositions -- as long as the extension does not contain any assertion that is inconsistent with the identification in question.

propositions are each reducible to the other, but it does not show which, if either, reduction ought to be performed.

All that separates the proposition-based from the world-based theory is a question of conceptual priority: which concepts should be taken as primitive, which as defined? In this, the final, section, I will attempt to answer this question by an appeal to methodological principles of metaphysical theorizing. As with any appeal to methodological principles, my conclusions will be tentative. For one thing, the interpretation, application, and justification of such principles rests upon controversial issues in the pragmatics of theory acceptance that are well beyond the scope of this work. Furthermore, such principles are only properly applied to one's total metaphysics, whereas I am not here in a position to consider in detail more than that fragment of metaphysics that I have called the standard theory. But tentative or not, I will argue in what follows that neither the proposition-based nor the world-based theory should be accepted, that neither worlds nor propositions should be reduced one to the other. If accepted, these arguments will close the book on reduction.

Reduction, Realism, and Pragmatism.

Before turning to the arguments against the proposition-based and the world-based theory, I want to outline the view that I take towards reduction, and to show how this view leads to what I call the pragmatic approach: decisions regarding reduction are to be based upon pragmatic considerations of overall theory. Roughly, the view is this: the decision whether or not to reduce, say, propositions to worlds is not a decision as to what to believe about entities -- "propositions" and "worlds" -- already picked out in advance; it is in part a decision as to what the references of the terms 'proposition' and 'world' are to be. On this view of reduction, I will claim, the pragmatic approach is compatible with a realist interpretation of metaphysical theories.

First, it will be useful to have a distinction between concepts and kinds.[2] There is a sense in which the proposition-based and the world-based theory have the same subject matter; in some sense, they are both about worlds and propositions. I will say that the same concepts are involved in both theories; in particular, both theories have to do with the concept of world and the concept of proposition. I suppose that the concept of world includes everything that the standard theory asserts about worlds, and that the concept of proposition includes everything that the standard theory asserts about propositions. But, on this use of 'concept', a concept does not in general determine its extension; the concept of world, for example, does not determine the reference of the term 'world'. The concept of world and the concept of proposition can meaningfully be discussed prior to and independently of a determination of the reference of the terms 'world' and 'proposition'; indeed, this has been done in sections 1 through 5.[3]

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[2] I do not claim that this distinction is customary, or that I have always held to it in previous sections.

[3] Note also that, for example, the concept of set of world is distinct from the concept of proposition. Does this beg the question against the world-based theory? I think not. The world-based theorist can maintain that these are distinct, but coextensive, concepts. He need not take concepts to be ontologically fundamental: perhaps concepts

Although in some sense the proposition-based and the world-based theory have the same subject matter, in another sense they presumably do not. The proposition-based theorist and the world-based theorist presumably do not assign the same extension, or class of entities, to the term 'world' or to the term 'proposition' (unless they are also disappearance theorists); indeed, it may be that what one theorist takes to refer to sets, the other takes to refer to nonsets, and vice versa. In this case, I will say that different kinds are involved in the two theories, that the theories associate different kinds with the term 'proposition' (or the concept of proposition) and different kinds with the term 'world' (or the concept of world). On this use of 'kind', a kind uniquely determines its class of members. Indeed, for present purposes, it will not matter if one chooses simply to identify a kind with its corresponding class. In any case, I will speak indifferently of the kind or the class as the reference of the term in question.

Both the proposition-based and the world-based theory are committed to there being some kinds of entity associated with the concepts of world and proposition. They differ, however, in their ontological commitment to basic kinds: the proposition-based theory is committed to there being a basic kind associated with the concept of proposition, but only to there being a constructed kind associated with the concept of world; the world-based theory has its priorities reversed. I say, for short, that the proposition-based theory takes propositions as basic, worlds as constructed; the world-based theory takes worlds as basic,

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are incomplete sets of properties; perhaps they exist only in the head.

propositions as constructed. By comparison, the standard theory takes both worlds and propositions as basic. In assessing the ontological parsimony of a theory, it is only the number of commitments to basic kinds that counts; constructed kinds are there for the asking, so to speak, free of ontological cost. At any rate, constructed kinds are free of cost if the tools used in the construction have already been paid for; such tools, I take it, include set theory, and whatever ideological commitments are needed for the construction. Note, however, that the number of commitments to basic kinds had by a theory might be greater than the number of commitments to basic kinds had by a more inclusive theory if the more inclusive theory is reductionist; witness the standard theory and the more inclusive proposition-based and world-based theories. For this reason, in assessing the ontological parsimony of a philosopher (or philosophy), one should look to the commitments to basic kinds had by the most inclusive theory accepted.

I can now begin to say what I mean by "perform a reduction", and in what sense reduction "eliminates" a kind from one's ontology. Given the above distinction between concepts and kinds, it is best to speak of reducing one concept to another, rather than reducing one kind to another. To <u>reduce</u> one concept to another is to <u>identify</u> the kind associated with the one with a kind that is constructed out of the kind associated with the other. To <u>identify</u> one kind with another is, at least, to assert that the kinds are identical, and thus that the members of one kind are identical with the members of the other. Accepting these assertions of identity results in a more parsimonious theory: it replaces a commitment to a basic kind by a commitment to a constructed kind, thus reducing the number of commitments to basic kinds.

Is the decision whether or not to perform a reduction simply a decision whether or not to believe that the identities in question are true? Are we somehow supposed to discover that the identities hold? This view leads straight to anti-reductionism with respect to metaphysical concepts and kinds: concepts and kinds generally classified as modal, intensional, or mathematical.[4] Take, for example, the proposal that propositions are to be reduced to worlds by identifying propositions with sets of worlds. How are we supposed to discover whether or not propositions are identical with sets of worlds unless this is merely stipulated at the outset? Do we somehow directly intuit whether or not propositions are sets of worlds? Is it a matter of thinking long and hard enough about the concepts in question? This is nonsense. When metaphysical kinds of entity such as propositions and sets of worlds are originally given to us in language or in thought as distinct, no examination of the meaning of our terms or the nature of our concepts will result in the discovery that, contrary to intitial appearances, they are identical after all. Once we have discovered that propositions are reducible to sets of worlds, and therefore that all talk about propositions can be faithfully reinterpreted as talk about

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[4] Exceptions. The concept of set of world is trivially reduced to the concept of world by whoever accepts set theory as a mode of ontological construction. Similarly, the concept of maximal consistent set of proposition is trivially reduced to the concept of proposition by whoever accepts set theory and is ideologically committed to the notion of consistency. I will ignore such trivial cases in what follows.

sets of worlds, there do not seem to be any further relevant metaphysical facts left to become cognizant of. Thus, the view that reduction is simply a matter of deciding which identities to believe would bring ontological endeavor to a standstill, and require that the metaphysician leave his ontology of metaphysical kinds in whatever state of disarray he first encounters it.

That the view in question leads to anti-reductionism does not, of course, show that it is wrong. Indeed, the following simple but effective argument can be used to defend it. Metaphysical kinds, like everything else, either are or are not identical with one another. The above discussion suggests only that these facts about identity can never be discovered, that we can never have adequate grounds for believing them to be true, not that there are no such facts of the matter. It follows that whoever identifies metaphysical kinds makes assertions of identity that he can have no adequate grounds for believing to be true. Moreover, pragmatic considerations can play no role in justifying these assertions unless the dubious assumption is made that metaphysical reality just happens to conform to our desire for economy, simplicity, and the other pragmatic virtues. Thus, the argument concludes, the only proper attitude is one of skeptical restraint: since one should not assert what one cannot have adequate grounds for asserting, one should not identify one metaphysical kind of entity with another.[5]

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[5] A related path to this conclusion goes by way of the problem of multiple reductions. For a mathematical analogue, see Paul Benacerraf, "What Numbers Could Not Be", Philosophical Review, 74 (1965), pp. 47-73.

This argument can be disarmed, needless to say, by rejecting its realist presupposition that entities of metaphysical kinds exist independently of our theorizing. If, for example, these entities are created by us, then we can simply choose whether or not to create them equal. I have presented qualified arguments against specific nonrealist positions in the preceding sections, but of course there still remain numerous nonrealist ways out.

What I want to emphasize, however, is that even a staunch realist (like myself) need not be swayed by the above argument for anti-reductionism. The argument is based upon a view of reduction that need not, and should not, be accepted. It assumes that the identification involved in a reduction consists of nothing but the assertion that certain identities hold. But identification may also involve something more. Identification may also be in part a linguistic act by means of which the meanings of the kind terms involved in the identities are further determined. This further determination of meaning serves to fix the reference of the kind terms, or, at least, to tie the reference of one kind term to the reference of the other. I claim that the meaning and reference of metaphysical kind terms such as 'proposition' and 'world' are not fully determined by any commonly accepted use, ordinary or philosophical, to which these terms are put. Sentences expressing identities between entities of such kinds may indeed lack truth values, but not because metaphysical reality is somehow indeterminate. They lack truth values simply because the meanings of some of their constituent terms are not fully determined. In this respect, there is no more of a puzzle than with sentences involving vague predicates: they too may lack truth values, I hold, because the meanings of the predicates are not fully determined.[6]

I said that the act of identification is in part a linguistic act. But focusing upon its linguistic aspect tends to obscure what I take to be its more fundamental purpose, which is not the assignment of meaning to terms, but the organization and systematization of the concepts expressed by those terms. Even if there were no words to express those concepts, the problem of reduction would remain as a problem about the concepts themselves. Identification and reduction have an ontological role to play that is quite independent of language. In addition to being linguistic acts, they are peculiarly ontological acts by which we articulate the ontological foundation of our conceptual scheme.[7]

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[6] I hold to the Fregean principle that meaning determines reference (at least in cases without context-dependence); thus, if the reference is undetermined, the meaning cannot be fully determined. I do not mean to deny, however, that there are perfectly good notions of meaning that violate the Fregean principle, and according to which the meaning of metaphysical kind terms (and vague predicates, for that matter) is fully determined.

[7] These ontological acts might be taken to be part of the process of explication, in Carnap's and Quine's sense of explication. See, for example, Quine's "Two Dogmas of Empiricism", From a Logical Point of View, 2nd ed. (Cambridge, Mass.: Harvard University Press, 1960), p. 25. But I prefer to reserve explication for the process by which the concepts of ordinary language are made precise, and to think of identification and reduction as further acts by which those concepts, once made precise, are provided with an ontological scaffolding. Thus, the process by which the ordinary language terms 'world' and 'proposition' are made precise by the standard theory is explication; the process by which we decide whether to accept the proposition-based or world-based theory is ontology.

Now, on the account of identification just sketched, realism need not lead to anti-reductionism. The reductionist need not deny that, prior to the reduction, there are facts about the identity and difference of entities of metaphysical kinds. He need only deny that, prior to the reduction, there are facts about the coextensiveness of our metaphysical kind terms and of the concepts they express, and this denial, certainly, is consistent with realism about metaphysical kinds of entity. Moreover, on this account, identification does involve the assertion of identities; but since these assertions are based upon stipulation rather than discovery, the adequacy of the grounds for such assertions cannot be called into question.

If the ontologist's decisions regarding reduction are not guided by his discovery of prior metaphysical fact, then what considerations will be relevant to his decisions? That will depend upon what qualities he wants in the finished product, the final theory. There is less than universal agreement as to exactly what these qualities ought to be, and it appears that individual taste will always play a significant role. Thus Quine has contrasted those, like himself, who have a taste for desert landscapes, with those (among whom I would count myself) who prefer overpopulated slums.[8] But, in any case, it is generally agreed that the desired qualities are, in some proportion or other, the familiar pragmatic ones such as simplicity, unity, economy, and explanatory adequacy. It is to qualities such as these, then, that the ontologist should turn in attempting to decide whether or not to perform

[8] In "On What There Is", From a Logical Point of View, 2nd ed. (Cambridge, Mass.: Harvard University Press, 1960), p. 4.

a reduction; that is, he should take what I call the pragmatic approach.

Before turning to the pragmatic approach, however, I want to briefly summarize my argument, and emphasize its limited scope. I began by assuming that we already accept a theory that carries ontological commitment to metaphysical kinds of entity, for example, the standard theory with its commitment to propositions and sets of worlds. I then argued that there is no problem justifying the use of pragmatic criteria in deciding whether or not to identify one kind with another. For in identifying one kind with another, we are not making assertions about entities that have already been picked out as the objects of discourse, but we are determining in part what the objects of discourse are to be. We are deciding what to talk about. And there is no reason why the decision what to talk about should not be based upon the practical consequences of making that decision, any more than, say, there is reason why the decision what to eat should not be based upon its practical consequences. In neither case do we commit ourselves to a pragmatist theory of truth.

The use of pragmatic criteria is uncontroversial in deciding whether or not to perform a reduction because the reduction serves only to limit the universe of discourse of our theories, to reduce their ontological commitments. But I have said nothing that would justify the use of pragmatic criteria in deciding whether or not to accept the theory in the first place, and to accept the ontological commitment to kinds of entity that goes with it. Here it might still be objected that the realist must unjustifiably assume that metaphysical reality has features corresponding to the features that we most desire in our theories. But the realist need not make this assumption. He need only assume that metaphysical reality is rich and varied enough to accommodate his final theory as what will presumably be but a tiny part. That slice of metaphysical reality that he singles out for description by his theory is, indeed, simple, unified, and so on. But there is no assumption that metaphysical reality as a whole must have these qualities. I want to say: the metaphysician, ultimately, could no more construct a theory that failed to correspond to some part of metaphysical reality than a craftsman could construct an object that failed to be a possible object. But the analogy is imperfect in many ways, and problems for the realist remain.

The Pragmatic Approach.

According to the pragmatic approach, decisions regarding reduction are to be made by way of pragmatic considerations of overall theory. Roughly, the pragmatic approach holds that:

(I) One concept should be reduced to another if and only if to do so would benefit one's total metaphysics with respect to simplicity, unity, economy, explanatory adequacy, and the other pragmatic virtues.

Of course, as it stands (I) is too vague to be of much use as a methodological principle. Its application to particular cases will depend upon what qualities are included among the pragmatic virtues, how these qualities are individually characterized, and how they are weighed against one another. There is considerable disagreement over how these questions should be answered, and, in what follows, I will have to assume certain answers with little or no argument. With the aid of such assumptions I will argue, first, that worlds should not be reduced to propositions, and, second, that propositions should not be reduced to worlds. The first argument focuses upon the notion of explanatory adequacy; the second involves more general considerations of pragmatic benefit.

Against the Proposition-Based Theory.

The argument that worlds should not be reduced to propositions is, in first approximation, as follows. The standard theory attributes a certain structure to the propositions: the propositions under implication form a complete, atomic Boolean algebra. This structure is of a sort that I will call ontologically relevant: in virtue of the structure, from the existence of some propositions the existence of other propositions can be derived. Ontologically relevant structure, I will claim, cries out for explanation. Moreover, I will argue that the structure on the propositions can be explained in terms of possible worlds and the relation of truth-at-a-world, but only if possible worlds are not in turn reduced to propositions. If worlds are reduced to propositions, then the possibility of explaining the propositional structure is forfeited. Now, assuming that explanatory adequacy is one of the pragmatic virtues, the following would seem to be an uncontroversial methodological principle:

(II) Other things being equal, a theory that explains ontologically relevant structure is to be preferred to a theory that merely posits that structure.

Thus, other things being equal, a theory that does not reduce worlds to propositions is to be preferred to a theory that does reduce worlds to propositions. In other words, proposition-based theories should be rejected because they merely posit structure that is better explained.

Unfortunately, the principle (II) and the argument based upon it have a fatal flaw connected with the rider that other things be equal. This rider needs to be inserted to ensure that (II) is consistent with (I); for if things are not equal between the theories with respect to the other pragmatic virtues, there is no guarantee that the gain in explanatory adequacy will not be offset by losses somewhere else. But if the rider succeeds in making (II) uncontroversial, it also makes it useless for the case at hand. Other things are never equal when reduction is at stake. To perform a reduction is always to save on ontology and score points for economy. Thus, whenever considerations of explanatory adequacy would have one abstain from reduction, as in the case at hand, the principle (II) will simply not apply: other things are not equal.

The easiest way to repair the above argument is to replace (II) by a stronger principle that gives expanatory adequacy precedence over economy with respect to the entities needed for explanation, at least in cases having to do with ontologically relevant structure. Thus,

(III) Other things being equal, a theory that explains ontologically relevant structure, even if the explanation requires the positing of additional entities, is to be preferred to a theory that merely posits that structure.

This principle is certainly more controversial than (II); but it seems plausible to me that whoever values explanation will be willing to sacrifice economy in order to attain it. Indeed, the real controversy will involve trying to decide what does and what does not count as a genuine explanation. For (III) does not license all manner of positing unheard-of entities wherever one has unexplained structure. The entities in question will have to satisfy various constraints if they are to be allotted explanatory value; and there is no presumption that entities can always be found satisfying these constraints. Although I will have a few words to say about the notion of explanation that is relevant to metaphysical theories, I plainly concede that I do not know how to make this notion precise, or how to divorce it from subjective considerations. In the places where my argument depends upon judgments involving explanation, I can do no more than try to make my judgment plausible to like-minded metaphysicians.

As a first step towards elaborating and defending the above argument that worlds should not be reduced to propositions, let me characterize in more detail the notion of ontologically relevant structure. Such structure is had by the kind of entity as a whole, rather than by its members individually. It is a sort of external structure that binds together the members of the kind. Specifically, a kind of entity has ontologically relevant structure just in case, in virtue of the structure, from the existence of some entities of the kind the existence of other entities of the kind can be derived. Whenever a kind of entity has ontologically relevant structure, one can speak of the set of entities generated by a given set of entities; and one can speak of the kind in question as being closed under the operations that do the generating. The Boolean structure of the propositions is ontologically relevant, as follows from the generating powers of the Boolean operations. The worlds, on the other hand, have no ontologically relevant structure, or, at any rate, none has been attributed to them by the standard theory.[9]

Why have I chosen to focus upon the notion of ontologically relevant structure? Perhaps other sorts of structure are also better explained than posited, even at the cost of economy; but with ontologically relevant structure, the demand for explanation seems especially acute. Whenever there is ontologically relevant structure, there are, in a sense, necessary connections between distinct entities, and these necessary conncections confer upon some entities the power to generate others. Necessary connections and generating powers tend to be mysterious. It seems appropriate to seek the source of these connections and powers, to try to explain them in terms of something less mysterious.

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[9] One way the worlds would have ontologically relevant structure is if some version of logical atomism were true, for example, if there is a set of independent propositions (so-called elementary or basic propositions) out of which all atomic propositions are (finite or infinite) Boolean combinations.

Take the case of propositional conjunction. Given the existence of any two propositions, the existence of a proposition that is their conjunction can be derived. But in what sense does the existence of the two conjuncts necessitate the existence of the conjunction? By virtue of what power can the two propositions generate a third? If the propositional structure is merely posited, these questions will go unanswered. But they can be answered, and the mysteries explained, by appealing to possible worlds and the role that possible worlds play in the standard theory.

The explanation of the ontologically relevant structure of the propositions runs as follows. Recall that it follows from the standard theory, which all parties to the present dispute accept, that the algebra of propositions is isomorphic to the full subset algebra of worlds, each proposition being mapped by the isomorphism onto the set of worlds at which it is true (Theorem 7.2). This isomorphism allows the ontologically relevant structure of the propositions to be explained in terms of the presumably less mysterious ontologically relevant structure of the corresponding sets of worlds. Take again the case of propositional conjunction. According to the standard theory (Thesis 3), propositional conjunction is truth-functionally standard at every world, and thus is mapped by the isomorphism into set-theoretic intersection. Now, the fact that from the existence of the conjuncts the existence of their conjunction can be derived is explained, given the isomorphism, by the fact that from the existence of the corresponding sets the existence of the intersection of those sets can be derived. More generally, the generating powers of all the Boolean operations on the propositions can be explained by way of the generating powers of the corresponding set-theoretic operations on sets of worlds.

Is this progress? I think that it is, provided that three conditions are met without which the above attempt at explanation would lose its explanatory power. First, worlds must not be defined in terms of the propositional structure that we trying to explain. Otherwise, that structure is simply explained in terms of itself, which is to say, not explained at all. Second, the ontologically relevant structure had by sets must either already have been posited for other purposes, or must be considered less in need of explanation than the ontologically relevant structure had by the propositions. I find this condition relatively uncontroversial on both counts. Finally, the notion of possible world must have sufficient clarity and intuitive content, at least upon reflection, to be able to play an explanatory role in our theories. This third condition, I take it, will be the chief bone of contention between those who accept and those who reject the above attempt at explanation. Indeed, one motivation often adduced for holding the proposition-based theory is that the notion of possible world is unintelligible unless it can be explained in terms of something else. How are we to adjudicate between such competing demands for explanation?

It would be convenient to be able to drop this third condition, and to turn the explanation of ontologically relevant structure into a purely formal affair. Then Boolean structure could always be explained in terms of set-theoretic structure because, by Stone's Representation Theorem for Boolean algebras, every Boolean algebra is isomorphic to a field of sets. But, clearly, in explaining why a kind of entity has a given Boolean structure, one cannot make use of just any isomorphic field of sets. At the least, the field of sets must be what I called a material representation of the Boolean algebra.[10] Otherwise, whenever two kinds of entity had the same Boolean structure, one could always give identical explanations for why they had that structure, which is absurd. Perhaps a given Boolean structure, when thought of in abstraction from the kind of entity that has it, can in some sense be explained by any isomorphic field of sets; but not any isomorphic field of sets can explain why that structure should be had by the particular kind of entity in question.

There is no guarantee that an appropriate material representation can be found for explaining a given instance of Boolean structure. In the case of the algebra of contingent propositions with which we have been concerned, the full subset algebra of worlds provides a material representation that, I claim, has explanatory power because the notion of possible world is intelligible without reference to the propositional structure. But in the case of the full algebra of propositions, both contingent and noncontingent, no acceptable explanation of the Boolean structure seems available. For any material representation of this algebra would have to make use of impossible as well as possible worlds, and the notion of impossible world, it seems to me, can only be

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[10] See section 2, pp. 40-41.

understood by reference to inconsistent sets of propositions, or something similiar, which would make the notion unavailable for purposes of explaining propositional structure. Possible worlds are capable of playing an explanatory role; impossible worlds are not.[11]

Of course, the proposition-based theorist will claim that, just as I do not allow the notion of impossible world to have explanatory value, so he does not allow the notion of possible world to have explanatory value. This leads to stalemate. It would do no good for me to say: look at all the analyses that can be given in terms of possible worlds, in so many different areas of philosophy. For I know that such analyses can also be given without possible worlds by making use of complex surrogates; and, anyway, the notion of possible world is not made legitimate merely by being shown to be useful. I do not know how to convince someone who finds the notion of possible world unintelligible that the introduction of possible worlds is not an *ad hoc* device devoid of explanatory value. Explanation, it appears, cannot be entirely divorced from philosophical understanding, and philosophical understanding, it has become all too clear, depends upon the psychological profile of the individual philosopher.

If, however, it is agreed that possible worlds can be used to explain the propositional structure, does it then follow from principle (III) that worlds should not be reduced to propositions? No, there is one more obstacle. If a theory can be found that explains the propositional

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[11] The use of impossible worlds in providing a representation of the full algebra of propositions was discussed above in section 5, pp. 99-100.

structure without having to posit an independent notion of world, then worlds can be reduced to propositions without forfeiting explanation. I cannot demonstrate that there is no such theory because I cannot canvass all possible extensions of the standard theory, or all possible modifications of our conceptual scheme. But let me at least say why I find it implausible that any explanation of propositional structure could bypass the worlds. The ontologically relevant structure of the propositions, if it can be explained at all, must be explained either in terms of the ontologically relevant structure of some other kind of entity, or in terms of the ontologically relevant structure of sets; such structure, once posited, can never completely be eradicated. But I do not see how the ontologically relevant structure of the propositions can be explained in terms of the ontologically relevant structure of some other kind of entity. The only candidates I can think of are other intensional kinds of entity, such as properties, whose ontologically relevant structure is at least as much in need of explanation as that of the propositions. Thus, if the ontologically relevant structure of the propositions is to be explained, it will have to be explained in terms of the ontologically relevant structure of sets. For the case of Boolean structure, I have claimed that this can sometimes be accomplished by way of an appropriate isomorphic field of sets; moreover, this is the only way of which I am familiar. Finally, I have argued that a field of sets will be appropriate for explanation only if it is a material representation of the Boolean structure, and that any material representation of the Boolean structure will have the set of possible worlds as its space (or equally controversial world-surrogates such as sets of possibilia). If all this is correct, then worlds (or world-surrogates) will have to play a role in any explanation of the propositional structure. It then follows from principle (III) that worlds (or world-surrogates) should not be reduced to propostions. I do not claim that this line of reasoning is conclusive: there are undoubtedly possible theories and modes of explanation of which I am unaware. On the pragmatic approach, all decisions regarding reduction are subject to revision in the light of future theory.

That completes the argument that worlds should not be reduced to propositions. It rests upon at least three controversial assumptions: first, that explanatory adequacy takes precedence over economy, at least with respect to ontologically relevant structure; second, that the notion of world is sufficiently intelligible to have explanatory value; and third, that the propositional structure cannot be explained without making use of an independent notion of world. Controversies such as these seem to be an inevitable consequence of taking the pragmatic approach. But the pragmatic approach, I claim, provides the only feasible approach to the problem of reduction.

Against the World-Based Theory.

The argument that worlds should not be reduced to propositions leaves it open whether or not propositions should be reduced to worlds. If propositions are identified with sets of worlds, then the explanation of propositional structure is particularly direct; but the explanation does not require, it seems to me, that this identification be made. Indeed, I think that there is a quite simple argument for not reducing propositions to worlds, although again the argument rests upon controversial assumptions. In brief, the argument is as follows. According to the main tenet of the pragmatic approach, principle (I), propositions should not be reduced to worlds unless doing so would result in some pragmatic benefit for our total metaphysical theory. I claim that no such benefit would result for the following reason. The reduction being contemplated is concerned only with the algebra of contingent propositions, not with the full algebra of propositions; and if the arguments of section 5 are correct, the former is a proper subalgebra of the latter. Our total metaphysical theory, however, will perforce be concerned with the full algebra of propositions, and will be ontologically committed to the all-inclusive kind of proposition that is represented within the full algebra. Any attempt to reduce only a subalgebra of the full algebra of propositions, and to eliminate ontological commitment only to the corresponding subkind of propositions, will not result in any pragmatic benefit for our total metaphysical theory.

In order to defend this argument, it suffices to examine the pragmatic virtues one by one and to show that, in each case, no benefit would result from the reduction. Consider first explanatory adequacy. The reduction being considered does nothing to explain the structure of the full algebra of propositions. Moreover, as already noted, it is not needed to explain the structure of the subalgebra of contingent propositions. The notion of consistency (or possibility) by means of which that substructure is determined can be explained in terms of possible worlds with or without the reduction.

Consider now the question of unity. The proposed reduction would add nothing to the unity of our total metaphysical theory. Singling out the algebra of contingent propositions for separate treatment will not lead towards a more unified theory. Perhaps the desired unity cannot be achieved; certainly the world-based theory does not help to achieve it.

But doesn't the world-based theory at least provide some benefit with repect to ontological economy? As long as the full algebra of propositions remains unreduced, no ontological benefit results from reducing the subalgebra of contingent propositions to possible worlds. The reduction does not eliminate our ontological commitment to any kind of entity. Certainly, it does not eliminate our commitment to the all-inclusive kind of proposition. Nor does it *eliminate* our commitment to the subkind associated with the subalgebra of contingent propositions. There is no such further commitment to be eliminated. That is, there is no ontological commitment to the subkind over and above the ontological commitment to the all-inclusive kind; the subkind is not a basic kind, since it can be constructed out of the all-inclusive kind (given the relevant ideological commitments). Thus, nothing short of reducing the all-inclusive kind can result in any diminution of our ontological commitment to kinds. Moreover, if we had a reduction of the all-inclusive kind, it would *a fortiori* provide a reduction of the subkind, and render a separate decision as to the world-based theory unnecessary.

Although the reduction does not eliminate our ontological commitment to any kind of entity, it does eliminate our commitment to certain particular entities: we are no longer committed to the thoroughly contingent propositions as anything over and above sets of possible worlds.[12] In the terminology of section 6, it makes our total theory quantitatively, although not qualitatively, more parsimonious. Moreover, I argued in section 6 that quantitative parsimony might be a good thing if the entities eliminated are somehow more dubious than the other entities of their kind. But that does not seem to be the case with the thoroughly contingent propositions. If anything, they seem to be less troublesome than the other contingent propositions that are not eliminated by the reduction. For example, the thoroughly contingent proposition that Jupiter has seventeen moons does not seem in any way more dubious than the nonthoroughly contingent proposition that either Jupiter has seventeen moons or 2+2=5. But then the gain in quantitative parsimony does not result in any gain in ontological economy with respect to the total metaphysical theory.

In conclusion, the reduction of propositions to possible worlds does not seem to benefit our overall theory with respect to any of the pragmatic virtues. It follows from principle (I) that the reduction should be rejected. This conclusion can be challenged, however, by rejecting the part of principle (I) that requires that decisions regarding reduction are to be evaluated within the context of our total metaphysical theory. If one is allowed to evaluate the world-based

[12] For the characterization of the thoroughly contingent propositions, see section 5, pp. 102-104.

theory, say, within the limited context of a theory of modality, then the above verdict will presumably be reversed. Quine has endorsed such piecemeal ontological activity: "It is generally conducive to understanding ... to welcome ontological economy in connection with one project even if a more lavish ontology is needed for the next."[13] I find this view puzzling. Would Quine say that if I do mathematics on Monday and modal metaphysics on Tuesday, my ontology changes in the interim? I do not want to deny that one can speak of the ontology of a limited theory, and then speak derivatively of a person having that ontology when and only when he is working on that theory. But it is the ontology of our total theory that is relevant to the metaphysical task of articulating our conceptual scheme. It is that task that I have been engaged in here: the attempt to determine our ontological commitments, not for this project or for that project, but once and for all.

That completes the argument that propositions should not be reduced to worlds. It rests upon at least two controversial claims: first, that the algebra of contingent propositions is a proper subalgebra of the full algebra of propositions; and second, that a decision concerning reduction should be evaluated within the context of our total metaphysical theory. The argument of this subsection together with the argument of the previous subsection lead to the conclusion that neither the world-based nor the proposition-based theory should be accepted. In sections 8 and 9, the prospects for eliminating either worlds or propositions in some other way looked dim. Both worlds and

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[13] Word and Object (Cambridge, Mass.: The MIT Press, 1960), p. 270.

propositions, it appears, have a fundamental and irreducible role to play in our conceptual scheme.

APPENDICES

First Appendix.

In this appendix, I show that Theses 1, 2, and 3 entail that the truth-functional principles (3.1) through (3.8) hold at every world. It will be convenient to do this by establishing some general principles about maximal consistent sets of propositions, one for each of (3.1) through (3.8). Thus, I will prove that for every maximal consistent set of propositions M:

(3.1') $T \varepsilon M$.

(3.2') It is not the case that $\emptyset \varepsilon M$.

(3.3') For all p and q, if $p \rightarrow q$ and $p \varepsilon M$, then $q \varepsilon M$.

(3.4') For all p and q, $p \& q \varepsilon M$ iff $p \varepsilon M$ and $q \varepsilon M$.

(3.5') For all p and q, $p \vee q \varepsilon M$ iff $p \varepsilon M$ or $q \varepsilon M$.

(3.6') For all p, $p \varepsilon M$ iff not $-p \varepsilon M$.

(3.7') For any set of propositions P, $\Lambda P \varepsilon M$ iff $p \varepsilon M$ for all $p \varepsilon P$.

(3.8') For any set of propositions P, $V P \varepsilon M$ iff $p \varepsilon M$ for some $p \varepsilon P$.

Since, according to Thesis 3, for any world, the set of propositions true at that world is a maximal consistent set, the unprimed principles follow from the primed principles together with Thesis 3. Moreover, since the primed principles hold solely in virtue of Theses 1 and 2, the unprimed principles are entailed by Theses 1, 2, and 3.

Now for the proofs (arranged in a convenient order):

(3.6') For all p, pεM iff not -pεM.

Proof. Assume that both pεM and -pεM for some maximal consistent set M. By definition, the greatest lower bound of M, ΛM, implies every member of M; so $\Lambda M \rightarrow p$ and $\Lambda M \rightarrow -p$. By Boolean algebra, the only proposition that implies both p and -p is $\emptyset$. So $\Lambda M=\emptyset$ and M is not consistent. Contradiction. Conversely, assume that neither pεM nor -pεM. Since M is maximal consistent, $\Lambda(M \cup \{p\})=\emptyset$ and $\Lambda(M \cup \{-p\})=\emptyset$. Since conjunction is associative, these can be rewritten as: $\Lambda M \& p=\emptyset$ and $\Lambda M \& -p=\emptyset$. By Boolean algebra, the only proposition that gives $\emptyset$ both when conjoined with p and when conjoined with -p is $\emptyset$. So $\Lambda M=\emptyset$ and again M is not consistent. Contradiction.

(3.2') It is not the case that $\emptyset \varepsilon$M.

Proof. For if it were, $\Lambda M=\emptyset$. Contradiction.

(3.1') TεM.

Proof. By (3.2') and (3.6').

For the rest it will be useful to have the lemma:

(3.9) For any maximal consistent set of propositions M, pεM iff $\Lambda M \rightarrow p$.

Proof. If pεM, then by definition the greatest lower bound of M implies p; that is $\Lambda M \rightarrow p$. Conversely, assume that it is not the case that pεM. Then, by (3.6'), -pεM, and so $\Lambda M \rightarrow -p$. But since the only proposition that implies both p and -p is $\emptyset$, it follows from the consistency of M that it is not the case that $\Lambda M \rightarrow p$.

(3.3') For all p and q, if $p \rightarrow q$ and $p \varepsilon M$, then $q \varepsilon M$.

Proof. Assume $p \rightarrow q$ and $p \varepsilon M$. By (3.9), $\Lambda M \rightarrow p$. By the transitivity of $\rightarrow$, $\Lambda M \rightarrow q$. Thus, again by (3.9), $q \varepsilon M$.

(3.7') For any set of propositions P, $\Lambda P \varepsilon M$ iff $p \varepsilon M$ for all $p \varepsilon P$.

Proof. $\Lambda P \varepsilon M$ iff (by (3.9)) $\Lambda M \rightarrow \Lambda P$ iff (by the definition of Λ and the transitivity of $\rightarrow$) $\Lambda M \rightarrow p$ for all $p \varepsilon P$ iff (by (3.9)) $p \varepsilon M$ for all $p \varepsilon P$.

(3.8') For any set of propositions P, $VP \varepsilon M$ iff $p \varepsilon M$ for some $p \varepsilon P$.

Proof. Follows from (3.7') using the DeMorgan laws for set-conjunction and set-disjunction, and a couple of applications of (3.6').

Finally, (3.4') and (3.5') can be seen to follow as special cases of (3.7') and (3.8'), respectively.

Each of the principles (3.1) through (3.8) now follows directly from Thesis 3 together with its primed counterpart. Thus, it holds in the present theory that the propositions are truth-functionally standard at every world.

Second Appendix.

Each of the five theses is logically independent of the others; no four logically imply the fifth. Independence can be shown in the usual way by exhibiting a structure that fails to satisfy the thesis in question, but satisfies each of the other four. In this appendix, I will briefly sketch proofs of the independence of the five theses, and of a couple of the more important subtheses. Although these proofs are mostly trivial, they help show the role that each thesis plays in determining the structure of modal or logical space. In each of these proofs, the interpretation of 'world' will be given by the set W, and the interpretation of 'proposition' by the set P.

Thesis 1 is independent. Theses 2 through 5 are not sufficient to ensure that the propositions with respect to implication form a Boolean algebra. For example, let $\langle P, \rightarrow \rangle$ be any finite lattice with a unique minimal element. Then all the terms in Theses 2 through 5 are well-defined. A structure in which Theses 2 through 5 are satisfied can be given as follows: W is the set of all atoms of the lattice, that is, strongest nonminimal elements; a proposition is true at a world just in case it is implied by the world. Although Theses 2 through 5 are all satisfied, as is easily checked, Thesis 1 will not be satisfied unless the lattice is Boolean. It need not be Boolean: any or all of the postulates (1.6), (1.7), (1.8), and (1.10) might fail to hold. Thus, Thesis 1 is independent.

As an example more relevant to the discussion in the text, I will show that postulate (1.3) is independent of Theses 2-5 and the other nine Boolean postulates. But as noted in section 6, (1.3) is only independent if the defined function symbols and constants occurring in Theses 3 and 5 and in the Boolean postulates are replaced by appropriate corresponding predicates that lack any implication of uniqueness: constants are replaced by unary predicates; n-place function symbols by n+1-place relation symbols. This does not affect the theory in any essential way, since in the complete theory all such assertions of uniqueness can be proved. Now, let W be any set, and t any element not in W. Let P be the set of all subsets of $W \cup \{t\}$. A proposition p implies a proposition q just in case all the worlds contained in p are contained in q. Under this interpretation, any set of worlds Z has two propositions that are true at all and only its members -- namely, Z and $Z \cup \{t\}$. Perhaps the second asserts that the first is true, as with Adams's semantical propositions (discussed in section 6); but there are numerous other possibilities. Since these two propositions imply one another, they provide a counterexample to postulate (1.3). On the other hand, it is easy to check that Theses 2-5 and the other nine Boolean postulates are satisfied by the interpretation (when they are written out in primitive notation in the manner suggested). Thus, it is possible to accept all of the standard theory except for postulate (1.3).

For the following independence proofs, let implication be interpreted by the set-inclusion relation, and truth-at-a-world by (the converse of) the set-membership relation, unless otherwise specified.

Thesis 2 is independent. Let W be the set of natural numbers: 0, 1, 2, ... Let P be the set of all finite and cofinite subsets of natural numbers. Thesis 1 is satisfied because <P, ↠> is a field of sets. Thesis 3 is satisfied because the set of propositions true at a world is a principal ultrafilter over the field. Theses 4 and 5 are trivially satisfied. But Thesis 2 is not satisfied: <P, ↠> is not a *complete* Boolean algebra. Consider, for each natural number i, the following cofinite set in P: {0,2,4,...2i,2i+1,2i+2,...}. This set contains the even numbers up to and including the number 2i, and then all numbers thereafter. Let K be the set of all such sets. Every finite set of even numbers is in P, and so is a lower bound for K; but K has no greatest lower bound in P. Thesis 2 is thus independent of the other four theses.

Thesis 3 is independent. Let W = {w, v, e}. Let P = {{}, {w}, {v}, {w,v}}. A proposition is *true at* w, respectively v, just in case it contains w, respectively v. All four propositions, however, are *true at* the "extra" world e. Since P is just the set of all subsets of {w, v}, Theses 1 and 2 are satisfied under the usual set-theoretic interpretation of a Boolean algebra. Moreover, Theses 4 and 5 are clearly satisfied. But Thesis 3 fails because the set of propositions true at the extra world e is not consistent.

A slight modification of the above example shows that the maximality requirement of Thesis 3 is independent of the consistency requirement together with the other four theses. Let everything be as above except that only one proposition is true at the extra world e, the proposition {w, v}. Then, the set of propositions true at e is consistent, but not maximal consistent. The other four theses still hold.

Thesis 4 is independent. Let W = {w, v}. Let P = {{}, {w, v}}. No proposition separates the two worlds w and v, so Thesis 4 fails. But all the other theses clearly hold.

Thesis 5 is independent. This will be shown twice in order to show, in addition, that Theses 5' and 5^+ are each independent of the other and the first four theses. In the first structure to be defined, Theses 1-4 and 5' hold but Thesis 5^+ does not. Let W contain only one world w, but let there be also a pseudo-world not in W called z. Let P be the set of all subsets of {w, z}. Since the proposition {z} is true at no world, Thesis 5 fails. Moreover, its failure can be traced to a failure of Thesis 5^+: there is a maximal consistent set of propositions -- {{w, z}, {z}} -- that is not realized by any world. Thesis 5', on the other hand, is satisfied, since in any finite Boolean algebra every consistent set is contained in a maximal consistent set. Theses 1-4 are easily checked to hold.

I now want to define a structure in which Theses 1-4 and 5^+ hold but Thesis 5' does not. Thesis 5' holds in a Boolean algebra if and only if the algebra is atomic, so let $\langle P, \twoheadrightarrow \rangle$ be any complete, nonatomic Boolean algebra, for example, the Lindenbaum-Tarski algebra for propositional logic, LT, discussed in section 1. Let W be the set of atoms of the algebra (or the empty set if there are no atoms, as in LT). A proposition is true at a world just in case the world implies it.

Theses 1 and 2 are satisfied by definition; and it follows immediately from Theorem (3.13) that Theses 3, 4 and 5^+ are satisfied. But Thesis 5' does not hold because there are consistent propositions that are not contained in maximal consistent sets of propositions: just take any nonnull proposition not implied by an atom, as must exist since the algebra is not atomic. Thesis 5' is thus independent of Theses 1-4 and 5^+.

PRINCIPAL THESES AND DEFINITIONS

Only theses and definitions that are mentioned outside of the section in which they first appear are listed. Those theses and definitions that are not part of the standard theory are prefixed by a '?'.

Theses.

THESIS 1. The propositions form a Boolean algebra with respect to implication.

THESIS 2. Every set of propositions has a greatest lower bound with respect to implication.

THESIS 3. For any world, the set of propositions true at that world is a maximal consistent set of propositions.

THESIS 4. For any two worlds, there is a proposition true at one of the worlds but not the other.

THESIS 5. Every proposition distinct from the null proposition is true at some world.

THESIS 3*. For any world, the set of propositions true at that world is a maximal finitely consistent set of propositions.

?THESIS 5*. Every finitely consistent set of propositions is realizable.

THESIS 5'. Every nonnull proposition is a member of a maximal consistent set of propositions.

THESIS 5^+. Every maximal consistent set of propositions is realizable.

THEOREM (3.13). The operation of set-conjunction maps the maximal consistent sets of propositions one-to-one, onto the atomic propositions.

THEOREM (6.2). The set of worlds is in one-to-one correspondence with the set of maximal consistent sets of propositions.

THEOREM (7.2). The algebra of propositions is isomorphic to the full subset algebra of worlds.

LEMMA (3.10). For any maximal consistent set of propositions, the conjunction of that set is an atomic proposition.

LEMMA (3.11). Distinct maximal consistent sets of propositions have distinct conjunctions.

LEMMA (4.2). For any set of worlds, there is a proposition true at all and only the worlds in that set.

LEMMA (7.1). For any two propositions, there is a world at which one is true but not the other.

(1.2) For all p, q, and r, if $p \to q$ and $q \to r$, then $p \to r$.

(1.3) For all p and q, if $p \to q$ and $q \to p$, then $p = q$.

(3.3) For all p, q, and w, if $p \to q$ and p is true at w, then q is true at w.

(3.6) For all p and w, p is true at w if and only if $-p$ is false at w.

(3.7) For any set of propositions P and world w, $\bigwedge P$ is true at w if and only if every member of P is true at w.

(5.3) Every consistent set of propositions is realizable.

(5.5) Logical space is not compact unless it is finite.

Definitions.

(D1) For all p, q, and r, $p\&q=r$ if and only if $r\rightarrow p$ and $r\rightarrow q$ and, for all s, if $s\rightarrow p$ and $s\rightarrow q$, then $s\rightarrow r$.

(D2) For all p, q, and r, $p\vee q=r$ if and only if $p\rightarrow r$ and $q\rightarrow r$ and, for all s, if $p\rightarrow s$ and $q\rightarrow s$, then $r\rightarrow s$.

(D3) For all p, $p=T$ if and only if, for all q, $q\rightarrow p$.

(D4) For all p, $p=\emptyset$ if and only if, for all q, $p\rightarrow q$.

(D5) For all p and q, $-p=q$ if and only if $p\vee q=T$ and $p\&q=\emptyset$.

(D6) For all p and q, $p\supset q=-p\vee q$.

(D7) For any proposition p and set of propositions Q, $\bigwedge Q=p$ if and only if $p\rightarrow q$ for all $q\varepsilon Q$, and, for any proposition r, if $r\rightarrow q$ for all $q\varepsilon Q$, then $r\rightarrow p$.

(D8) For any proposition p and set of propositions Q, $\bigvee Q=p$ if and only if $q\rightarrow p$ for all $q\varepsilon Q$, and, for any proposition r, if $q\rightarrow r$ for all $q\varepsilon Q$, then $p\rightarrow r$.

?DEFINITION (6.3). Something is a <u>possible</u> <u>world</u> if and only if it is a maximal consistent set of propositions.

?DEFINITION (6.4). One thing is <u>true</u> <u>at</u> another if and only if the first is true at the second, and the second is a maximal consistent set of propositions.

?DEFINITION (7.4). Something is a <u>proposition</u> if and only if it is a set of worlds.

?DEFINITION (7.5). One thing <u>implies</u> another if and only if the first is included in the second, and both are sets of worlds.

?DEFINITION (7.6). One thing is <u>true</u> <u>at</u> another if and only if the second is a member of the first, and the first is a set of worlds.

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