IS THERE A HUMEAN ACCOUNT OF QUANTITIES?

Phillip Bricker
University of Massachusetts Amherst

1. *Introduction*. Humeans have a problem with quantities. A core principle of any Humean account of modality is that fundamental entities can freely recombine. But determinate quantities, if fundamental, seem to violate this core principle: determinate quantities belonging to the same determinable necessarily exclude one another. Call this the *problem of exclusion*. Prominent Humeans have responded in various ways. Wittgenstein (1929), when he resurfaced to philosophy, gave the problem of exclusion as a reason to abandon the logical atomism of the *Tractatus* with its free recombination of elementary propositions. Armstrong (1978) and (1989) promoted a mereological solution to the problem of exclusion; but his account fails in manifold ways to provide a general solution to the problem. Lewis studiously avoided committing to any one solution, trusting simply that, since Humeanism was true, there had to be *some* solution. Abandonment; failure; avoidance: we Humeans need to do better. It is high time we Humeans confronted and dispatched this elephant in the room.

It won't be easy. Whether it is possible at all I leave to the reader to judge. In this paper, I present what I take to be the best account of quantities, tailoring it where needed to meet Humean demands as well as my own prior commitment to quidditism, and my own comparativist inclinations. In short: determinables, not determinates, are the fundamental properties, and freely recombine; determinates arise from the instantiation of determinables in an enhanced world structure; determinate quantities may be *local* (in a sense to be explained), but they are *not intrinsic*. Is the account I end up with Humean? Not, unfortunately, as it stands: the problem of exclusion still rears its ugly head. After dismissing a failed attempt at a solution, I consider in the final section the two viable Humean options. One attributes the source of the necessary exclusions to conventional definition, the other attributes it to logic. The first is safe and familiar, but not a response I can accept given my other commitments. The second is more radical and less familiar; but I am convinced it is on the right track. I don't have space to develop it much here, but I put it out for future research.

2. *Quantities.* By quantities, I will mean quantitative *properties*. Perhaps there are also quantitative relations; but they are beyond the scope of the present inquiry. I understand 'quantitative' broadly: what matters for my purposes is that quantities come in families with a determinate-determinable structure; much of what I say could be applied also to the problem of exclusion for qualitative determinates, such as determinates of color. I call both the

determinates and the determinables "quantities".[1] On my usage, 'determinate' is an absolute, not a relative, term: determinates are maximally specific.

I will use mass as a paradigm example. There is the *determinable* property, *having mass*. And there are the *determinate* properties: *having 2 kg mass, having 5 kg mass*, etc. In the case of mass, the determinates have definite *magnitudes* that are represented by the non-negative real numbers (relative to the chosen unit). But there are other possibilities: for example, the determinates of *charge* may have just two magnitudes, one positive and one negative. Both mass and charge are *scalar* quantities. There are also *vector* quantities, such as *force*, the determinates of which have both a magnitude and a direction. And there are other, more complex, sorts of quantities. Although I have space here only to discuss scalar quantities, I take it to be a *desideratum* of any account of scalar quantities that the account generalize naturally to vectors, and more complex quantities.

I will confine my attention to worlds that are quantitative at the fundamental level, although whether it is the determinate quantities or the determinable quantities that are fundamental, or something that undergirds them both, is yet to be decided. As a realist, I suppose that fundamental quantities, no less than other fundamental properties, correspond to immanent universals or tropes. (In fact, I am a trope theorist, but I will remain neutral in what follows.) As I note below, however, the Humean does not escape the problem of exclusion by endorsing nominalism. I will also confine my attention to worlds at which the fundamental quantities are instantiated by pointlike entities, entities that, though they may not be simple, have no finite spatiotemporal extent. Thus, for the case of mass, I have in mind something like the ideal point particles of classical physics. But, perhaps unlike classical physics, I will also confine my attention to worlds at which there is at most one object located at any point of spacetime. That allows me to speak indifferently of a fundamental property being instantiated by an object or by the spacetime point occupied by the object.

3. *Humeanism*. Humeans, in the sense relevant to the metaphysics of modality, accept what is often called Hume's Dictum:

(HD) There are no necessary connections between distinct existents.

The necessity in question is some sort of *metaphysical* or *broadly logical* necessity, not *nomological* or *epistemic* necessity. Entities are *distinct* if they have no parts or constituents in common. Hume famously applied his dictum to events in his arguments about causation. Contemporary Humeans may apply it to concrete objects, or to properties, or both.

---

[1] On an alternative usage, 'quantity' refers, not to the individual determinates and determinables, but to the family or type to which they belong, as when we say that mass is a quantity; I trust when I use 'quantity' in this way it will cause no confusion.

Hume's Dictum, on its face, is an assertion of modality *de re*. But when applied to objects it must be given a special interpretation to be plausible. It is not meant to rule out, for example, the necessity of origins, that I could not have existed without my parents. (Although for the Humean such *de re* necessity involves nominal, not real, essences.) When applied to objects, Hume's Dictum must be understood in terms of *intrinsic duplicates*: a *duplicate* of me could have existed without any *duplicate* of my parents existing. When applied to tropes or universals, no special interpretation is needed: the posited possibilities are naturally understood to involve duplicates of the tropes, or the universals themselves.

It has become standard to interpret Hume's Dictum as a *principle of recombination* stated in terms of possible worlds. I do not need to give the principle in its most general form.[2] The part of the principle that will be relevant to my discussion is just the part that rules out *necessary exclusions*, as follows:

(HPR) Consider any class of distinct elements (perhaps taken from different worlds). Consider any possible world structure, and (category-preserving) arrangement of those elements within that world structure. There is a possible world in which (duplicates of) those elements are arranged in that way.[3]

We can say that the elements in question can be *freely recombined*. Of course, the import of this principle depends on what the "elements" are; all that is presupposed by (HPR) is that the elements are fundamental entities. If the elements are all simple, then 'distinct' can be omitted; if the elements are all universals, then 'duplicate' can be omitted.

The qualification 'category-preserving' will be needed if the Humean countenances more than one fundamental category that the elements belong to. For example, if there are particulars and universals among the elements, then the Humean will only allow that the particulars all freely recombine with one another and the universals all freely recombine with one another. Or, for another example, if there are relations of different adicity among the elements, the Humean will only allow that relations of the same adicity freely recombine. This qualification requires that each place in a world structure be assigned the category of an entity that occupies that place in a world having that world structure. (HPR) then says: for any arrangement that assigns elements to places that match their category, there is a possible world in which the elements are arranged in that way.

---

[2] On how to formulate general principles that together rule out all necessary connections and exclusions, see Bricker (forthcoming a).

[3] An *arrangement* is a many-many mapping from the elements to be recombined into the places of the world structure. The left 'many' is to allow that different elements may occupy the same place; the right 'many' to allow that a single element may occupy, or have duplicates that occupy, multiple places. See below for more on world structures.

Note that Humean principles of recombination are only part of an account of modal plenitude, of what possibilities there are. We also need an account of what structures are possible world structures, and what elements are possible inhabitants of worlds, in particular, whether there are elements that are *alien* to our world. Then the recombination principle tells us that all ways of assigning the elements to (appropriate) places in a possible world structure represent a possible world.

An account of modal plenitude that permits violations of Humean recombination principles will, on its face, be committed to primitive modality. But there are other ways to be so committed, for instance, by providing modal characterizations of the possible worlds, or possible elements, or possible structures. A Humean account of modal plenitude, I will assume, eschews all forms of primitive modality. Or, being a bit more careful, the Humean eschews all modality that cannot be reduced to logic and definitions. A weaker version of Humeanism would accept (HD) and (HPR), but allow that primitive modality may be needed to *express* the Humean account. On this weaker version, although there is no primitive modality "in the world", primitive modality is needed for our theorizing about the world. It is the stronger version that I accept, and will attempt to defend below.[4]

Humeanism is sometimes taken to include Lewis's doctrine of Humean supervenience, that worlds that agree with respect to the distribution of local qualities over spacetime agree with respect to the truth or falsity of all (qualitative) propositions. But Humeanism, as I understand it, is known *a priori* if known at all, whereas the doctrine of Humean supervenience, for all we know, is false at the actual world. Indeed, even for the region of logical space that is my target, where classical physics reigns, I do not accept that part of the doctrine that asserts: the only fundamental relational structure is spatiotemporal structure. I do however hold that, at such classical worlds, all truth supervenes on the distribution of fundamental local qualities over the fundamental relational structure, whatever that structure may be. In particular, all facts about quantities so supervene. In this sense, one can say that facts about quantities are *local* facts.

4. *The Problem: Quantities Appear to Stand in Necessary Connections and Exclusions.* First off, there appear to be necessary connections between the determinates of a family and their determinable. For example, necessarily, if something has the determinable of mass, then it has some determinate mass property; and necessarily, if something has a determinate mass property, then it has the determinable of mass. This problem is easily solved, however, by noting that Humean recombination applies only to *fundamental* properties (if it applies to properties at all), and so does not prohibit necessary connections between

---

[4] Lewis (1986, p. 179) seems to recognize this distinction, holding that some ways of accepting primitive modality—for example, accepting immanent modal relations—are "especially repugnant". On different sorts of primitive modality, see Bricker (2008, pp. 114-8).

non-fundamental properties, or between fundamental properties and non-fundamental properties. So if either the determinate properties or the determinable properties are not fundamental, the above necessities are no violation of (HPR). The natural response—the response I will explore at the start—is to say that the determinate properties are fundamental, and the determinable property is analyzed as a(n infinite) disjunction of the determinate properties: to have mass is either to have 1 kg mass or 2 kg mass or … (where the ellipsis covers disjuncts corresponding to every non-negative real number).[5]

But, second, the determinates themselves appear to stand in necessary connections (or, more precisely, necessary exclusions). For example, necessarily, nothing instantiates both *being 1 kg mass* and *being 2 kg mass*. This seems to be a clear violation of the recombination principle if fundamental properties are among the elements to which it applies,[6] and the determinate properties are taken to be fundamental. My focus for the remainder of this paper will be on whether the Humean can develop an account of quantity that successfully responds to the problem raised by these necessary exclusions.

5. *First Humean Response*. According to the first response I consider, the Humean recombination principle does not apply to *properties*; it applies only to *objects* (or *things*, or so-called *thick particulars*.) This was Lewis's response in *On the Plurality of Worlds*. His recombination principle was explicitly restricted to "spatiotemporal parts" of worlds. Universals or tropes, which (if they exist) are non-spatiotemporal parts of worlds for Lewis, are no counterexample to free recombination because they do not recombine at all.[7] This also seems to be Russell's response in his logical atomist period. At any rate, he never endorses

---

[5] Rosen (2010, p. 129) suggests for epistemic reasons an alternative analysis that quantifies over properties and does not require an infinite *analysandum*: to have mass is to have some determinate of mass. Both analyses leave open the hard question (if determinates are fundamental): in virtue of what are two determinate properties determinates of the same determinable?

[6] This is loose talk. Following Lewis (1986, pp. 59–69), I take properties to be abundant and not immanent. Fundamental properties are just some of the abundant properties, and so also not immanent. Strictly speaking, then, it is the universals or tropes that are among the elements to which (HPR) applies, not the properties to which the universals or tropes correspond. For convenience I persist in this loose talk throughout, trusting that it will not mislead.

[7] Lewis does not explicitly characterize this restriction on recombination as a response to the problem of exclusion, but I suspect the problem was an unstated motivation. The only reason he gives for not endorsing a recombination principle that applies to all parts of worlds is not cogent. He writes: "such a principle, unlike mine, would sacrifice neutrality about whether there exist universals or tropes." Lewis (1986, p. 92) But I fail to see why accepting a principle that applies to all parts of worlds (or all fundamental parts) should commit one to non-spatiotemporal parts. By the time we get to Lewis (2009), recombination is taken to apply to all the elements, whatever the elements are.

the logical independence of properties or elementary propositions, only of particulars. He writes: "each particular has its being independently of any other and does not depend upon anything else for the logical possibility of its existence." Russell (1918, 203) At most, this entails that the properties had by *different* particulars are logically independent of one another.[8]

This response, however, is inadequate, for two reasons. First, if one is a realist about (sparse) properties, taking them to be immanent universals or tropes, then it is arbitrary to apply the principle of recombination to some fundamental entities and not others. As a trope theorist, I cannot avail myself of this response. But, second, even for the nominalist this response is inadequate. Recombinatorial reasoning applies to determinates of *different* determinables. For example, if determinates of mass and determinates of charge are fundamental properties, then for any determinate of mass and determinate of charge, it is possible that something have that combination. This places the following demand on any acceptable Humean account of quantities: it must entail

(D1) There are no necessary exclusions between determinates of *different* determinables.

The first Humean response does nothing to meet this demand and, therefore, should be rejected.

6. *Second Humean Response.* A second Humean response is to baldly deny that there are *metaphysically* necessary exclusions between determinates of a single determinable. This response comes in two versions. The first takes the exclusions to be only *nomically* (or *causally*) necessary. Thus, it is *metaphysically* possible for something to be both 1 kg mass and 2 kg mass, just not *nomically* possible; it is ruled out by the laws of nature. This response is floated, though not endorsed, by Lewis. What he endorses is the unknowability of whether the exclusions are metaphysically, or only nomically, necessary. He writes:

> "On some other questions … we just have to confess our irremediable ignorance. I think one question of this kind concerns incompatibility of natural properties. Is it absolutely impossible for one particle to be both positively and negatively charged? Or are the two properties exclusive only under the contingent laws of nature that actually obtain? I do not see how we can make up our minds …" Lewis (1986, p. 114)

But I do not find this response at all plausible. The problem isn't that it accepts metaphysical possibilities that go beyond anything we ordinarily take to be possible. The principle of recombination already commits the Humean to a slew of bizarre possibilities. For example, it allows one to patch together parts of

---

[8] For discussion of how Russell's logical atomism compares to Wittgenstein's post-*Tractarian* view, see Bell and Demopoulos (1996).

worlds with different laws, resulting in a schizophrenic world in which induction would be utterly unreliable. But *distinguish*: for all these bizarre worlds, it seems right to say that it is an *a posteriori* matter whether or not our world is one of them. Worlds at which two determinates of a determinable are co-instantiated, on the other hand, seem to be ruled out *a priori*. Do I really need to make observations to discover whether or not something is both 2 kg and 3 kg? That sounds absurd. I know that nothing is both 2 kg and 3 kg simply in virtue of understanding the concepts involved. I conclude, then, that another demand on an acceptable Humean account is that:

(D2) The exclusions between determinates of a single determinable are knowable *a priori*.

This suggests a better way to deny that the exclusions are metaphysically necessary: take them to be *conceptually* necessary, and thus *a priori*, and claim that a conflation between conceptual and metaphysical necessity is responsible for our mistaken judgment that the exclusions are *metaphysically* necessary. One must distinguish between the *concepts* of determinate quantities, and the determinate quantities themselves, that is, the properties picked out by those concepts. Thus, consider the concepts of being 1 kg mass and being 2 kg mass, and the determinate properties that these concepts pick out (at a classical world). On the response now being considered, there will be a possible world in which these two determinate properties are co-instantiated in accord with the principle of recombination; but in this world, the concepts do not apply to the properties, the properties are not properly called "determinates of mass." This could be because it is analytic to 'determinate' and 'determinable' that distinct determinates of a determinable are never co-instantiated, or because it is analytic to 'mass' that distinct determinates of mass are never co-instantiated. Either way, on the response being considered the necessary exclusion is *de dicto*, not *de re*.

Although I will revisit a version of this Humean response below, it should be rejected in its present form. Any attempt to explain away the necessary exclusion of mass determinates by packing the exclusions into the concept of determinate or the concept of mass is bound to fail because the problem of exclusion persists when expressed using different concepts that lack any exclusionary nature. Suppose that your favorite property is *being 5 kg mass* and my favorite property is *being 17 kg mass*. There is a clear sense in which it is true to say: my favorite property and your favorite property are necessarily not co-instantiated. But no plausible implementation of the current strategy can capture this sense. And that is just to say that the necessity in question is *de re*, not *de dicto*: the properties themselves, independently of how we pick them out, necessarily exclude one another. If the Humean is to attribute the source of the necessary exclusions to some sort of conceptual necessity, it must derive not from the concepts used to pick out the determinates of mass, but from the concept of necessity itself. I return to consider this sort of response in the final section.

7. *Third Humean Response.* The first two responses, in one way or another, denied that there are necessary exclusions. The third response instead accepts the necessary exclusions, and tries to *explain* them. It takes the fundamental determinates to be *structural properties*, properties that are instantiated by composite entities in virtue of the properties of and relations among their parts. The structural properties are themselves complex entities that have those properties and relations as constituents. The idea, then, is that in some way this complexity can be used to explain the exclusions.

One version of this response is due to Armstrong. He focuses on the case of extensive quantities, such as mass. He holds that whenever an object has a determinate mass, it divides into distinct parts whose masses sum to the mass of the whole. Thus, the determinate, *being 5 kg*, will be associated with the structural property, *being composed of five distinct 1 kg proper parts*. But, of course, it is no less associated with infinitely many other structural properties, such as *being composed of two distinct proper parts of 2 kg and 3 kg*. This suggests that, to avoid arbitrariness and capture all relations between the mass determinates, the determinate, *being 5 kg mass*, should be identified with the conjunction of all of these associated structural properties. It then follows that, for any two determinates of mass, the smaller mass determinate and all of its constituents are also constituents of the larger mass determinate. This sharing of constituents, according to Armstrong, can then be invoked to explain the resemblance between the determinates of mass.

My concern here, however, is not with Armstrong's attempted explanation of the resemblance between determinates of a determinable, but his attempted explanation of their incompatibility.[9] In this respect, the account is clearly inadequate. As best I can tell, the explanation is simply this. Consider, for *reductio*, an object that instantiates two determinates of mass, say, 5 kg and 2 kg. In virtue of being 5 kg, it has a proper part that is 2 kg. But then the object will share a structural property, the property of being 2 kg, with one of its proper parts, and that, Armstrong thinks, is impossible.[10] But why? Granted, if an object shares a structural property with one of its proper parts, the object will have to contain an infinite sequence of smaller and smaller proper parts. But that is to be expected, on Armstrong's mereological account, if there is no quantum of mass. (And note that this infinite nesting of proper parts doesn't require that the objects be gunky, and not composed of extensionless points; it only requires that

---

[9] The account of resemblance has many problems, not least of which is its failure to generalize to fundamental quantities that differ in their structure from mass: intensive quantities, vector quantities, quantities with both positive and negative magnitudes. For a decisive critique of Armstrong's account of resemblance between quantities, see Eddon (2007).

[10] See, for example, Armstrong (1978, p. 123). In Armstrong (1989, p. 79) he says only this: "it becomes clear why the very same thing cannot be both five and one kilogram in mass. To attempt to combine the two properties in the one thing would involve the thing's being identical with its proper part."

all (non-zero) mass determinates be instantiated by extended objects.) I conclude that Armstrong's account fails to provide an explanation for the incompatibility of determinates.[11]

Could Armstrong instead claim that it is the sharing of constituents that explains why determinates of mass are incompatible? Indeed, according to Humean recombination principles, only elements that are *distinct*, that share no parts or constituents, freely recombine. But failure of distinctness only explains necessary *connections*, not necessary *exclusions*. When elements are not distinct, the existence of (a duplicate of) one necessitates the existence of (a duplicate of) a part or constituent of the other. But I see no reason why the existence of (a duplicate of) one should necessitate the non-existence of (a duplicate of) the other, or any part or constituent of the other. If this third Humean response is to solve the problem of exclusion, it needs an explanation of incompatibility that does not rest on whether or not the determinates are distinct.

Or does it? Could Armstrong claim that he does not need to provide an explanation of the exclusions, that it is enough if his account does not entail any violations of the Humean principle that *distinct* entities freely recombine? But an account of modal plenitude is incomplete if it does not say, for entities that are *not* distinct, how they recombine. A demand on a successful account of Humean recombination is that it entails all the possibilities for recombining entities, whether distinct or not. We need, for example, an account of why *being 2 kg* and *being 5 kg* do not exclude one another if they are instantiated at different spacetime locations. More generally, any acceptable Humean account of quantities must entail:

(D3) There are no necessary exclusions between a determinate being instantiated at one spacetime location and a different determinate being instantiated at a different location.

If Armstrong's account of Humean recombination fails to entail anything about the recombination of entities that are not distinct, his Humean account of quantities will not be able to meet this demand.

Perhaps, however, there is a better version of the response that determinates are structural properties. Surely sometimes entities necessarily exclude one another in virtue of their differing structure. Consider shapes. A cube and a sphere could not be co-located: in virtue of their differing spatial structure, they make incompatible demands on the regions of space that they occupy. So a cube occupying a region necessarily excludes a sphere occupying that very same region. Similarly, if *being 2 kg* and *being 5kg* are differently structured entities, then the Humean is not committed to holding that they freely recombine. It may be that no arrangement could assign them to the same place in a world structure. Complex elements, as well as the places they occupy, must first be divided into *categories*: elements belong to the same category iff they have matching internal

---

[11] This problem for Armstrong's explanation of incompatibility is briefly raised in Lewis (1992, p. 197).

structure. We then invoke the qualified version of (HPR) that requires only that elements of the same category freely recombine: any arrangement that assigns elements to places that respect their category represents a possible world. Determinates of the same category, with matching structure, freely recombine, but not determinates of different categories. This is the natural way to apply Humean recombination to elements that are complex. Although it allows that there may be necessary connections between complex elements, the core of the Humean view is that all necessary connections *be explained*. The focus has often been put too much on explanations that invoke distinctness.[12] But explanations that invoke differing structure may be no less in accord with the Humean view.

There is a problem, however: how, on this view, can we satisfy the *desideratum*, (D1), that there are no necessary exclusions among determinates of *different* determinables, whether or not those determinates have a different internal structure? There is no way of dividing determinate properties into categories such that a determinate of one determinable freely recombines with every determinate of any other determinable without it immediately following that all determinates belong to a single category, and so freely recombine with one another.

In any case, taking determinates to be structural properties faces a decisive objection. Structural properties can only be instantiated by composite entities. But simples can have determinate quantities, at least if classical physics with its attribution of mass to point particles is not impossible. The only way to make the account of determinates as structural properties compatible with classical physics is to suppose that its point particles are really mereologically complex, indeed, have infinitely many parts. That is to demand too much. It is one thing to hold that an account of quantities demands that there be additional structure to the world; I will endorse that below. But to demand that the additional structure be *internal* to what instantiates the quantities, with the accompanying multiplication of entities, does not seem to me to be defensible. I conclude: the internal structure of determinates is not what explains the exclusions.

8. *Fourth Humean Response.* A fourth Humean response claims that *neither* the determinates *nor* the determinables are fundamental entities. In that case, (HPR) simply won't apply to quantities. Since determinates are not fundamental, there is no reason, in principle, why they cannot necessarily exclude one another. Determinates will still be properties in the abundant sense. But abundant properties, of course, can necessarily exclude one another without violating Humeanism; just consider a property and its negation.

---

[12] Indeed, if Humean recombination is not understood in terms of duplicates, and mereological essentialism is rejected, then failure of distinctness is irrelevant to explaining necessary connections. Ross Cameron (2010) has argued that the core of Humeanism is not that there are no necessary connections *between distinct existences*, but that all necessary connections *can be explained*. I concur, but would add: can be explained *without invoking primitive modality*.

What combinations of quantities are possibly co-instantiated will be a consequence of some *theory* of quantities that we posit. Let us simply stipulate that it will follow from the theory that determinates of the same determinable *do*, and determinates of different determinables *do not*, exclude one another. As long as we take this theory of quantities to be necessary, we can say that it explains the recombinatorial facts. In order to be strongly Humean, the necessity of the theory will somehow need to be reduced. But set that aside for now.

Indeed, there are many such relational accounts of quantities on the market, so-called *comparativist* views, that implement this strategy. A familiar version comes from measurement theory.[13] What is fundamental are *relations* between objects such as *x is at least as massive as y* and *x together with y is equally massive as z.* The facts about determinate properties, including how they can be represented by numbers, are then derived by way of proving representation and uniqueness theorems. On this view, an object has a determinate property, such as *being 2 kg mass*, in virtue of the mass relations it stands in to other objects in the world. We can still say that mass is fundamental in a derivative sense if the mass relations that ground the facts about mass are fundamental.

But here I sense a confusion between how we get evidence for statements about determinate mass, and what the content of those statements should be taken to be. The mass relations are indeed closer to the empirical operations by which we measure mass, for example, using balances; we measure the mass of one body by comparing its behavior to the behavior of other bodies. But that is not a good reason to take the mass relations to be fundamental. In any case, the measure-theoretic approach has unacceptable consequences. It allows that an object's mass would be different if the masses of objects elsewhere had been different. Indeed, if there are not enough objects in the world to prove the uniqueness theorem, then no object has a determinate mass. We need an account of determinate mass that makes it *local* in this sense: the determinate mass of one object does not depend upon the determinate mass of its worldmates, or even whether it has worldmates at all. More generally, I claim that an account of quantities should affirm this:

(D4) Determinates of fundamental quantities are local properties.

(D4) does not entail that the determinate quantities are intrinsic: to say that the determinate properties had by one object do not depend on the existence or nature of other objects is not to say that it does not depend on anything external to the object. Should we take the further step of affirming that determinates are intrinsic? On the one hand, if determinates are intrinsic, that would explain why material objects *within the same world* can be compared with respect to their mass independently of the masses of other material objects in that world. The mass relations would be internal relations, supervening on the intrinsic natures of their *relata* taken separately. But it would also require that material objects *in*

[13] For measurement-theoretic approaches to quantities, see Krantz, *et al.* (1970).

different worlds can be compared with respect to their mass; and that seems to me a step too far. For example, I do not allow that two worlds could differ only in that the objects of one have double the mass of the corresponding objects of the other.

How can these claims all be reconciled? Say that a relation is *world-internal* iff it supervenes on the intrinsic natures of its *relata* taken separately, *together with the structure of the world the relata inhabit and their location in that structure*. World-internal relations need not be internal because the structure they depend on is not intrinsic to the *relata*. But whether a world-internal relation holds does not depend on the intrinsic nature of any material objects beyond the *relata*. Plausible examples of world-internal relations come from physical geometry. Consider the relation of having the same orientation, or handedness. This relation is not plausibly taken to be internal: whether it holds depends on the structure of the surrounding space. A right- and left-handed glove are differently oriented in three-dimensional Euclidean space, but not if embedded in a four-dimensional Euclidean space in which one can be "flipped" onto the other. Similarly, having the same length is arguably not internal: it depends on whether the surrounding space would allow one object to be superimposed on another. For objects in different worlds, there is no fact as to whether or not they have the same length. Similarly, we can say that there may be a determinate fact as to whether objects in the same world have the same mass, or as to what the ratio is between their masses. But for objects in different worlds, there are no such determinate facts: they can only be compared by introducing some sort of counterpart relation that is based on global similarities between the worlds, or based on stipulation.[14]

This leads to the following demand on an account of quantities:

(D5) The relation *having the same determinate quantity* is world-internal, but not internal. Determinate quantities are not intrinsic.

Taking determinate quantities in general, and determinate masses in particular, not to be intrinsic is controversial to be sure. I have space here just to say a few words in its defense. Note, first, that it won't do to say that determinate quantities are *always* intrinsic because determinate *vector* quantities are not. Vector quantities have a magnitude and a direction. It might seem, intuitively, that both the magnitude and direction could be intrinsic features of a vector. But that can't be right: it makes no sense to say that vectors in different worlds are pointing in the same direction. It only makes sense to compare the directions of vectors if they are embedded in a spatial (or spatiotemporal) structure with an affine connection. Then, in terms of that connection, one can define a notion of "parallel transport" along paths connecting the locations of the vectors, and one can compare the direction of the vectors by parallel transporting one to the

[14] On the use of counterpart relations for accommodating the intuition that objects in different worlds can be compared with respect to their mass, see Dasgupta (2013).

other. Note, second, that in a non-Euclidean space whether two vectors have the same direction is *path-dependent.* Two vectors may agree in direction when brought together by parallel transport along one path, but disagree in direction when brought together by another path. Now, it seems to me clearly possible for magnitudes to be path-dependent as well. For example, it seems to me that there are worlds where two remote objects have the same mass relative to one path for transporting one object to the other, but different masses relative to another path. If a physical theory were to propose this, I would not respond: "But that's impossible!" A general account of quantity, then, needs to allow for this possibility. And that in brief is why I endorse (D5). On this approach, the intuition that mass is intrinsic can be explained by the fact that, in classical worlds, having the same mass is not path-dependent, just as an intuition that vector quantities are intrinsic can be explained by the fact that, in Euclidean worlds, having the same direction is not path-dependent. These intuitions can be accommodated within a general account of quantity that endorses (D5).[15]

Let's take stock. When objects are embedded within a world structure, they may have intrinsic properties that, together with the world structure and their location in that structure, determine their determinate quantities. For example, supposing mass is not path-dependent (as I will henceforth), the determinate mass of an object supervenes on the object's intrinsic nature and how the object is embedded in its world structure. Duplicate objects within the same world may have the same or different quantities of mass depending upon the surrounding structure in which they are embedded. On this account, whatever the determinate mass properties are, they are not fundamental. What is fundamental is the intrinsic property that together with the world structure determines the determinate mass properties. But this intrinsic property, it seems, is just the determinable *having mass*. And this leads to a rejection of the fourth Humean response, which held that neither the determinates nor the determinables are fundamental.

9. *Fifth Humean Response.* We have arrived at a fifth Humean response, some version of which I accept:

(D6) Determinables are intrinsic and fundamental.

I have argued that we can't take *both* determinates and determinables to be fundamental, and that we can't take determinates to be fundamental; but now there is no reason not to reconsider our original assumption that determinables should be analyzed in terms of determinates, and to take the determinables to be

---

[15] See Maudlin (2007, pp. 86-96) for some discussion of path-dependence within gauge theory. Maudlin claims that fundamental quantities at the actual world, such as quark colors, are path-dependent. Note that once we allow that determinate quantities may be path-dependent, we must distinguish between *local* determinates and *global* determinates. There are no global determinates for path-dependent quantities.

what is fundamental.[16] This accomplishes two things. First, it allows us to explain why any two massive objects are qualitatively similar, and any two charged objects are qualitatively similar, but an object that only had mass and an object that only had charge would not be qualitatively similar. The sharing of fundamental properties grounds qualitative similarity. Second, the determinables freely recombine, just what the Humean expects of the fundamental properties. It is possible for an object to have one or both or neither of the determinables *having mass* and *having charge*. Moreover, some of the features traditionally taken to characterize the fundamental properties apply to determinables, not determinates. For example, when we say that physics posits only a few fundamental properties, such as mass, charge, and spin, we seem to be taking the determinables, not the determinates, to be fundamental.[17]

In taking the determinables to be intrinsic and fundamental, I mean to endorse a form of *quidditism*: two worlds can be structurally isomorphic but qualitatively discernible in virtue of having different fundamental properties occupying corresponding places within the world structure.[18] The Humean could, in theory, accept combinatorialism with respect to fundamental properties without endorsing quidditism. In that case, permuting or replacing fundamental properties within a possible world would always result in a possible world—just not always a *different* possible world. But such a Humeanism would be toothless. The Humean recombination principles applied to fundamental properties would not generate *new* possibilities, just new ways of representing the same old possibilities. I will assume, henceforth, that the Humean endorses quidditism.

It should be noted that the move to take determinables to be fundamental has been driven thus far not specifically by Humean concerns, but by my own commitments to quidditism and comparativism. In the last section of this paper, however, it will be apparent that taking determinables to be fundamental is integral to the solution to the problem of exclusion that I prefer.

Claiming that it is the determinables that are fundamental, and not the determinates, is just the beginning of this fifth response. The Humean still needs an account of determinate quantities, and needs to show how the account

---

[16] Wilson (2012) defends the view that determinable properties are fundamental, although she allows that determinates may be fundamental as well.
[17] Hawthorne (2006) discusses ways in which Lewis's characterization of fundamental, or perfectly natural, properties sometimes better applies to determinables rather than determinates. But I don't think there can be any doubt that Lewis's considered opinion was that the determinates are fundamental. Lewis (2009, p. 204) gives this characterization of the fundamental properties: "They are not at all disjunctive, or determinable, or negative."
[18] I distinguish *quidditism* from the weaker *haecceitism about properties* which holds only that structurally isomorphic worlds may differ by a permutation of properties, not that they may differ qualitatively. See Hildebrand (2016) for discussion. (He calls the two views *qualitative quidditism* and *bare quidditism*.)

explains the necessary exclusions. And it is natural at this point to try this: determinates arise from the *way* in which the fundamental determinables are instantiated, where the different ways of being instantiated correspond to the determinable occupying different locations in the world structure. But now it is high time I say more about world structures.

10. *World Structure: Horizontal and Vertical.* For any possible world, we can ask what the (total) structure of that world is. A complete description of the world is given by saying how the fundamental elements, whatever they may be, are arranged in the world structure. This shouldn't be controversial. Indeed, world structures are tacitly presupposed by all recombination principles. It isn't enough to say just that, for any elements, there is a world in which those elements all coexist. To capture the full plenitude of possibilities, one must also say that, for any *arrangement* of the elements, there is a world that arranges those elements in that way. And talk of arrangements only makes sense against a backdrop of structures within which to do the arranging.

What is controversial is how world structures are to be understood. In particular, one can ask: Does the world structure supervene on facts about the relations among the elements? Or is structure an independent feature of worlds, in which case places in the structure may be unoccupied by any element? To grasp what is at stake, consider the case of spatial structure and responses to the problem of *empty space*, that is, space unoccupied by matter or energy. I suppose that it is possible for a Euclidean world to have a region of empty space. The problem is: in virtue of what can we say that that empty region is itself Euclidean in shape? There are three main views. One view grounds the shape of empty space in brute modality of some sort: *were* an object located in that empty region, it *would* be Euclidean in shape. But that view, of course, is anathema to Humeans. A second view grounds the shape of empty space in the spatial relations, not between material objects, but between unoccupied points of physical space that compose the empty region. To be empty of matter is not to be empty of any substance. This is the familiar response of the substantivalist about space. It upholds the supervenience of structure by positing additional elements. A third response grounds the shape of empty space in irreducible facts about the structure of the world. On this view, the world structure does not supervene on relations among the elements; structures are *sui generis* and abstract.[19] Here I will remain neutral between these two views of world structure. Either view is compatible with an epistemology of geometry according to which we may have good reason to posit facts about the geometric structure of empty space. Similarly, either view is compatible with an epistemology of quantities according to which we have good reason, as I

---

[19] The distinction between these latter two views tracks the debate between *in re* and *ante rem* structuralism in the philosophy of mathematics; see Shapiro (1997). There is also a mixed view according to which points of physical space exist only where no matter or energy exists.

suppose, to posit a quantity structure whose places may or may not be occupied by determinable elements.

The spatial or spatiotemporal structure of a world are instances of what I call *horizontal* structure, the structure that unites the objects of a world to make a single world. In our world, according to general relativity, it is four-dimensional spatiotemporal structure that is locally Minkowskian, but with variable curvature. According to Lewis, the horizontal structure had by *all* worlds is spatiotemporal structure, structure determined by fundamental relations that are either spatiotemporal, or analogous to the spatiotemporal relations. I have argued elsewhere (Bricker (1996)) that this conception of horizontal structure is too narrow. There are possible worlds whose horizontal structure is not spatiotemporal but is instead provided by other sorts of fundamental external relations. But I will continue to focus on worlds with spatiotemporal structure in what follows.

Vertical structure is, perhaps, less familiar; but it must be posited by any account of the world that is realist about properties, and so allows that there are multiple elements located at a single point of spacetime. Vertical structure unites the properties, or the properties and a particular, into a single object. In the simplest case, the vertical structure could just be a cardinal number assigned to each point of spacetime that represents how many properties are, or might be, instantiated at that point. (Compare Armstrong's metaphor of a "layer cake", where the properties are "stacked up" one on top of the other.) If the properties in question are scalar quantities, however, more structure is needed: at each spacetime point, we need the structure of the non-negative reals to give the magnitude of the determinable that is instantiated at that point. For vector quantities, it seems, we need even more vertical structure: at each spacetime point, we need a four-dimensional space in which vectors can live, and have determinate magnitude and direction, the tangent space associated with the point. And for more complex quantities, perhaps more structure is needed. But let us focus, for now, on the case of (extensive) scalar quantities.

The simplest way to think about how vertical and horizontal structure make up the world structure is to take the world structure to be the *product* of vertical and horizontal structure. For example, if horizontal structure is a four-dimensional spacetime and vertical structure (to account for scalar quantities) is a one-dimensional half real line, then the location of a scalar quantity is given by giving five coordinates: four to give where the quantity is located in spacetime, and one to give its magnitude. Thus, taking account of vertical structure (for scalar quantities) makes the world structure five-dimensional. More generally, every location in the world structure can be divided into two components, a location in the horizontal structure and a location in the vertical structure.

This simple product account of horizontal and vertical structure, however, is insufficiently general; it posits too much structure. On this account, whether or not two instantiations of a determinable at different spacetime points have the same magnitude supervenes on the locations of those two instances. I want an account of world structure according to which locational properties carry less information.[20] Such an account must distinguish between the *local* vertical structure, the structure at each spacetime point, and a *global* structure that connects all the local vertical structures. The local structure, we are supposing, must at least have the structure of the half real line so as to capture the magnitudes of extensive quantities. The connection then tells us when one point of a half real line in one local structure is to be identified with a point of a half real line in another local structure, thereby calibrating units across different points of spacetime. But without the calibration, there are no facts as to whether, when the mass determinable is instantiated at two different spacetime points, the determinate masses are the same or different. In other words, the relation, *having the same magnitude*, does not supervene on the locational properties of its *relata*. Now, in terms of this account of world structure, we can give an analysis of what it is for a point object *o* located at a point of spacetime *p* to have the determinate property, *having magnitude m*, of some determinable *d*. Identify magnitudes with classes of vertical locations that, according to the connection, have the same value. Then, *o* has *m* of *d* iff, for some vertical location *v*, *d* is located at <*p*, *v*> and *v* is in *m*.[21]

How does this account relate to comparativist accounts that take relations such as *having the same mass*, *having the same charge*, etc. to be primitive relations holding between objects or properties? For one thing, the relevant relations on my account hold between locations in the world structure, and so do not depend on the existence of objects or properties to provide the *relata.* Moreover, speculatively, I would argue that, at each spacetime point, there is a single local vertical structure in which all quantities live, and there is a single global connection; there are not separate structures and relations for each quantity. Thus, *having the same mass* reduces to *having mass* and *having the same magnitude.* Even more speculatively, I would identify the single local vertical structure with the tangent space associated with a point of spacetime interpreted as the infinitesimal neighborhood of that point, and the global connection with the connection as characterized by the spatiotemporal structure. On this account, the vertical structure is reinterpreted as an enhanced horizontal structure; at worlds with spacetime, all world structure is spatiotemporal.[22] Much more would need to be said about the conceptual and

---

[20] Often it won't matter to the quantitative truths at the world because that information will be available by other means. But when comparisons of quantities are path-dependent, it matters.

[21] Note two things. First, the magnitudes are not numbers; rather, we *represent* a magnitude by a number, its coordinate, by conventionally choosing one of the magnitudes to serve as the unit. Second, if a quantity is path-dependent, then there are no determinate magnitudes for that quantity. In that case, we can say there are *local* determinates in virtue of the determinable for that quantity occupying a location in the vertical structure at spacetime points; but there are no *global* determinates for that quantity.

[22] I argued for this view on somewhat different Humean grounds in Bricker (1993).

technical underpinnings of such as account. But for reasons of space, I need now to return to the problem for Humeanism with which we began: the problem of exclusion.

11. *Refining the Fifth Response: Identifying Arrangements.* Whatever the merits of the account I have given of determinable and determinate quantities, with respect to the problem of exclusion it appears to be a total failure. The Humean must claim that the fundamental properties, the determinables, can be freely recombined within the world structure. And that gives the wrong results. For there are multiple distinct vertical locations corresponding to each location in spacetime, and the Humean will be committed to saying that a single determinable can occupy more than one such vertical location. But that would correspond to a location of spacetime instantiating different determinates of a determinable. We have made no progress with the problem of exclusion. We have no explanation, for example, for why the determinable of mass can occupy different horizontal locations, but not different vertical locations at the same horizontal location.

The Humean seems to be in a bind: either there are multiple locations at a spacetime point for the determinable to occupy, or there is only one. If there are multiple locations, then it seems that the Humean is committed to allowing different determinates of a single determinable to be instantiated at a spacetime point. If there is only one, then it seems the Humean will have to posit fundamental relations holding between instances of a determinable at different spacetime points to capture the ratios of their magnitudes, relations such as *having the same mass as*, *having twice the mass as*, etc. But then these posited fundamental relations will violate (HPR): if two instances of the mass determinable stand in one mass ratio, they cannot stand in any other. Or perhaps, to avoid a violation of (HPR), the Humean could go adverbial: the uncountably many ways for the mass determinable to be instantiated at a spacetime point do not correspond to different recombinations of the elements. But one still needs modality to say why, if a mass determinable is instantiated *2-kg-ly* at a spacetime point, it cannot be instantiated *3-kg-ly* at that point. Strong Humeanism is still threatened.

Let's take stock once again. The problem with the present account is that there are too many arrangements, resulting in too many possibilities. Now, there are two general strategies for cutting down on the arrangements. The most obvious strategy is to rule out some arrangements as impossible. That strategy immediately raises the specter of primitive modality. I will return to it below. But there is a different general strategy that I want to consider first. On this strategy, we say that distinct arrangements need not correspond to distinct possibilities. This is no violation of (HPR) which says only that each arrangement corresponds to *some* possibility. The arrangements, after all, were introduced to *represent* possibilities; they are not the possibilities themselves. On this strategy, we can say that we are "identifying" arrangements that correspond to the same possibility.

It will be useful first to see how this strategy of identifying arrangements is needed to solve a different problem with the present account. The problem is this. On some arrangements, determinables for *different* quantities will occupy the same place in the vertical structure at a spacetime point (assuming, as I do, that there are not separate vertical structures for the different quantities). But that is too much information: there isn't any genuine fact as to whether, say, the mass at one point does or does not have the same magnitude as the charge at that point; and the global connection gives no information as to how the magnitude of mass at one point compares to the magnitude of charge at another point. To eliminate this fake news, we need to identify arrangements that differ only in that all the instantiations of a single determinable have had their magnitude uniformly scaled up or down. Arrangements that differ only in this way represent the same possibility.[23, 24]

Now, the thing to notice about this solution for our purposes is that it cuts down on the arrangements without invoking modality. In the course of identifying arrangements, no arrangement is deemed impossible. Can something similar work to solve the problem of exclusion? Call an arrangement *bad* if it ever assigns a determinable to multiple places at a single spacetime point; call it *good* if it doesn't. What we need is a way to divide the arrangements into equivalence classes so that every bad arrangement gets identified with exactly one good arrangement. To illustrate: consider the case where the vertical structure at each spacetime point is a half real line. Then we could say: arrangements are equivalent iff, for any spacetime point and any determinable, the magnitudes assigned to the determinable at the point by the arrangements have the same greatest lower bound. We can then say that equivalent arrangements represent the same possibility, the possibility represented by the good arrangement that assigns each determinable at each point to that greatest lower bound. So no possibilities are represented in which a determinable takes

---

[23] *Note bene*. If different arrangements represent the same possible world, then the underlying structure of the world is not the same as the structure within which we do the arranging. In this case, the underlying world structure is sensitive to the number and types of quantities instantiated at the world. I see no way to avoid this consequence without an unfortunate multiplying of possibilities.

[24] The strategy of identifying arrangements can also be used to solve the following problem: how can a single vertical structure be used to represent both scalar and vector quantities? For the vector quantities, we need the vertical structure at a spacetime point to be the tangent space at the point. But that is too much structure for the scalar quantities. If we assign a scalar determinable to a location in the tangent space, we wrongly attribute a direction to the scalar quantity. But the strategy in question gives us a natural way, in effect, to ignore the direction. We can identify arrangements that assign a scalar determinable to locations in the tangent space with the same magnitude but with different directions.

on multiple values at a single spacetime point. The problem of exclusion is thus solved without invoking modality.

Or is it solved? I hope it is obvious that, in this case, the method is a cheat. What we have is a gimmick for ignoring bad arrangements. There is no natural sense in which the bad arrangements represent the same possibility as the good arrangement. The proffered representation relation between arrangements and possibilities depends on an arbitrary choice. Moreover, when we switch to consider vector quantities, there will be nothing analogous to the greatest lower bound that would allow this method to be adapted to the problem of exclusion for directions. The strategy of identifying arrangements will not solve the problem of exclusion.

12. *Refining the Fifth Response: Omitting Arrangements.* Time for the Humean to face the music. The Humean must remove the bad arrangements from the scope of (HPR). And that implicitly invokes modality. We can say *which* arrangements are bad without invoking modality, but not *why* they are bad. What makes an arrangement bad is that it is not metaphysically possible; it doesn't represent the way any possible world could be. But all is not lost: to invoke modality is not to be committed to *primitive* modality. The Humean does not reject modality when it can be reduced to logic and definitions. And the Humean can allow that there are substantive, objective facts as to what the true logic is, and what the linguistic conventions are.

Indeed, I am a Humean with a robust conception of logic, and a robust notion of absolute modality that comes with it. I am a realist about mathematical systems and possible worlds and mixtures and generalizations of the two. Whatever is not ruled out by logic (in a broad, non-formal sense of 'logic') exists somewhere in reality.[25] Quantifying over this expansive reality leads to a Humeanly acceptable notion of absolute modality according to which only logic and definitions are absolutely necessary. The question before us is: does this realist account make the exclusions between determinates absolutely necessary? That depends. There are two ways for the Humean to approach this question. One approach takes the exclusions to be *metaphysically* necessary in virtue of conventions that govern the meaning of 'metaphysical necessity'; but they are not *absolutely* necessary. The other approach understands logic in a way that makes the exclusions absolutely necessary. I consider these approaches in turn.

On the first approach, it is not a deep fact that determinates necessarily exclude one another; it tracks no joint in reality. It is a superficial fact that reflects a particular conventional definition of 'necessity', what philosophers refer to as 'metaphysical necessity'. Some philosophers, ancient and modern, have thought that there are deep facts about 'metaphysical necessity' having to do with real essences, and what not; but they are deluded in this, and the delusion has become widespread. The Humean succumbs to no such delusion.

---

[25] See Bricker (forthcoming b). One thing that makes my logic "broad" is that it includes mereology and higher-order plural quantifiers. Another is introduced below.

Starting from absolute necessity, she can define a restricted notion of necessity to match these deluded philosophers' usage. That determinates necessarily exclude one another can simply be a part of this definition, and in that way be true by convention.

I need this strategy to be available in any case, whether or not I use it to solve the problem of exclusion. My notion of absolute modality does not agree with any notion of modality typically used by philosophers or the folk. On my notion, much more is possible, much less is necessary. Mathematical truths, for example, are true at some mathematical systems and not others, and therefore not absolutely necessary. Moreover, I believe that there are world-like entities that have any sort of mathematical structure as their horizontal structure. The existence of such entities is *absolutely* possible—they are part of reality—but their existence would not be deemed *metaphysically* possible by most modal metaphysicians. To speak their language, I need to define a notion of modality that matches their usage.[26] Why not just include as part of the meaning of 'metaphysical necessity' that determinates of a determinable necessarily exclude one another? In that way, I get the necessary exclusions for cheap.

Note that this is a version of the second Humean response considered above: the exclusions are conceptually necessary, following from conventional definitions, but not absolutely necessary. There are world-like entities in reality at which, say, the determinable of mass is badly arranged. But since these world-like entities are beyond the range of our defined necessity operator, we can truly say that, necessarily, determinates of a determinable exclude one another. Moreover, because the necessity of good arrangements is analytic to 'necessary', not to 'determinable' or to terms for quantities such as 'mass', we can truly say *de re* of a determinable that it necessarily never has two determinate values at a single point.[27] What more could a Humean want?

I am not content. The badly arranged world-like entities are now deemed impossible by definition, but they are still a part of reality. And so it still makes sense to wonder whether I am located in one of them. I cannot know *a priori* that that is not the case, and so this response fails to meet (D2), one of the *desiderata* on an acceptable account. Nor would it help to say: I know I am located in a *possible* world *a priori* in virtue of knowing *a priori* that I am actual and that whatever is actual is possible. For on the view in question, I cannot know *a priori* that whatever is actual is possible. Would it help if the Humean rejects my realism about the absolutely possible? Realism makes the problem more vivid, perhaps. But any Humean who accepts a robust notion of absolute possibility according to which the bad arrangements are absolutely possible will be in my boat; the bad arrangements cannot be ruled out *a priori*. At best, only a Humean who takes *all* logic and modality to be conventional should consider availing herself of this response.

---

[26] That is what I took myself to be doing, for example, in Bricker (1991).

[27] Sider (2005, pp. 684–5) makes a similar point in connection with Armstrong's reductive account of modality.

That leaves the second approach: take the logic that governs the instantiation of determinables to be the source of necessity. On this approach, we don't divide arrangements into the good and the bad and then throw out the bad. We use a non-traditional logic to reconfigure what the possible world structures are so that it follows that there are no bad arrangements. The problem of exclusion turns out to be an artifact of our reliance on traditional logic. It results from the overly restrictive assumption that properties (and relations) are all or nothing, that there is only one way for a property to be instantiated. Instead, I propose, we should take determinables to belong to different logical categories depending on how they are instantiated. In the simplest case, determinables of extensive scalar quantities, we can make use of a many-valued logic whose values come from the non-negative reals. Thus, the determinable of mass is assigned a value at each spacetime point that, in ways already discussed, gives rise to a determinate magnitude. The exclusions come, not from any linguistic conventions, but from logic itself.[28]

I expect the following objection. "The problem of exclusion has not been solved, but merely shifted to another domain. We can now ask: why is it necessary that a determinable have *exactly one* value assigned by the logic relative to a point in the domain? What explains that necessity?" But this objection misunderstands the Humean's commitments. It was no part of Humeanism that combinatorialism be applied even to the elements of logic, thereby generating alternative logics. No Humean principle of recombination entails, for example, that a proposition can be both true and false. The Humean is entitled to hold, as I do, that there is one true logic, and that the values assigned by that logic hold exclusively. Since a determinable having multiple values relative to a point in the domain is ruled out by logic, it is absolutely impossible.

There is a terminological issue that needs to be resolved. Up to now, 'determinable' has been taken to refer to all-or-nothing properties. These all-or-nothing properties are still around (on an abundant conception), but they are not what our present account takes to be fundamental. So perhaps it would be better to call the fundamental, many-valued elements *ur-determinables*: they ground the instantiation of the all-or-nothing determinables that, necessarily, are co-instantiated with them. Terminology aside, what matters is that the fundamental quantities, on the present account, are of a different logical type than the fundamental quantities posited by traditional accounts.

Will this account generalize to vector quantities, and more complex sorts of quantity? I see no reason why not. The values that the logic assigns to a determinable, say, of a vector quantity at a point of spacetime will have both a

---

[28] Interestingly, this appears to be Wittgenstein's solution to the problem of exclusion, in which case he was not abandoning Humeanism (as I understand it) when he abandoned logical atomism. He claims that the truth table assignment <*t*, *t*> to two elementary propositions that exclude one another will be seen not to be a possible assignment once we have developed the proper logic, the proper "syntax" of the elementary propositions. See Wittgenstein (1929, pp. 169–171).

magnitude and a direction; there will be not just a continuum of values, but a space of values with a more complex structure. Whatever vertical structure we posited previously to account for some quantity can now be taken to derive from the logic of the determinable associated with that quantity.

There are delicate and substantial questions, however, that I leave here unanswered, questions as to how logic relates to world structures, and, in particular, as to how vertical and horizontal structure are integrated with one another. For this approach to work, the Humean must hold that logic alone determines what world structures, and what arrangements of elements within those structures, are absolutely possible. Answers to these questions about logic and structure will be needed to configure the boundary of the absolutely possible.

Acknowledgements

Versions of this paper were presented at the Seven Hills Workshop at the College of Holy Cross and at a conference, "Modal Metaphysics: Issues in the (Im)Possible," before the Slovak Metaphysical Society in Bratislava. For comments and discussion, I thank Cameron Gibbs, Chris Meacham, and Alejandro Perez-Carballo.

References

Armstrong, D. M. (1978). *Universals and Scientific Realism (Volume 2).* Cambridge: Cambridge University Press.

——— (1989). *A Combinatorial Theory of Possibility.* New York: Cambridge University Press.

Bell, John L. and William Demopoulos (1996). "Elementary Propositions and Independence." *Notre Dame Journal of Formal Logic* 37: 112–24.

Bricker, Phillip (1991). "Plenitude of Possible Structures." *Journal of Philosophy* 88: 607–19.

——— (1993). "The Fabric of Space: Intrinsic *vs*. Extrinsic Distance Relations." *Midwest Studies in Philosophy* 43: 271–94.

——— (1996). "Isolation and Unification: The Realist Analysis of Possible Worlds." *Philosophical Studies* 84: 225–38.

——— (2008). "Concrete Possible Worlds." In *Contemporary Debates in Metaphysics*, edited by Theodore Sider, John Hawthorne, and Dean Zimmerman, 111–34. Blackwell.

——— (Forthcoming a). "Principles of Plenitude." In *Modal Matters: Essays in Metaphysics*. Oxford: Oxford University Press.

——— (Forthcoming b). "Realism Without Parochialism." In *Modal Matters: Essays in Metaphysics*. Oxford: Oxford University Press.

Cameron, Ross (2010). "From Humean Truthmaker Theory to Priority Monism." *Noûs* 44: 178–98.

Dasgupta, Shamik (2013). "Absolutism vs. Comparativism about Quantity." In *Oxford Studies in Metaphysics* 8, edited by Karen Bennett and Dean Zimmerman. Oxford: Oxford University Press.

Eddon, Maya (2007). "Armstrong on Quantities and Resemblance." *Philosophical Studies* 136: 385-404.

Hawthorne, John (2006). "Quantities in Lewisian Metaphysics." *Metaphysical Essays*, 229–38. Oxford: Oxford University Press.

Hildebrand, Tyler (2016). "Two Types of Quidditism." *Australasian Journal of Philosophy* 94: 516-32.

Krantz, D., Luce, R., Suppes P., and Tversky, A. (1970). *Foundations of Measurement, Volume 1.* New York: Academic Press.

Lewis, David (1986). *On the Plurality of Worlds.* Oxford: Blackwell.

——— (1992). Critical Notice of Armstrong, *A Combinatorial Theory of Possibility*, *Australasian Journal of Philosophy* 70: 211-24. Reprinted as "Armstrong on Combinatorial Possibility" in *Papers in Metaphysics and Epistemology* (1999): 196-214. Cambridge: Cambridge University Press.

——— (2009). "Ramseyan Humility." In *Conceptual Analysis and Philosophical Naturalism*, edited by D. Braddon-Mitchell and R. Nold, 203-22. Cambridge, MA: MIT Press.

Maudlin, Tim (2007). *The Metaphysics Within Physics*. Oxford: Oxford University Press.

Rosen, Gideon (2010). "Metaphysical Dependence: Grounding and Reduction." In *Modality: Metaphysics, Logic, and Epistemology*, edited by Bob Hale and Aviv Hoffmann. Oxford: Oxford University Press.

Russell, Bertrand (1918). *The Philosophy of Logical Atomism.* In *Logic and Knowledge: Essays 1901-1950*, edited by R. Marsh, 1956. London: George Allen and Unwin.

Shapiro, Stewart (1997). *Philosophy of Mathematics: Structure and Ontology*. Oxford: Oxford University Press.

Sider, Theodore (2005). "Another Look at Armstrong's Combinatorialism." *Noûs* 39: 679-95.

Wilson, Jessica (2012). "Fundamental Determinables." *Philosophers' Imprint* 12.

Wittgenstein, Ludwig (1929). "Some Remarks on Logical Form." *Proceedings of the Aristotelian Society, Supplementary Volume IX*: 162-71.