All Worlds in One: Reassessing the Forrest-Armstrong Argument (2011/2018)[*]

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1. INTRODUCTION

In 1984, Peter Forrest and David Armstrong published "An Argument Against David Lewis' Theory of Possible Worlds." In that article, they assume that Lewis is committed to an unrestricted principle of recombination. When the principle is applied to Lewis's plurality of non-overlapping possible worlds, it leads to the conclusion that "there can be neither the aggregate, nor the set, of all possible worlds" (Forrest and Armstrong: 164). David Lewis responds to the argument in *On the Plurality of Worlds* (Lewis 1986: 101-4). Because he accepts that there is both an aggregate and a set of all possible worlds, he recasts the argument as a *reductio* against the unrestricted principle of recombination. The argument, he thinks, is sound. But he denies that it targets *his* theory of possible worlds because he accepts only a qualified version of the principle of recombination. Lewis concludes: "The Forrest-Armstrong *reductio* stands; but a *reductio* against the unqualified principle of recombination is not a *reductio* against my theory of possible worlds, as they say it is. I do not accept the unqualified principle, and there is no strong reason why I should" (p. 104).

In 1996, Daniel Nolan published an influential critique of the Forrest-Armstrong argument. He pointed out, correctly, that there is a gap in the argument, a gap that it appears can only be bridged by assuming some form of haecceitism. But Lewis, of course, is no haecceitist, and so the argument has no force, Nolan claims, against a Lewisian theory that accepts unrestricted recombination. (This leaves a residual question: how could Lewis have thought the

argument was sound?) Nolan thus concludes that Lewis need not qualify the principle of recombination; and that is all to the good since Lewis's qualification appears to be unacceptably *ad hoc*. Nolan writes: "The realization that Forrest-Armstrong style cardinality arguments are not successful allows us to have a simpler and more intuitive principle of recombination …" (Nolan 1996: 259). Nolan's verdict appears to have gained widespread acceptance, and there was no serious discussion of the Forrest-Armstrong argument in the following fifteen years.[1]

I do not think, however, that Nolan's conclusion withstands scrutiny: Forrest-Armstrong style cardinality arguments cannot be so easily dismissed. There is indeed a crucial gap in the argument for an anti-haecceitist. But that gap can be bridged in various ways, by invoking various plausible principles of plenitude. Moreover, the argument cannot be disarmed—as Forrest and Armstrong suggest—by accepting that there is no aggregate or set of possible worlds; for the argument can be formulated without assuming that there is such an aggregate or set as long as one allows irreducibly plural quantification over the possible worlds.

Here is the plan. In Sections 2 and 3, I present some needed background on Lewisian realism and principles of plenitude. In Sections 4 and 5, I present (Lewis's version of) the Forrest-Armstrong argument, focusing on the crucial gap in the argument. In Sections 6 through 8, I consider ways that the Forrest-Armstrong argument can be fortified and recast. I conclude, in agreement with Lewis, that the unrestricted principle of recombination must be rejected by a realist about concrete possible worlds. But it is not enough simply to switch to a qualified principle of recombination. For the qualification will do no good if one endorses an overly liberal account of what structures are possible, that is, what structures are instantiated in some possible world. Unless there is a principled way to say why intuitively possible structures are not genuinely possible, the resulting theory will appear *ad hoc*. In Section 9, I consider the prospects

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[1] That is, no serious discussion when a first (incomplete) draft of this chapter was written in 2011, and presented at conferences. Since then, however, there have been a number of important publications on the topic of recombination and paradox. See, for example, Hawthorne and Uzquiano (2011), Uzquiano (2015), Fritz (2017), and Hawthorne and Russell (2018). When I revised the first draft for publication in this volume in 2018, I added Section 10 which briefly responds to the arguments in Hawthorne and Uzquiano (2011). But I did not take into account Hawthorne and Russell (2018) which I had just become aware of. That far-ranging paper discusses some of the issues of this chapter in a more general and formal setting. It also has the effect of vindicating Forrest-Armstrong style arguments against unrestricted recombination by invoking (what I call) plenitude of structures, something I do in Section 7.

for providing such an explanation. In Section 10, I critically examine arguments in Hawthorne and Uzquiano (2011) against the position that I endorse. Finally, in Section 11, I discharge assumptions and state my conclusions.

2. LEWISIAN REALISM: THE STRUCTURE AND SIZE OF (CONCRETE) REALITY

For most of what follows, I will assume Lewis's version of realism about possible worlds as presented in *On the Plurality of Worlds.* My own version of realism about possible worlds differs in substantial ways from Lewis's. (See Bricker 2008 for a summary of some of these differences; see also Bricker forthcoming b, §3.3 for a further difference that is relevant below.) Most of these differences, however, do not affect the main thrust of the Forrest-Armstrong argument. I will mention them in passing when they may be relevant. By the time I get to Section 9, however, I will be focusing on my own view rather than Lewis's. Other realist accounts of possible worlds also succumb to the argument if they endorse an unrestricted recombination principle, adapted to fit their view. But in this chapter, I set them aside.

Lewisian realism about possible worlds starts with the following three assumptions. First: *worlds are concrete.* Lewis shied away from using the term 'concrete' owing to its ambiguous usage by philosophers. But for what follows it will be enough if we take the claim to include the following: worlds are individuals, not sets or classes; worlds are particulars, not properties or universals; and worlds are fully determinate in all qualitative respects. Call the fusion of all the worlds *modal reality*. Second: *worlds are maximal spatiotemporally interrelated wholes.* We can then say that a *possible individual* is any spatiotemporal part of a world. (Lewis (1986) is agnostic as to whether there are also non-spatiotemporal parts, such as universals and thin particulars, or tropes.) Worlds, then, are themselves possible individuals, the biggest possible individuals. Every world has a *spacetime structure* determined by the spatiotemporal relations among its parts. We can say that the possible individuals in a world occupy *places* in the world's spacetime structure.[2] Third: *worlds do not overlap* at least in this sense: no *possible individual* is part of more than one world.

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[2] How this second assumption is to be understood will depend crucially on what structures count as "spatiotemporal." Lewis calls the relevant structures "analogically spatiotemporal" and gives

I will also need to make assumptions, following Lewis, about the framework. First, I assume with Lewis that plural quantification is not reducible to singular quantification over sets or classes (or "set-like" entities), and that one can quantify, plurally or singly, over absolutely everything whether or not there is a universal class (or set-like entity). Second, I assume classical mereology, and in particular, unrestricted composition. Lewis (1993) calls the framework that adds plural logic and classical mereology to first-order predicate logic *megethology*. Third, I assume (second-order) Zermelo-Fraenkel set theory with choice (ZFC).[3] The pure sets that serve as cardinal numbers in ZFC—the natural numbers and the alephs—provide the measure of size for pluralities of things, such as (mereological) atoms or worlds. A plurality of things has size κ iff the things are in one-one correspondence with the members of κ. A plurality of things *do not form a set*, and do not have a cardinal size, iff, for any cardinal number κ, there is a subplurality that is in one-one correspondence with the members of cardinal number κ.[4] When a plurality do not form a set, I will say that they are *many;* when a plurality do form a set, I will say that they are *few*.[5] Fourth, I assume that there is an objective relation between possible individuals of being *intrinsic duplicates*. This relation, as we will see, is crucial to understanding Lewisian principles of recombination.

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four conditions: the structure is based on relations that are *natural*, *pervasive*, *discriminating*, and *external*. See Lewis (1986: 75-6) for the details.

[3] I do not, however, take ZFC to be a foundation for mathematics. Rather, the mathematical systems in which (second-order) ZFC holds true are just some among the myriad mathematical systems. Moreover, I (speculatively) interpret all ordinary and philosophical talk of impure sets in terms of higher-order plural logic: see Bricker (forthcoming a, §2). But nothing I say about impure sets here will depend on accepting this interpretation.

[4] Two important notes. First, because second-order ZFC is not categorical, quantification over the cardinals should, I think, be understood to involve implicit quantification over systems (or models): for some system of ZFC, for some cardinal in that system … . I do not suppose that there is any one system whose cardinals are all the cardinals. Second, I will assume that talk of one-one correspondence, with its implicit quantification over relations, makes good sense even for pluralities that do not form sets. I want to stay non-committal about how best to understand such quantification over relations. But the methods developed in the appendix to Lewis (1991) establish that this can be done within megethology, without any dependence on sets or classes.

[5] Although 'plurality' is grammatically singular in English, I distort English grammar and use the pronouns 'they' and 'them' as if it were grammatically plural. This is to emphasize that pluralities are not set-like entities. To speak of "a plurality of atoms" is just to speak plurally of some atoms.

To simplify the presentation, I will make two further assumptions; I will revisit these assumptions at the end of the paper to check whether they affect my conclusions. First, I will assume that there are no universals or tropes or other non-spatiotemporal parts of worlds. That rules out co-located universals or tropes or entities of any sort. If a possible individual has no other possible individuals as parts, then it has no proper parts and is a mereological atom. Second, I will assume there are worlds composed entirely of atoms, and I will ignore worlds (if any) that contain atomless gunk. (An object is *atomless gunk* if and only if every one of its parts has proper parts.) That allows us to measure the *size* of any thing—be it a world, or modal reality as a whole—by the number of atoms it contains. I will say that a thing is *large* iff its atoms are many; *small* iff its atoms are few. Note well that, given unrestricted composition, there will always be more parts of a (composite) thing than atoms in the thing, more fusions of atoms than atoms. In particular, there are more possible individuals in a world than there are atoms in the world. If there are κ atoms in a world, then there are $2^{\kappa} - 1$ possible individuals, and by Cantor's theorem, $2^{\kappa} - 1$ is greater than κ, for any cardinal κ greater than 1. Thus, the size of a world cannot be identified with the number of possible individuals that exist in the world.

Given these assumptions, what does Lewisian realism say about the shape and size of modal reality? With respect to shape, Lewisian realism accepts what I call *Structural Pluralism* in the following restricted form: modal reality divides into *spatiotemporally* isolated components—the worlds—whose parts stand in no spatiotemporal relations to one another. Each world has a spacetime structure, but no spacetime structure encompasses multiple worlds, let alone modal reality as a whole. Lewisian realism does not, however, rule out that some *non-*spatiotemporal structure encompasses multiple worlds, or modal reality as a whole. My own realism accepts an unrestricted version of *Structural Pluralism*: modal reality divides into *absolutely* isolated components whose parts stand in no (non-logical) external relations to one another. Worlds are maximal *externally* interrelated wholes.[6] Which version of structural pluralism is accepted affects how some principles of recombination are interpreted, and this in

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[6] See Bricker (1996) for a defense of this view. Some of these isolated components might not properly be called "worlds"—see Bricker (forthcoming a)—but that won't alter any of the conclusions of this chapter.

turn may be relevant to an evaluation of the Forrest-Armstrong argument (see Section 6). But none of my main conclusions will depend on which version of structural pluralism is accepted.

What about the size of modal reality? Given our assumptions, the size of modal reality is determined by two factors: how many worlds there are and how many atoms there are in each world. With respect to the first, we can ask: do the worlds form a set? If so, what is that set's cardinality? Lewis (1986) maintains that the worlds form a set, but the reasons he gives are not very convincing (see Section 9 below). In Bricker (1991) (and, more briefly, in Section 7 below) I argue that it follows from a plausible principle of plenitude for possible structures that there is no set of worlds.[7] But, contrary to what some have thought, this does nothing to disarm Forrest-Armstrong style arguments. (See Section 8 below.) With respect to the second factor, we can ask: do the atoms in any world form a set? If so, is there a cardinal bound on the size of worlds? The two factors that determine the size of modal reality are *logically* independent of one another. For example, modal reality could contain only one world whose atoms do not form a set; or modal reality could contain too many worlds to form a set, each of which is itself one atom. But plausible principles of modal plenitude may constrain how these questions about size can be jointly answered. That indeed, I claim, is the chief lesson of the Forrest-Armstrong argument. I will argue that the best response to the argument is to endorse the following combination of answers: there is no set of worlds, but for any world, there is a set of the atoms (and therefore the possible individuals) that compose that world.[8]

3. PRINCIPLES OF PLENITUDE FOR WORLDS

If possible worlds are to serve in an analysis of modality, there will have to be enough of them: for any way a world could possibly be, there will have to be a world that is that way. Otherwise,

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[7] Cf. Nolan (1996: 246-7) who argues on the basis of an unrestricted principle of recombination (once he has dismissed the Forrest-Armstrong argument) that there is no set of possible individuals; but he thinks this leaves it open whether or not there is a set of worlds.

[8] I hold that modal reality is only one part of reality. There is also mathematical reality composed of isolated mathematical systems in much the same way that modal reality is composed of isolated worlds. (See Bricker forthcoming a.) What I say about the shape and size of modal reality I also say about mathematical reality: there is no set of mathematical systems, but, for any system, the elements in the domain of that system form a set.

there will be possibilities that lack worlds to represent them. Lewis asks: what general principles would be sufficient to guarantee an appropriate plenitude of worlds? He focuses on what he calls a "principle of recombination," which he initially states, roughly, as "anything can coexist, or fail to coexist, with anything else" (Lewis 1986: 88). The principle naturally divides into two independent claims:

> *Denial of Necessary Exclusions* (DNE). Anything can coexist with anything else.

> *Denial of Necessary Connections* (DNC). Anything can fail to coexist with anything else.

In Bricker (forthcoming b), I consider precise generalizations of each of these claims. For now, it will be enough to focus on two ways of making (DNE) more general and precise. (DNC) will have a role to play later when we evaluate the Forrest-Armstrong argument.

We want to turn (DNE) into a principle of plenitude for possible worlds. The underlying idea is that, given individuals from different possible worlds, they can always be "patched together" in a single possible world. The quantifiers occurring in (DNE), then, are possibilist quantifiers: they range over all possible individuals. But how should we understand 'can coexist'? Because worlds do not overlap, the individuals themselves cannot be patched together; each individual exists in only one world. This "patching" together must instead be understood either in terms of counterparts or (intrinsic) duplicates. But (DNE) is problematic if interpreted in terms of counterparts: it would rule out exclusionary essential properties. Arguably, possible worlds themselves have exclusionary essences: it is of the nature of a possible world to be a world, a maximal unified whole. In that case, distinct worlds could not coexist.[9] It has become standard, therefore, to interpret "can coexist" in (DNE) in terms of duplicates: there is a world where a duplicate of one and a duplicate of the other coexist. Finally, Lewis does not want to restrict (DNE) to just two possible individuals. If we put no restriction on which pluralities of possible individuals can coexist together, we get an unrestricted principle of recombination. Let us say that a world *w recombines xx* iff, for some *yy* (1) each of *yy* is part of *w*, (2) no two of *yy*

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[9] A counterpart theorist might allow that under one counterpart relation worlds are essentially worlds, under another counterpart relation they are not. But noting that does not by itself solve the problem of interpreting (DNE) in terms of counterparts.

overlap, and (3) there is a one-one correspondence between *xx* and *yy* such that each one of *yy* is a duplicate of that one of *xx* that corresponds to it.[10] Then, the first precise generalization of (DNE) is:

> *Unrestricted Principle of Recombination* (UPR). For any possible individuals *xx*,
> some world recombines *xx*.

Note that by quantifying plurally over possible individuals in expressing (UPR), no assumption is made as to whether the individuals that are recombined form a set.

(UPR) is not the principle of recombination that Lewis endorses, however. (UPR) needs to be both strengthened and qualified. It needs to be strengthened because plenitude requires not just that any things can be recombined, but that they can be recombined *in any possible spatiotemporal arrangement*. To use Lewis's example: if one has a unicorn from one world and a dragon from another world, then there is a third world in which a duplicate of the unicorn and a duplicate of the dragon coexist *side by side*. Let us say that a *spatiotemporal arrangement* of possible individuals *xx* *within* a possible spacetime structure is a one-many mapping of *xx* into places in the structure that takes each *xx* into a place (or places) with the same shape and size. It is *non-overlapping* iff the places the *xx* are mapped into do not overlap. World *w recombines xx according to an arrangement 𝒜* iff 𝒜 is an arrangement of *xx* within the structure of *w*, and 𝒜 maps each *x* that is one of *xx* to a place (or places) in the structure occupied in *w* by a duplicate of *x*. (Note: the reason to allow the mapping to be one-many is to incorporate multiple duplication; for example, starting from a single individual, (LPR) provides worlds that contain multiple duplicates of that individual.) Then we have the following principle of recombination:

> *Lewis's Principle of Recombination* (LPR). For any possible individuals *xx* and
> any non-overlapping[11] spatiotemporal arrangement of *xx*, there exists a
> world that recombines *xx* according to that arrangement.

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[10] As is standard, I use double letters—'*xx*', '*yy*'—as plural variables. '*xx*' may be pronounced as 'the *x*s' where appropriate.

[11] In Bricker (forthcoming b), I give a stronger formulation of (LPR), quantifying over "consistent" arrangements rather than "non-overlapping" arrangements, where an arrangement is *consistent* iff whenever the arrangement maps two possible individuals to two overlapping

(UPR) also needs to be qualified: things can be recombined only if there is a possible (spacetime) structure into which they all fit. (A structure is (metaphysically) *possible* if and only if it is instantiated in some possible world.) Thus, Lewis adds to his principle of recombination the proviso: "size and shape permitting." But no adjustment to (LPR) is needed: it automatically incorporates the proviso by quantifying over *possible* arrangements within *possible* (spacetime) structures. If there is no possible structure within which *xx* can be arranged, perhaps because there are more *xx* than there are places in any possible (spacetime) structure, then (LPR) does not require that there be a world that recombines *xx*.

The Forrest-Armstrong argument, once bolstered in one of the ways I will suggest, shows that (UPR) must be rejected. But it isn't enough just to switch to (LPR). (LPR) together with an overly liberal principle of plenitude for possible (spacetime) structures might entail (UPR). An overly liberal principle of plenitude for world structures threatens to make the proviso moot: size and shape will always be permitting. An adequate response to the Forrest-Armstrong argument, then, must consider whether there is a principled way to say what structures are possible, one that doesn't allow the derivation of (UPR).

4. THE FORREST-ARMSTRONG ARGUMENT, LEWIS'S VERSION

I turn now to a presentation of the Forrest-Armstrong argument. I will focus on Lewis's version of the argument, slightly altered. Lewis formulated the argument as a *reductio* on (UPR). First, I present the argument unadorned as a series of fourteen numbered steps. Then, in the remainder of this section, I add some brief commentary on steps of the argument that require clarification. I note where the argument I present diverges from Lewis's version, and where Lewis's version diverges from Forrest and Armstrong's version. In the next section, I focus on the "gap" in the argument, the inference from (F7) to (F8) below. I show that, for an anti-haecceitist who interprets recombination in terms of duplicates, the gap cannot be bridged by any straightforward

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places, the parts of those possible individuals that are mapped to the place of overlap are duplicates of one another. Lewis would no doubt accept the stronger principle, but he explicitly endorses only the weaker one.

generalization of (DNC), the denial of necessary connections. Some stronger principle will be needed to bridge the gap.

Here is the argument.

(F1) Assume (for *reductio*): *The Unrestricted Principle of Recombination (UPR): for any possible individuals xx, some world recombines xx.*

(F2) *Worlds are possible individuals.*

(F3) Therefore: *Some world recombines all the worlds, and thus contains duplicates of all the worlds as non-overlapping parts.* Following Nolan, call one such world *Giganto*. (Lewis calls it "the big world." But note that I do not assume that only one world recombines all the worlds.)

(F4) Assume: *For any world, the atoms in that world form a set.*

(F5) Then: *The atoms in Giganto form a set.* Call this set A and let its cardinality be κ. (κ can safely be assumed to be greater than 1.)

(F6) *There are $2^\kappa - 1$ non-empty subsets of A* (by elementary set theory).

(F7) *For each non-empty subset of A, there is a world in which just the atoms in that subset remain and the rest have been deleted.* (Lewis calls these worlds *variants* of the big world, Giganto.)

(F8) *There are at least $2^\kappa - 1$ such variants of Giganto.*

(F9) *Each variant of Giganto contains at least one atom.*

(F10) *There are at least $2^\kappa - 1$ non-overlapping duplicates of these variants within Giganto* (by (UPR)).

(F11) *The duplicate of a world with at least one atom contains at least one atom.*

(F12) Therefore: *There are at least $2^\kappa - 1$ atoms in Giganto.*

(F13) *Contradiction*. (By Cantor's Theorem, $2^\kappa - 1$ is greater than κ for all cardinals κ greater than 1.)

(F14) Therefore: *(UPR) is false.*

I begin with six brief comments on various steps of the argument.

(1) One might wonder whether Lewis, in his presentation of the argument, has in mind a formulation of the unrestricted principle of recombination that is weaker than the formulation (UPR) I

gave in Section 3 and take as the assumption for *reductio*, (F1). For Lewis's formulation of the unrestricted principle seems to be based on a weaker definition of 'recombines'. (He says 'copies' where I say 'recombines'. In this paragraph, I follow Lewis in using 'copies'; but I will introduce a different sense of 'copies' in the next section.) Lewis writes:

> "Say that a world *copies* a class of possible individuals, perhaps from various possible worlds, iff it contains non-overlapping duplicates of all the individuals in that class. The [unrestricted] principle of recombination … says that, given a class of possible individuals, there is some world which copies that class" (Lewis 1986: 101).[12]

On one natural reading, this does not require a one-one correspondence between the individuals being copied and the duplicates that copy them. For example, a world could copy a pair of duplicate globes, it seems, by containing a single duplicate of the globes. But Lewis's use of the unrestricted principle in his rendition of the Forrest-Armstrong argument suggests that he has the stronger notion requiring a one-one correspondence in mind. In particular, (F10) in the above argument would not follow from the weaker version of the unrestricted principle. I will suppose that Lewis intended the stronger version given by (UPR) in what follows.

(2) Forrest and Armstrong (1984: 164) appear to take Lewis's principle of recombination to apply only to *sets* of possible individuals. (They say: "Given any number of worlds … ," and I suppose only sets have a "number.") In that case, the "unrestricted" principle of recombination that features in their argument is (logically) weaker than (UPR), and this difference will matter if the worlds are many and do not form a set. For, in that case, their argument never gets off the ground. But, as Lewis (1986: 104) rightly points out, taking the principle of recombination to apply only to sets of possible individuals is already to impose a restriction on it. An unrestricted principle should apply to any plurality of possible individuals, whether or not they are "too big" to form a set. Thus, I

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[12] Note that, in Lewis (1986), quantification over classes is explicitly given a non-standard interpretation: it is to be understood as plural quantification, not singular quantification over set-like entities. See Lewis (1986: 50-1). The difference between plural quantification and quantification over classes didn't much matter given Lewis's views in 1986 which did not include any commitment to classes over and above sets. But the difference matters in a big way beginning with Lewis's (1991) account of classes.

follow Lewis in formulating the unrestricted principle using plural quantification.

(3) (F3) follows directly from (UPR) applied to all worlds. Since Giganto is itself a world, it follows that Giganto contains a duplicate of itself as a proper part. Moreover, that proper part of Giganto contains another duplicate of Giganto as one of its proper parts, and so on *ad infinitum*. This may seem odd, but it is not yet a contradiction. For example, worlds of one-way eternal recurrence contain duplicates of themselves as proper parts: the world minus its first epoch is a duplicate of the world as a whole. (On worlds of eternal recurrence, see Lewis 1986: 63.) Let us say that modal reality is *reflective* iff some world contains a duplicate of every world, and so every possible individual, as a proper part. (UPR) demands that modal reality be reflective.

(4) In (F4), and throughout the rest of the argument, I use the cardinality of the set of *(mereological) atoms* in a world as a measure of the size of the world. The original argument given by Forrest and Armstrong used the cardinality of the set of *electrons* in a world as a measure of its size; and Lewis followed them in this. I have modified Lewis's argument here so as to skirt issues having to do with whether *being an electron* is an intrinsic property. If *being an electron* is not intrinsic, then the duplicate of an electron need not be an electron, and (F7) will not follow from any principle of recombination expressed in terms of duplicates.

(5) (F4) is an unstated assumption of Lewis's argument. In 1986, he accepted the stronger assumption that the possible individuals altogether form a set, and that may be the assumption he is implicitly invoking; but only this weaker assumption is needed to derive (F5). (He later changed his view; see Bricker 1991 expanded version, n. 17.) In Section 8, I consider loosening this assumption.

(6) Instead of (F7), Forrest and Armstrong assert: for some property F and each subset of A, there is a world in which just the atoms (for them, electrons) in that subset have F. But this raises issues as to what counts as a property, and what sorts of property could be used in the argument to play the role of F. These issues are best left to one side. Lewis, in effect, is using the property of existence to play the role that variable F plays for Forrest and Armstrong.

5. THE GAP

When Lewis gets to the step in the argument where (F7) is asserted, he writes: "I take this to be a subsidiary appeal to recombination" (p. 101). But it is not an appeal to (UPR). Rather, some principle that includes a generalization of the Humean denial of necessary connections (DNC) is required as an

additional premise.[13] Although Lewis never gives a precise formulation of any such generalization, there is not much doubt as to how such generalizations should go. In Bricker (forthcoming b), I present and motivate various principles that would do the job. Here I will consider the two most natural candidates, one that strengthens (UPR), the other that strengthens (LPR). These two principles lead to different reformulations of (F7), and raise different considerations in evaluating the inference from the reformulated (F7) to (F8). But first I ask: why do we need to reformulate (F7)?

Taken at face value, (F7) could only be true if the domains of worlds overlap: otherwise there will be no world in which some but not all of the atoms in *A* "remain." By the *domain* of a world, I mean the entities that *exist at* the world in the broad sense of 'exist at', where for the Lewisian an individual exists at a world just in case it has a counterpart that is part of the world. (Elsewhere in this chapter, I use 'exist in' to capture the narrow notion, where an individual exists in a world just in case it is a part of the world.) So taking (F7) at face value does not require that the worlds themselves (literally) overlap, which would violate our characterization of Lewisian realism. It only requires that the domains of worlds overlap. Now, suppose for the sake of argument that there is a viable *haecceitist* version of Lewisian realism.[14] A *haecceitist* holds that there are distinct qualitatively indiscernible worlds that differ with respect to which individuals are in their domains or which individuals instantiate which fundamental properties and relations. Haecceitists and anti-Haecceitists differ with respect to how they individuate and count possible worlds, and such differences are directly relevant to the inference from (F7) to

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[13] Nolan (1996) is aware that Lewis cannot be appealing only to (UPR), but he does not attempt to provide an adequate formulation of a principle Lewis might be appealing to. He writes: "this flaw in the argument could be repaired by invoking a more complicated principle of recombination, but to state such a principle is not a straightforward matter" (p. 245). He goes on, however, to suggest that there is a gap in the argument however (F7) is formulated in terms of duplicates. The argument he gives for this, however, applies to the reformulation based on strong (UPR) below, but not to the reformulation based on strong (LPR). And it is the latter reformulation, I believe, that Lewis most likely had in mind. These problems of exegesis arise because Lewis (1986) is not consistent with what he calls "recombination" or "the principle of recombination." So when he speaks of "a subsidiary appeal to recombination," all that can be inferred is that he thinks some principle of recombination or other can be invoked to support the argument. For some of the exegetical issues, see Bricker (forthcoming b).

[14] McDaniel (2004) and Cowling (2012) both develop haecceitist versions of modal realism that allow overlapping worlds. I have in mind a version that uses a non-qualitative counterpart relation without overlap. To get the logic of identity right (for a haecceitist), the non-qualitative counterpart relation will have to be an equivalence relation.

(F8). (See Lewis 1986: 220-35 for Lewis's characterization of haecceitism; see Bricker (1987, §5) on haecceitist vs. anti-haecceitist modes of counting.)

Now suppose that recombination principles are interpreted, not in terms of duplicates, but in terms of the things themselves. The unrestricted principle of recombination, then, will say simply: for any *xx*, there is a world at which every one of *xx* exists and no two of *xx* overlap. Call this (UPR)[H] because it is the version of unrestricted recombination that a haecceitist might endorse. The propounder of the Forrest-Armstrong argument can then affirm (F7) at face value by appealing to the following strengthening of (UPR)[H]: for any *xx* and any *yy* such that no one of *xx* is one of *yy*, there is a world in which every one of *xx* exists, no two of *xx* overlap, and no one of *yy* exists. It then follows immediately from (F7) that each non-empty subset of *A* corresponds to a *distinct* variant of Giganto, and so there are at least $2^\kappa - 1$ such variants. There is no gap between (F7) and (F8) for the haecceitist. When the entire Forrest-Armstrong argument is reformulated in this way—replacing talk of duplicates by talk of the things themselves—it leads to the conclusion that (UPR)[H] is false. But invoking the Forrest-Armstrong argument to reject (UPR)[H] is overkill. (UPR)[H] should be rejected by a Lewisian for a more basic reason: it requires that some world contain *itself* as a proper part (if there is more than one world), not just a *duplicate* of itself as a proper part. And that, I think, should be rejected by a haecceitist version of Lewisian realism, even when interpreted in terms of counterparts.

I will say no more about haecceitist readings of principles of recombination. Lewis is no haecceitist and there is no question that he intended (F7) to be interpreted in terms of duplicates. But how (F7) should be reformulated depends on what strengthened principle of recombination is appealed to at this step of the argument. Let's first consider a strengthening of (UPR) that incorporates (DNC). The way to strengthen (UPR) is to construct a principle that says, not only that any possible individuals can be recombined, but that any possible individuals can be recombined *without any other possible individuals*. But we need to be careful how we state this because when some of *xx* (or parts of *xx*) are duplicates of some of *yy* (or parts of *yy*), then a world that recombines *xx* may thereby also recombine some of *yy*. What we need to say is that, for any *xx* and any *yy*, some world recombines *xx* and contains only those duplicates of *yy* that it contains in virtue of recombining *xx*. Here are the requisite definitions. Say that some individuals *zz copy* at *w* some individuals *xx* iff (1) *zz* are non-overlapping and all exist in *w*, and (2) there is a one-to-one correspondence between *xx* and *zz* that maps each one of *xx* to a duplicate of itself in *zz*. (Note that this notion of "copies" differs

from Lewis's: it relates some individuals to some other individuals, not a world to some individuals.) Say that a world *w recombines xx without yy* iff *w* contains some *zz* that copies *xx,* and if *w* contains any duplicate of any one of *yy*, then that duplicate is part of the fusion of some *zz* that copies *xx*. Finally, we take the principle of recombination that Lewis appealed to as an unstated premise to be this:

> *Strong* (UPR). For any possible individuals *zz*, and any division of *zz* into *xx* and
>
> *yy*, some world recombines *xx* without *yy*. (This principle is called (C2) in
>
> Bricker forthcoming b.)

Strong (UPR) entails (UPR),[15] and so is not a principle that Lewis accepts. Does appealing to Strong (UPR) undermine the *reductio* against (UPR), turning it into a *reductio* against Strong (UPR)? No, for Strong (UPR) is equivalent to the conjunction of (UPR) and a version of (DNC) that Lewis clearly accepts. (Namely, principle (B2) in Bricker forthcoming b.) So the *reductio* is still squarely directed at (UPR). Now, applying Strong (UPR) to the set *A* of atoms in Giganto, we get the following reformulation of (F7) in terms of duplicates:

> (F7)* For each non-empty subset *B* of *A*, there is a variant of Giganto that
>
> recombines the members of *B* without the members of *A − B*. (*A − B* is the
>
> relative complement of *B* in *A*.)

Finally, we get to the crucial "gap" in the argument, the transition from (F7) to (F8), the claim that there are $2^\kappa - 1$ variant worlds, one for each non-empty subset of *A*. To show that there is a gap in the argument, it suffices to give some description of modal reality according to which Strong (UPR), and hence (F7)*, hold (along with the other premises of the argument), but nonetheless there are only κ variants of Giganto. The description of modal reality that provides the counterexample does not need to be plausible as an account of modal reality. The role of the counterexample is to demonstrate that Strong (UPR), by itself, is not enough to bridge the *logical* gap in the argument.

For the counterexample, suppose that modal reality is Democritean in this sense: the atoms of which it is composed are all duplicates of one another. Worlds differ, then, only in how many atoms

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[15] By letting *zz* be a (trivial) division of itself, in which case *yy* drops out and Strong (UPR) reduces to (UPR).

they contain and how those atoms are arranged. In this case, modal reality is *homogeneous*: any two possible individuals have duplicate parts. (See Bricker forthcoming b, §2.3.) Now, follow the argument up to (F7), where we have the world Giganto whose set of atoms *A* has cardinality κ. Consider any subset *B* of *A*, and suppose the cardinality of *B* is λ. Strong (UPR) demands that there be a variant of Giganto with *exactly* λ non-overlapping duplicates of atoms in *B*. The world must contain *at least* λ non-overlapping duplicates because Strong (UPR) entails (UPR). It cannot contain *more than* λ non-overlapping duplicates lest it contain duplicates of atoms not in *B* that it is not required to contain in virtue of recombining the atoms in *B*. But, by our assumption that all atoms are duplicates of one another, it follows that a single variant of Giganto containing exactly λ atoms suffices to satisfy Strong (UPR) with respect to all subsets *B* of size λ. Since there are only κ cardinal numbers less than or equal to κ, it follows that only κ worlds are needed to satisfy Strong (UPR) with respect to every non-empty subset of *A*. So (F8) is false, even supposing Strong (UPR), and we have a counterexample to the inference from (F7)* to (F8).

I am doubtful, however, that Strong (UPR) and (F7)* is what Lewis had in mind when he spoke of a "subsidiary appeal to recombination." I think it likely that Lewis intended to be appealing to a stronger principle of recombination, a principle that quantifies over arrangements. For he says: "There is a world *rather like the big world* in which just those electrons [atoms, for us] remain and the rest have been deleted" (Lewis 1986: 102, my emphasis). Strong (UPR) does not give any reason to say that the world that recombines some of the atoms without the others is "rather like" the big world, Giganto. A natural way to make Lewis's vague demand precise is a principle that says that when we recombine *xx* without *yy*, we can do so while keeping the (spacetime) structure of *xx* in Giganto unchanged. Here are the requisite definitions. Say that arrangements 𝒜 and ℬ *agree on xx* iff every one of *xx* is in both the domain of 𝒜 and the domain of ℬ, and the structure within which 𝒜 arranges and the structure within which ℬ arranges have a common substructure such that, for any one of *xx*, 𝒜 and ℬ map that one to the same place in that common substructure. Say that a world *recombines xx according to* an arrangement 𝒜 but *without yy* iff the world recombines *xx* according to 𝒜 and the world recombines *xx* without *yy*. Now, the principle of recombination that we are supposing Lewis appeals to as an unstated premise is this:

> *Strong* (LPR). For any possible individuals *zz*, for any division of *zz* into *xx* and
>   *yy*, for any non-overlapping arrangement 𝒜 of *zz*, there is an arrangement ℬ

that agrees with $\mathcal{A}$ on *xx* and a world that recombines *xx* according to $\mathcal{B}$ but without *yy*. (This principle is called (C4) in Bricker forthcoming b.)

We can now apply Strong (LPR) to Giganto, resulting in a reformulation of (F7) in terms of duplicates:

(F7)** Let $\mathcal{A}$ be the arrangement of the atoms in A within Giganto, and let B be a non-empty subset of A. There is an arrangement $\mathcal{B}$ that agrees with $\mathcal{A}$ on the members of B, and a world that recombines the members of B according to $\mathcal{B}$ but without the members of $A - B$.

(F7)** is a reformulation of (F7) in terms of duplicates that incorporates one way of understanding Lewis's proviso that the variants of Giganto be "rather like Giganto." In moving to a variant of Giganto based on the subset B, we hold fixed how the atoms in B are arranged in Giganto. We do not specify, however, whether the atoms in $A - B$ are replaced by non-duplicate atoms, thereby retaining the full structure of Giganto, or the atoms in $A - B$ are simply deleted, resulting in the variant having a reduced structure (or something in between).

The argument we used above to show the inadequacy of Strong (UPR) to bridge the gap won't do for showing the inadequacy of Strong (LPR). It is no longer enough just to suppose that modal reality is homogeneous. For consider two subsets of A of the same cardinality. If the atoms in those subsets are differently arranged in Giganto, then different variants of Giganto will be needed to satisfy strong (LPR). It is no longer the case that a single variant can accommodate all subsets of the same cardinality. (See Bricker 1987, §5 where it is shown that $\beth_1$ atoms can be arranged in $\beth_2$ non-isomorphic ways within Euclidean spacetime.) To get a counterexample, we can add to the homogeneity assumption a strong restriction on what spacetime structures are instantiated in modal reality. Consider this. At every world, every atom is the same distance from every other atom. In other words, every world has a *simplex* structure, the structure had by the vertices of "generalized tetrahedra."[16] The simplex structure gives us just what we need: all substructures of a simplex

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[16] Note that these structures would not count as "analogically spatiotemporal" according to Lewis's account: they are not *discriminating*. Thus my argument that there is a gap on the assumption of Strong (LPR) may apply only to my own account of how worlds are unified, and not to Lewis's. (Again, see Bricker 1996 for my account, and why I think Lewis's account is too

structure with the same cardinality are isomorphic to one another. So different subsets of *A* with the same cardinality are similarly arranged in Giganto, and a single variant of Giganto can serve to satisfy strong (LPR) with respect to them all. We have again that only κ variants are needed *in toto* when *A* has cardinality κ, and the Forrest-Armstrong argument is stopped in its tracks.

The counterexamples presented above show that there is a logical gap between (F7) and (F8) even when (F7) is reformulated in a way that appeals to a stronger principle of recombination that incorporates a version of (DNC). But there is more to plenitude than recombination. Perhaps appealing to other plausible principles of plenitude can bridge the gap in the argument. Indeed, inspection of the counterexamples suggests what might be needed. The counterexample to employing strong (UPR) required, implausibly, that all atoms in modal reality are duplicates of one another. A principle of plenitude *for world contents* that requires more variety among the atoms in modal reality might rule this and all other counterexamples out, thus bridging the gap. The counterexample to employing strong (LPR) required, even more implausibly, that all worlds with the same number of atoms were isomorphic. A principle of plenitude *for world structures* might require enough structural variation between worlds to rule this and all other counterexamples out, thus bridging the gap. I do not say that adding these principles as additional premises counts as a defense of the original Forrest-Armstrong argument, which was supposed only to appeal to a principle of recombination to derive the contradiction. But it will show that a fortified Forrest-Armstrong argument is still a formidable weapon against realist accounts of possible worlds.

6. BRIDGING THE GAP: THE PLENITUDE OF WORLD CONTENTS

In this section, I consider whether and how a principle of plenitude for world contents can bridge the gap in the Forrest-Armstrong argument. By a "principle of plenitude for world contents," I mean a principle requiring that there be a plenitude of fundamental properties instantiated throughout modal reality. Some such principle is needed, I claim, to support our ordinary modal beliefs. For example: whatever fundamental properties happen to be instantiated in the actual

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restrictive.) I leave it as an open question whether on the assumption that worlds are unified by analogically spatiotemporal relations, there is a gap. Given the vagueness in Lewis's characterization of "analogically spatiotemporal," the question may have no determinate answer.

world, I believe that different, *alien* fundamental properties could have been instantiated instead, indeed, could have been instantiated within the spacetime structure had by the actual world.[17] In the current Lewisian framework, principles of plenitude quantify over possible individuals, not fundamental properties.[18] But we can require a plenitude of fundamental properties indirectly, by requiring a plenitude of possible individuals none of whose parts are duplicates of one another. And that in turn will require a plenitude of atoms that are not duplicates of one another, that differ in their intrinsic qualitative nature.

There are stronger and weaker principles of plenitude for world contents. The principle that I take to be fundamental to an account of modal plenitude is about as strong as they come. Say that an individual is *alien* to another individual iff no part of one is a duplicate of any part of the other. The principle I accept says that any part of any world can be replaced by an alien part (of the same shape and size) while holding the rest of the world fixed.

> *Principle of Interchangeable Parts* (PIP). For any world *w* and any individual *a* in
> *w*, there is a world exactly like *w* except that *a* has been replaced by an
> individual alien to *a*.[19]

(PIP) is a *pure* principle of plenitude for world contents. It has no implications for what world structures are possible. And it has no implications for what individuals can be recombined; it is independent of (LPR). But together with (LPR), it entails a principle that backs much of our modal reasoning about aliens:

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[17] See Bricker (forthcoming a, §5) for a defense of quidditism, the view that structurally indiscernible worlds may differ qualitatively. See Bricker (forthcoming b, §3.3) for the claim that the practice of science supports the plenitude of fundamental properties. See Bricker (1987) and Lewis (1986: 158-65) for how the possibility of alien fundamental properties makes trouble for certain "actualist" reductions of possible worlds.

[18] In the framework I prefer, fundamental properties are grounded in tropes, and possible individuals are sums of co-located tropes. (See Bricker forthcoming a, §5.) By quantifying over tropes, a principle of plenitude for fundamental properties can be expressed directly. On how to understand co-location, see Section 11 below.

[19] See Bricker (forthcoming b, §3.2) for discussion of (PIP) and proofs of various corollaries. The notion of two worlds being exactly alike except for a replacement of individuals can be defined precisely in terms of arrangements within the worlds' underlying spacetime structure.

Principle of Alien Individuals (PAI). For any world *w* and any individual *a* in *w*,
there is a world exactly like *w* except that *a* has been replaced by an
individual alien to *w*.

From (PAI) we can derive the principle that I want to use to bolster the Forrest-Armstrong argument, a principle requiring a plenitude of worlds that are maximally heterogeneous. Say that a world is *heterogeneous* iff no (spatiotemporal) part of the world is a duplicate of any other (spatiotemporal) part of the world. Then,

Principle of Heterogeneity (PH). For any possible world structure, there is a
heterogeneous world that has that structure.

Here is a sketch of the proof that (PH) follows from (PAI). (A formal proof is by transfinite recursion appealing liberally to the axiom of choice.) Consider any world structure and any world *w* that has that structure. Well-order all the atoms in *w*. Using (PAI), successively replace each atom by an atom that is alien to the world provided by the previous replacement. Continue until all atoms have been replaced. At the end of this process, one has arrived at a heterogeneous world with the same structure as *w*.

The Principle of Heterogeneity is just what we need to bridge the gap in the Forrest-Armstrong argument. We can amend the argument as follows. First, after (F4) we add:

(F4.1)^H. Assume: *Principle of Heterogeneity* (PH).

(F4.2)^H. Therefore: *There is a heterogeneous world, call it Hetero, with the same
structure as Giganto*.

The argument continues as before, substituting 'Hetero' for 'Giganto' and 'variant of Hetero' for 'variant of Giganto' in (F5) through (F10), and substituting a version of (F7)* in place of (F7). That gives the following:

(F5)^H. Then: *The atoms in Hetero form a set.* Call this set A_H and let its cardinality be κ.

(F6)^H. *There are $2^\kappa - 1$ non-empty subsets of A_H.*

(F7)^H. *For each non-empty subset B_H of A_H, there is a variant of Hetero that recombines the
members of B_H without the members of $A_H - B_H$.*

(F8)[H]. *There are at least $2^\kappa – 1$ such variants of Hetero.*

(F9)[H]. *Each variant of Hetero contains at least one atom.*

(F10)[H]. *There are at least $2^\kappa – 1$ non-overlapping duplicates of these variants of Hetero within Giganto.*

(F11) through (F14) are unchanged. (F8)[H] now follows straightforwardly from (F7)[H] because distinct subsets of Hetero's atoms correspond to distinct variants of Hetero—distinct because they differ with respect to which natures are instantiated by their atoms. I conclude that whoever agrees with me in accepting (PIP) will take the fortified Forrest-Armstrong argument to provide a *reductio* of (UPR).

But I dare not stop here, for two reasons. First, I have based (PH) on (PIP), and (PIP) is controversial. Lewis, for example, did not accept (PIP), or its corollary (PIA). He writes:

> A world to which no individuals, worlds, or properties are alien would be an
> especially rich world. There is no reason to think we are privileged to inhabit such
> a world. Therefore any acceptable account of possibility must make provision for
> alien possibilities. (Lewis 1986: 92)

(PIA) immediately entails that no world is "especially rich." If Lewis had based his belief in alien possibilities on (PIP) or (PAI), he would have rejected especially rich worlds out of hand. It is unclear what weaker principle Lewis would have accepted to support his belief in alien possibilities. (In Bricker forthcoming b, §3.3 I consider some possibilities, and offer my objections; I consider a different possibility below.) He needed some principle weaker than (PIP) that nonetheless could support alien possibilities. In any case, the question now before us is this: is there a weaker principle of plenitude for world contents that does not rule out especially rich worlds, but that is still strong enough to entail (PH), and thus support the fortified Forrest-Armstrong argument?

There is an even stronger reason, however, why I dare not stop here. An account of plenitude that accepts (PIP) and (LPR) has no need for a fortified Forrest-Armstrong argument: (PIP) and (LPR) entail (PIA), and (PIA) immediately refutes (UPR) by entailing that no especially rich world like Giganto can exist. No Cantorian cardinality argument is needed to

refute (UPR).[20] Now, *I* accept (PIP) and (LPR), so *I* have no need for the fortified argument. But for those who accept a weaker principle of plenitude for world contents than (PIP), a fortified argument may still be needed to demonstrate that (UPR) must be rejected. And perhaps the weaker principle they accept will still be strong enough to entail (PH). Moreover, in the conflict between (PIP) and (UPR), I want to convince the reader that (PIP) is not the guilty party. Initially, both (PIP) and (UPR) may seem like plausible candidates for principles of plenitude; both accord with our modal intuitions. One must go. But which one? A fortified Forrest-Armstrong argument that was not based on (PIP) would provide an independent argument against (UPR), leaving one free to accept (PIP). The dialectic is a bit peculiar. A fortified Forrest-Armstrong argument, by refuting (UPR) and supporting (PIP), may thereby persuade one that the fortified argument wasn't needed after all to refute (UPR).

There is a flatfooted method for guaranteeing that the gap in the Forrest-Armstrong can be bridged: posit that there can be as many *kinds* of atom as there can be atoms. Let us identify the *kind* of an atom *a* with the plurality of atoms across modal reality that are duplicates of *a*: duplicate atoms are atoms of the same kind; non-duplicate atoms are atoms of different kinds. Then we can express this principle of plenitude for world contents as follows:

Principle of Atomic Diversity (PAD): If some world contains κ atoms, then some world contains κ atoms no two of which are duplicates of one another.[21]

(PAD) constrains how many duplicates of atoms there are in any one world. If there are only κ *kinds* of atom throughout modal reality, then no world contains more than κ kinds of atom, and so, by (PAD), no world contains more than κ atoms of any one kind. That goes against the standard interpretation of Lewis's principle of recombination according to which the only constraint on the number of duplicates comes from the proviso "size and shape permitting." (See the discussion of what I call the "principle of duplication" in Bricker forthcoming b; see also

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[20] Fritz (2017) emphasizes that the fundamental "combinatorial" ideas that underlie principles such as (UPR) and (PIP) are in *direct* conflict with one another.

[21] Note that, because non-duplicate atoms can share fundamental properties, (PAD) doesn't entail that, if some world contains κ atoms, then some world has κ fundamental properties (of atoms) instantiated in it. Rather, it entails that, if some world contains κ atoms, then some world has λ fundamental properties (of atoms) instantiated in it, where $2^\lambda \geq \kappa$.

Nolan 1996.) To accommodate (PAD), (LPR) will have to be understood to be twice qualified: possible arrangements will have to be limited not only by what structures are possible, but by the number of possible kinds.

(PAD) together with (LPR) entails (PH). So if we assume (PAD), the fortified Forrest-Armstrong argument as presented above in terms of variants of Hetero bridges the gap in the original argument. But note also that (PH) together with (LPR) entails (PAD). In light of (LPR), these two principles of plenitude are equivalent. One might wonder, then, what advantage there is to assuming (PAD) to bolster the Forrest-Armstrong argument rather than just taking (PH) to be a fundamental principle of world contents. Indeed, (PH) seems to me more in accord with basic modal intuitions than (PAD). But starting from (PAD), rather than (PH), has the advantage that it allows us to stay closer to the Forrest-Armstrong argument as first presented above. We can bypass Hetero and again state the argument entirely in terms of Giganto and its variants. But now the variants of Giganto are determined by which *kinds* of atom remain and which are deleted. In effect, this results in a quidditist version of the Forrest-Armstrong argument; it contrasts with the haecceitist version by resting on the transworld identity of kinds of atoms, rather than the atoms themselves.

This fortified version of the Forrest-Armstrong argument replaces (F6) with the following three claims:

(F6.1)^K. Assume: *Principle of Atomic Diversity (PAD)*.

(F6.2)^K. *There are κ kinds of atom in Giganto* (by (PAD)). Let K be a complete set of representatives of the kinds of atom in Giganto, containing one atom from each kind. K has cardinality κ.

(F6.3)^K. *There are $2^κ – 1$ non-empty subsets of K.*

Then, one way of continuing the argument proceeds by applying strong (UPR) to the set K instead of the set A, thereby replacing (F7)* with:

(F7)^K. *For each non-empty subset L of K, there is a variant of Giganto that recombines the members of L without the members of K – L.*

A second way of continuing the argument avoids the need to reinterpret (F7) in terms of duplicates by quantifying directly over kinds:

> (F7)^{KK}. *For each non-empty subset L of K, there is a variant of Giganto in which*
> *just the kinds with representatives in L remain (are instantiated), and the rest*
> *have been deleted (are not instantiated).*

This suggests that—perhaps—the reason Lewis was content to present a "haecceitist" version of the Forrest-Armstrong argument is that he knew, given his acceptance of a plenitude of alien kinds, that a "quiddistic" version of the argument would result by merely switching from talk of individuals to talk of kinds. In any case, (F8), that there are at least $2^\kappa - 1$ variants of Giganto, now follows immediately from either (F7)^K or from (F7)^{KK}: distinct subsets of *K* correspond to distinct variants of Giganto, distinct because they differ in what *kinds* of atom they contain. Appealing to (PAD) has successfully bridged the gap.

7. BRIDGING THE GAP: PLENITUDE OF WORLD STRUCTURES

Some philosophers reject any principle of plenitude for world contents. Modal reality, they say, is composed entirely of individuals that instantiate actual, not alien, kinds. The fortified arguments of the last section are not for them. Do these philosophers have a way of bridging the gap in the Forrest-Armstrong argument? Indeed, they can turn instead to principles of plenitude for world structures. As the counterexample to the inference from (F7)** to (F8) suggests, the gap results in part because no premises of the argument rule out modal reality being composed entirely of worlds with very simple structures. If we have independent reason to think there are big worlds whose structures are appropriately complex, then the gap from (F7)** to (F8) can be bridged.

I presented my account of plenitude of world structures in Bricker (1991, forthcoming d). By 'world structure', I mean the underlying structure of a world over which the fundamental contents of the world—be they individuals, or universals, or tropes—are arranged. What I call the world structure, then, is independent of the instantial structure that arises from the pattern of instantiation, or occupation, of the world contents. Given our Lewisian assumptions in this chapter, the world contents are possible individuals and the underlying world structures are

always spatiotemporal (or analogously spatiotemporal). For example, four-dimensional Minkowski spacetime is a possible world structure. Note that when I say "possible world structure," the 'possible' is redundant (as it is for me in 'possible world'): to be a world structure is just to be the underlying structure of some world.

Two principles follow from my account of plenitude of world structures that in combination can be used to bridge the gap. Let us measure the size of a world structure by the number of atoms in a world that has that structure. Let us say that modal reality is $>\kappa$-*diverse* iff there are more than κ non-isomorphic possible world structures. Then, the first principle we need is:

> *Principle of Structural Diversity* (PSD): If there is a possible world structure of size κ, then modal reality is $>\kappa$-diverse.

The second principle we need is:

> *Substructure Principle* (SP): Any (spatiotemporally connected)[22] substructure of a possible world structure is a possible world structure.

These two principles allow us to reformulate the Forrest-Armstrong argument in the following way. After (F5), add:

> (F5.1)^D. Assume: *Principle of Structural Diversity (PSD) and Substructure Principle (SP).*

> (F5.2)^D. *The world structure of Giganto has size* κ (by (F5)).

> (F5.3)^D. Therefore by (PSD): *There are more than* κ *non-isomorphic world structures*. Let W_D be a set of more than κ worlds no two members of which have isomorphic world structures. Let G_D be a set of duplicates of these

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[22] I would say instead "externally connected" because I do not think that all worlds are unified by spatiotemporal relations; see Bricker (1996). Some qualification is needed because no world divides into spatiotemporally disconnected parts (on Lewis's account), or externally disconnected parts (on my account). For more on how I understand the Substructure Principle, see Bricker (forthcoming d).

worlds in Giganto. For each individual d in G_D, let A_d be the atoms in d, and let $\mathcal{B}_d$ be the arrangement of A_d in Giganto restricted to the substructure determined by A_d.

Then the argument continues as before, substituting a version of (F7)** in place of (F7). ((F6) is no longer relevant.) That results in the following:

(F7)**^{D}. Therefore by (SP) and Strong (LPR): *For each subset A_d of A, there is a variant of Giganto that recombines members of A_d according to $\mathcal{B}_d$ but without the members of $A - A_d$.*

(F8)^D. *There are more than κ such variants of Giganto.*

(F9) through (F14) are unchanged except that 'more than κ' is substituted for '$2^\kappa - 1$'. (F8)^D follows from (F7)**^{D} for the following two reasons: first, if $d \neq e$, for d and e in G_D, then A_d does not overlap A_e and so the variant based on A_d is different from the variant based on A_e (because these variants have non-isomorphic world structures); and, second, the cardinality of G_D is greater than κ. Appealing to (PSD) and (SP) has successfully bridged the gap.[23]

It remains to ask: why accept (PSD) and (SP)? As I said above, they follow from the account of plenitude of structures that I accept. I defended that account in Bricker (1991, forthcoming d), so here I will be brief. Roughly, I hold that a structure is possible—instantiated in some possible world—if either (S1) it plays, or has played, an explanatory role in our theorizing about the actual world, or (S2) it belongs to a natural (mathematical) generalization of structures that are possible in virtue of (S1). (Compare the principles (B) and (PPS) in Bricker 1991.) For example, a Euclidean space of three dimensions is clearly possible by (S1); and therefore so are Euclidean spaces of one and two dimensions (since they are instantiated

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[23] Have we strayed from the core of the original Forrest-Armstrong argument by no longer appealing to a Cantorian diagonalization argument? The argument is still a Cantorian argument that there can be no largest size. Or we could more closely follow the original Forrest-Armstrong argument by switching from worlds with ordinal time to worlds with linear time, and make use instead of the theorem that there are 2^κ non-isomorphic linear orderings of size k, for infinite κ. This theorem is proved by a straightforward Cantorian diagonalization argument. See Rosenstein (1982: 24).

whenever Euclidean space of three dimensions is instantiated). Euclidean spaces of any finite dimension are possible by (S2), being a natural generalization of Euclidean spaces of dimension one, two, or three. I also hold (S3) that any (spatiotemporally connected) possible structure is a possible *world* structure—the complete underlying structure of some possible world. (Compare the principle (PW3) in Bricker forthcoming d.) Returning to our example: it follows from (S3) that, for each finite n, there is an (entirely spatial) world with n-dimensional Euclidean space.

The Substructure Principle (SP) follows immediately from this account of the plenitude of structures. Any (spatiotemporally connected) substructure of a possible world structure is automatically a possible structure, and so, by (S3), a possible *world* structure. The Substructure Principle serves to ground a Humean principle of plenitude that I, and many others, accept, a principle I call the *Principle of Solitude*: for any (spatiotemporally connected) possible individual, there is a world containing a duplicate of that individual and nothing that isn't a part of that duplicate (not even empty spacetime). (For the derivation of this principle, see Bricker forthcoming d; for an application of the principle, see Bricker 2001.) The principle of solitude provides one way—though not the only way (see Bricker forthcoming b)—of supporting the Humean denial of necessary connections (DNC).

The principle of structural diversity (PSD) also follows from my account of plenitude. A simple way to illustrate this invokes what I have elsewhere called "worlds with ordinal time." (See Bricker 1991 (expanded version, §7) for a more detailed version of the argument below.) First, it follows from (S1) that temporal structures with discrete time are possible, including finite structures with n instants of time, for any (non-zero) natural number n. For, surely, both discrete time and finitism have been taken seriously in our theorizing about the actual world. It then follows from (S2) that, for any ordinal number α, temporal structures with discrete time where the instants are temporally well-ordered like the ordinal α are possible structures.[24] For, surely, the ordinal numbers are a natural generalization of the natural numbers. Finally, it follows

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[24] As is usual, I understand an ordinal α to be a well-ordered set every member of which is the set of its predecessors. An ordinal, then, is well-ordered by $\in$.

from (S3) that, for each ordinal α, there is a world structure, and so an (entirely temporal) world, with α instants of time.[25] We have then:

(OT) For any ordinal α, there is a world structure isomorphic to α.

We can use (OT) to argue for (PSD) as follows. Consider any cardinal κ. There are more than κ ordinals of size at most κ.[26] Different ordinals correspond to non-isomorphic world structures of worlds with ordinal time. Therefore, there are more than κ non-isomorphic world structures of size κ, which is to say that modal reality is >κ-diverse. This shows that, in the Forrest-Armstrong argument fortified with (PSD) and (SP), just focusing on the worlds with ordinal time and their duplicates in Giganto is enough to generate the contradiction. Again, we have successfully bridged the gap.

8. WORLDS WHOSE ATOMS ARE MANY?

Nothing stronger than (PSD) was needed to bridge the gap in the Forrest-Armstrong argument. But in supporting (PSD) with (OT), we have invoked a much stronger principle. For it follows immediately from (OT) that *for any* κ, modal reality is >κ-diverse. It might seem that this stronger principle undermines, rather than supports, the Forrest-Armstrong argument. Here's why. The principle (OT) leads directly to the conclusion that the worlds are many, that there is no set of worlds. Accepting that conclusion, however, suggests a way to protect (UPR) from the fortified Forrest-Armstrong argument. Thus far, we have treated the Forrest-Armstrong argument as a *reductio* with (UPR) as its target. But (UPR) wasn't the only assumption of the argument (in addition to the other principles of plenitude). There was also (F4), the assumption that the atoms in any world are few, and form a set. Given that the worlds are many, (UPR) is in direct conflict

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[25] One might wonder what distinguishes a temporal structure with α instants from a one-dimensional spatial structure with α points. I would say: they do not differ intrinsically; they differ, when they do, in virtue of the pattern of instantiation of qualitative features. But note that the present argument can be run without supposing the structure is intrinsically temporal or spatial.

[26] That is how the next cardinal number after κ is defined when cardinals are identified with initial ordinals. See, for example, Hartogs' Theorem and its proof in Enderton (1977: 195-6).

with (F4): the world Giganto guaranteed to exist by (UPR) will have non-overlapping duplicates corresponding to all the worlds, and so will itself have many atoms. That suggests two things. First, the Forrest-Armstrong argument wasn't needed to attack (UPR): (F4) and the fact that the worlds are many can do the job all by themselves. But, second, if we instead want to hold on to (UPR), we can use it and the fact that the worlds are many to reject (F4). In that case, we simply take the target of the fortified Forrest-Armstrong argument to be (F4) rather than (UPR). (UPR), it appears, is immune even to fortified versions of the Forrest-Armstrong argument.

But rejecting (F4) cannot provide a defense of (UPR). (F4) was not essential to the Forrest-Armstrong argument. It was merely a convenient posit that allowed the argument to be given a set-theoretic formulation. Even if we were to allow that worlds may have many atoms, we could still prove the contradiction that Giganto is bigger than itself. The proof can be carried out in various frameworks that invoke proper classes, such as Bernays-Gödel class theory with urelements.[27] Or it can be carried out in Lewis's "megethology," without ever mentioning proper classes (or sets, for that matter). I will here just sketch informally how such a proof would go.

Let us say that some things are *barely* many iff they correspond one-one with the cardinals of pure set theory.[28] (F4) and (F5) are replaced with the weaker:

(F4)⁺. Assume: *For any world, the atoms in that world are few or barely many*.

(F5)⁺. Then: *The atoms in Giganto are few or barely many.* Call the atoms in Giganto *aa*.

We then complete the argument as a simple dilemma. One fork, assuming that *aa* are few is unchanged. The other fork, assuming that *aa* are barely many, continues as follows:

(F6)⁺. *There are many, but not barely many, fusions of atoms among aa.*

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[27] See Uzquiano (2015: 9) for the coding trick that will be needed to carry out the reasoning in Bernays-Gödel class theory. Hawthorne and Russell (2018) give a detailed version of the Forrest-Armstrong argument where (F4) (what they call "Marble Set") is rejected, and worlds are allowed to be large.

[28] Lewis (1993) has instead this definition: *xx* are *barely many* iff they correspond one-one with all the atoms in reality. On the significance of this difference, see Section 9 below.

(F7)$^+$. *For each fusion of atoms among aa, there is a variant of Giganto in which just the atoms in that fusion remain and the rest have been deleted.*

(F8)$^+$. *There are many, but not barely many, such variants of Giganto.*

(F9) through (F14) are unchanged except that 'many, but not barely many' is substituted for '$2^\kappa - 1$'. (F8)$^+$ follows immediately from (F7)$^+$ for the *haecceitist* version of the argument.

The anti-haecceitist can fortify the argument by invoking principles of plenitude stronger than (PAD) or (PSD). Thus, we can strengthen (PAD) by adding:

(PAD)$^+$. If some world contains barely many atoms, then some world contains barely many atoms, no two of which are duplicates of one another.

(PAD)$^+$ together with (UPR) will guarantee that, if Giganto contains barely many atoms, then Giganto contains barely many *kinds* of atom. And then, by an argument paralleling the argument from (PAD) in Section 6, it will follow that there are many, but not barely many, variants of Giganto. Invoking (PAD)$^+$ bridges the gap on the "barely many" fork of the dilemma.

Alternatively, we can bridge the gap using the plenitude of world structures. we can strengthen (PSD) by adding:

(PSD)$^+$. If some world contains barely many atoms, then there are many, but not barely many, non-isomorphic world structures.

We can support (PSD)$^+$ in the same way that we supported (PSD) by considering worlds whose instants of time are well-ordered. If we were to allow worlds whose atoms are barely many, there could be no good reason not to allow a world whose instants of time are ordered like all the ordinals, one instant for each ordinal number. But we couldn't stop there. There would be a world with one more instant, and another, and so on. To turn this into a proper argument, we could introduce, on top of the ordinal numbers, the *super-ordinal* numbers. For every way of well-ordering barely many elements, let there be a *super-ordinal* number that represents that

well-ordering.[29] Just as the ordinal numbers are a natural generalization of the natural numbers, the super-ordinal numbers are a natural generalization of the ordinal numbers. Given the account of plenitude of world structures I endorsed above, we will have:

(OT)$^+$ For any ordinal or super-ordinal α, there is a world structure isomorphic to
   α.

(OT)$^+$ will entail (PSD)$^+$ by the same reasoning that (OT) entails (PSD). And then, by an argument paralleling the argument from (PSD) in Section 7, it will follow that there are many, but not barely many, variants of Giganto. Invoking (PSD)+ gives a second way of bridging the gap on the "barely many" fork of the dilemma.

I conclude that (F4) was not needed for the Forrest-Armstrong argument. A weaker assumption, (F4)$^+$, will do. But wait: why not take the +-version of the Forrest-Armstrong argument to be a *reductio* of (F4)$^+$ and allow that the atoms of some world may be many, but not barely many? Well, you know how the dialectic will go. Whatever imaginary über-size we allow worlds to have, the über-sized version of the Forrest-Armstrong argument will show that either (UPR) must be rejected, or we must allow worlds to have some über-über-size. If I thought that modal reality was *indefinitely extensible*, I suppose that this would allow me to hold on to a modalized version of (UPR). But if, as I have been assuming, modal reality has some determinate size, *whatever it may be*, then some version of (F4) that gives the size of worlds must hold, and the Forrest-Armstrong *reductio* will be aimed squarely at (UPR). There is no escape.

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[29] These super-ordinals should not be thought of as set-like entities. They are just well-ordered pluralities. Hazen (1996, 2004) would claim that it is not even *conceptually possible* to posit "super-ordinals," or, indeed, a world whose atoms are many. Any attempt to do so just extends the domain of sets so as to include the posited entities. It is constitutive of the distinction between sets and proper classes that proper classes always remain beyond the horizon of our speculations about reality. Hazen's view raises deep questions. In short, I would say that it rests on a conception of reality as indefinitely extensible that I reject.

9. MODAL REALITY AND LIMITATION OF SIZE

The upshot of the somewhat intricate argumentation of the last three sections is this. The Forrest-Armstrong argument, when fortified in various ways by plausible principles of modal plenitude, is alive and kicking. And so the unrestricted principle of recombination (UPR) must go. It must be replaced by a restricted principle such as (LPR). (Or strong (LPR). But if one follows me in accepting (PIP), strong (LPR) can be derived from (LPR) and (PIP); see Bricker forthcoming b.) According to (LPR), some things can be recombined *only if* there is a possible arrangement of those things. And what arrangements are possible will depend in part on what world structures there are within which to do the arranging. The question—what is the appropriate restriction on recombination?—is reduced, at least in part, to the question: what world structures are possible? For the purposes of this chapter, what matters most is just one aspect of that question: what are the possible *sizes* of world structures?

The principle (OT) gives part of the answer. For every cardinal κ, there is a world, and so a world structure, of size κ. There is no cardinal bound on the size of worlds, or world structures. That tells us that the worlds are many, but it leaves open *how many*. To get purchase on that question, we need to make some supposition as to what the possible sizes are that exceed every cardinal. I propose, in line with many others, that we endorse some version of the doctrine of Limitation of Size. By Limitation of Size, I mean more than the claim that some pluralities are "too big" to form a set. What I take to be essential to the doctrine is, first, that there is some maximal size, the size of all the parts of reality, and, second, that the cardinal numbers of ZFC determine this maximal size. That there is some maximal size follows from my view that reality is definite, not indefinitely extensible; and this follows from my rejection of primitive modality. In saying that the cardinal numbers determine this maximal size, I do not say that there is only one size that exceeds all the alephs. There are more fusions of cardinals than cardinals (if, as I think, the pure sets are atoms and therefore do not overlap). So there are at least two sizes that exceed all the alephs. Moreover, for all I know, there may be sizes between the size of the cardinals and the size of the fusions of the cardinals. (To deny that would be to endorse, for no known reason, an even more general General Continuum Hypothesis.) So, in saying that the cardinals determine the limit on size, I am saying that maximal size is given by the size of the

fusions of the cardinals, not by the size of the cardinals.[30] The possible sizes divide into three categories—few; barely many; and many, but not barely many—where the third category includes a maximum size. If a plurality has this maximal size, we can say that it is *maximally many*.

With Limitation of Size in place, we can give definite answers to the two questions about the size of modal reality raised in Section 2. First, we have:

Size of Reality. The worlds in modal reality are barely many.

For the worlds in modal reality are many, by (OT), and if the worlds were many, but not barely many, there would be more worlds than cardinals, and so more fusions of worlds than fusions of cardinals (since worlds do not overlap), contradicting Limitation of Size. Since worlds do not overlap, it follows from Size of Reality that the atoms in modal reality are many, and that modal reality is large. Second, returning to our question—what are the possible sizes of world structures?—we have what was earlier labelled '(F4)':

Size of Worlds. For every world, the atoms in that world are few; in other words, every world is small. Moreover, for every cardinal κ, some world has exactly κ atoms.

The second half, as noted above, follows from (OT). To see why the first half holds, suppose there was a world with barely many atoms. Then by (PSD)⁺ (which I defended in the last section on the assumption that some world has barely many atoms), there would be many, but not barely many, non-isomorphic world structures, and so many, but not barely many worlds, contradicting

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[30] If one accepted proper classes as parts of reality, then one could instead take the *classes* of cardinals to give the maximal size. But if, like me, one takes talk of proper classes to be best interpreted in terms of plural terms and plural quantifiers, then this would amount to taking the *pluralities* of cardinals to give the maximal size, and I would reject that because I do not apply Limitation of Size to pluralities. Indeed, since there are more pluralities of fusions of cardinals than there are fusions of cardinals, the size of the fusions of cardinals would not be the maximal size that a plurality could have. Could we instead take the pluralities of fusions of cardinals to give the maximal size? I think not. I speculatively endorse (in Bricker forthcoming a) an "ideological hierarchy" that allows taking, for any plurality, a plurality of its subpluralities, and so on without end. See Linnebo and Rayo (2012) on the distinction between ontological and ideological hierarchies.

Size of Reality and Limitation of Size. Size of Reality and Size of Worlds together entail that there are barely many atoms in modal reality. It then follows that, although there is more than one way for a plurality of parts of reality to be many, there is only one way for a part of reality to be large: a part of reality is large if and only if it contains barely many atoms.

This neat little package respects the fortified versions of the Forrest-Armstrong argument as well as all of the principles of plenitude that I have endorsed throughout the course of this chapter.[31] It ensures that there will be a restriction on recombination sufficient to avoid the paradox. For according to Size of Worlds, every world is small, and so no world is big enough to recombine all the many worlds. There is no one world such as Giganto that reflects all of modal reality, and so (UPR) can be safely rejected.

Is there any cost at all to rejecting (UPR)? One might think that (UPR) is needed to support an intuition that universal actualization is possible. How, without Giganto, do we get the possibility that the goings-on of every possible world could be actualized together? But (UPR) is not needed, nor wanted, to capture this possibility. In Bricker (2001), I argued that we need to divorce what is possible from what is true in some possible world. Rather, possibility should be analyzed in terms of plural, not singular, quantification over worlds. To be possible is to be true in some world *or some worlds*. The possibility of (absolutely isolated) island universes, for example, is the possibility that more than one world is actual. The possibility of universal actualization, then, is just the extreme case of the possibility of island universes: the possibility that every world is actual. So, if one rejects the orthodox (and Lewisian) analysis of possible worlds as I do, there is no problem accommodating universal actualization without (UPR). And that is to the good, because the intuition supporting this possibility has nothing to do with recombination. Even if there were such a world as Giganto, it would be a poor substitute for the possibility of universal actualization. For the situation in which Giganto is actualized and other worlds are not only indirectly represents the situation in which all of modal reality is actualized. Worse, it imputes additional structure—the world structure of Giganto—that was no part of the possibility in question. Recombination is needed to capture many of our beliefs about the

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[31] It is a package I have been promoting since the mid-80s. See especially Bricker (forthcoming b).

plenitude of possibilities. But our belief in the possibility of universal actualization is not one of them.

Is there a cost to founding an account of modal reality on a doctrine of Limitation of Size? Limitation of Size is a fundamental assumption that does not follow from more fundamental assumptions; but if that is a cost, it is a cost borne by all theories. Still, even fundamental assumptions need reasons to believe them. My version of Limitation of Size holds that the mathematical part of reality provides an absolute measure of size for all of reality. In particular, the size of modal reality is constrained by the size of the cardinals. There can be too many atoms in modal reality for them to form a set, but not too many for them to be in one-one correspondence with the cardinals. This must be included as a substantial "principle of the framework." (On "principles of the framework, see Bricker forthcoming c, postcript.) I confess I do not know how to defend the truth of this principle. No doubt there are pragmatic arguments based on the relative simplicity of the resulting account of reality, or based on some doctrine of philosophical conservatism. But I do not take such arguments to bear directly on truth. Limitation of Size is a reasonable supposition to make: it attributes to reality the minimal size that leaves room for all the principles of plenitude I have reason to believe; and selecting any other size for reality would be arbitrary. But I am doubtful that that provides much of an argument for its truth.

Lewis (1991) also endorses a version of Limitation of Size according to which there are at least two sizes beyond the alephs. On his account of classes, classes are fusions of singletons. It follows that there are more classes than there are singletons and, since the singletons are many, at least two ways for a plurality to be many. Lewis's account differs from the account I have given above, however, by defining 'many' and 'barely many' relative to reality as a whole. He does not take the cardinals of pure set theory to provide an absolute measure of size. Lewis asserts, as I do, that the atoms in reality are barely many, where for him the atoms include the singletons. But for him this is the *definition* of 'barely many', and so not a substantial assumption. Does that make his account preferable in virtue of carrying less baggage? I think not. Lewis's account leaves it open whether the cardinals of pure set theory are barely many, *or some lesser size*. For all that Lewis explicitly says, there may be fewer cardinals than there are

atoms in modal reality, atomic urelements.[32] No doubt Lewis did not think there were more atoms than cardinals. But his account does not seem to rule this out. He has two options. He can add the assumption that the cardinals are barely many, thus equating the size of the cardinals with the size of the atoms in modal reality. But then, definitions aside, the account would carry the same baggage as my account, and be no easier to defend. Or, he can decline to make the assumption, keeping all measures of size beyond the alephs relative, and not absolute. He can plead ignorance, leaving it entirely open how many atomic urelements there are in any absolute sense. But then I do not see how the account deserves to be called "Limitation of Size." No limit is being placed on the size of reality. It is whatever size it is, and all measures of size are determined relative to that.

Might it be possible to do away with Limitation of Size and rest solely on the iterative conception? We can still say, based on the iterative conception, that some pluralities are "too big" to form a set. But we put no limits on how big these pluralities can be. Indeed, Size of Worlds can be based on an iterative conception of world structures if we take world structures to be represented by sets (as I do). Let us say that a *mathematical* structure is an ordered pair whose first element is a pure set, the "domain" of the structure, and whose second element is a set of relations over that domain, where a relation over the domain is a set of ordered pairs of members of the domain, and ordered pairs are taken to be sets in the usual way. Then every mathematical structure is itself a set that shows up in the iterative hierarchy five levels beyond the level where the domain shows up. It follows that there are barely many mathematical structures. Now, I am not suggesting that we *identify* world structures with mathematical structures. For one thing, world structures have coarser individuation conditions. For another, I have not been supposing in this chapter that every mathematical structure corresponds to a world structure, only that the mathematical spacetime structures do.[33] But it is enough to say that each world structure is

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[32] By 'urelement', I mean any entity that is not a set, or class, or other set-like entity. Lewis's definition of 'urelement' is more restrictive, requiring that urelements be *individuals*, that is, members of some class. Mixed fusions of classes and individuals are not members of classes for Lewis, and so not urelements. But note that the usages agree on what counts as an atomic urelement.

[33] In fact, I hold to a very expansive principle of plenitude for structures: every mathematical structure is instantiated by some maximal isolated portion of reality; see Bricker (forthcoming c). And I hold that mathematical structures may be "higher order," and thus go beyond the relational

represented by some mathematical structure, where the (atomic) places in the world structure correspond one-one to the members of the domain. There is thus a mapping from some of the mathematical structures *onto* the world structures. It follows that there are barely many world structures, and that the (atomic) places of any world structure form a set. An iterative conception of mathematical structures thus leads to an iterative conception of world structures. And an iterative conception of world structures is all that is needed to get Size of Worlds. (Remember, we are still assuming that there is no co-location; I will revisit that assumption in the final section.) But an iterative conception of world structures does nothing to support Size of Reality. We need Limitation of Size to get any definite claim about the size of modal reality as a whole. For if any atom, or any world, has many, but not barely many, duplicates in modal reality, then Size of Reality is false. The iterative conception plausibly places a limit on the plenitude of world structures, but not on the plenitude of world contents. We need a limit on both to get a definite size for modal reality. We need Limitation of Size.

Of course, one can object in familiar ways to the iterative conception of mathematical structure. Consider the ordinals. On the usual construction of the ordinals as sets, the ordinals are well-ordered by $\in$, the membership relation. Doesn't that show that the ordinals form a mathematical structure, a structure beyond all the levels of the iterative hierarchy? But then shouldn't there also be a corresponding world structure, and a world with ordinal time whose instants of time are unbounded by any ordinal number? I think not. The pressure to accept large structures is really no different from the pressure to accept large set-like entities beyond all the sets, such as proper classes. I have nothing new to add to the debate. To accept proper classes, or large structures with proper class domains, where these classes are somehow forbidden from being members of other classes, is unacceptably *ad hoc*. To allow these large set-like domains to be members of other set-like domains gains nothing, and lands us back in the same dilemma. The best response is to reject the large set-like entities and structures. The intuition that whenever there are some things, there is a set-like entity with those things as members, or that whenever there are some things related in some way, there is a structure with a set-like domain with those

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structures mentioned in the text.. But all that is compatible with an iterative conception of structure. See Hawthorne and Russell (2018) for ways of regimenting principles of this sort within model theory.

things as members, is faulty, and has to go. It is a vestige of a general conflation in thought and language between pluralities and single entities. When there are some birds flying overhead, I may say that there is a flock of birds. I may add that the flock has the shape of a vee.. But I don't thereby refer to a set or set-like entity. What I say can be fully captured within a language with plural terms and plural quantifiers. It is no different with the ordinals, or other pluralities of *many* things. We can say everything we want to say about the ordinals plurally, including how they are well-ordered, using plural terms and irreducibly plural quantification. We can even compare the sizes of pluralities of many things by quantifying over relations between those pluralities where this is reduced to plural quantification over ordered pairs and ordered pairs are construed non-set-theoretically. (For example, we can quantify plurally over things that encode the ordered pairs mereologically; for one way of doing this, see Lewis 1993: 222-4.) I do not see any way that large set-like entities or structures will be missed. [34]

One might try to argue directly for a large world, and so a large world structure, whose instants of time correspond to the ordinal numbers. For although I deny that whenever there are some things, there is a set-like entity with those things as members, I accept that whenever there are some things, there is fusion with those things as parts. If there were worlds with ordinal time nested one within the next, with one world for each ordinal, then indeed taking the fusion of these worlds would lead to a large world with as many instants of time as all the ordinals. But here is where the thesis of structural pluralism introduced in Section 2 comes into play. The worlds with ordinal time are all isolated from one another, and do not overlap. They have a fusion all right; but their fusion is an aggregate of worlds, not a world. I conclude that there are no good reasons why an account that accepts the worlds with ordinal time, all of which are small, must also accept a large world whose instants of time match the entire sequence of ordinal numbers.

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[34] Hawthorne and Russell (2018: 20) introduce what they call "plural structures"—certain indexed families of pluralities—whose domains may be too big to form sets. If we allowed such "plural structures" to be instantiated at worlds, then Size of Worlds, of course, would be false. I have no objection to speaking of these "plural structures," but I do not count them as possible structures in the relevant sense: they are not candidates for providing the underlying or instantial structure at worlds. Worlds I take it are essentially unified. Instantiating a "plural structure" does not bestow the requisite unification.

10. Hawthorne and Uzquiano: Arguments Against Modal Realism

Hawthorne and Uzquiano (2011) reject the neat little package expressed by Size of Reality and Size of Worlds. They have two main arguments, one based on the Urelement Set Axiom, the other based on a version of Limitation of Size stronger than what I endorsed. They conclude that a "modal realist" will be forced to hold that there is some cardinal bound on the size of possible worlds, the view endorsed by Lewis (1986). I argued against there being any such cardinal bound in Bricker (forthcoming b, 1991). It would require that, for some ordinal α and beyond, there are no worlds with ordinal time with α instants of time. In this section, I say why I do not find their arguments persuasive. In short: one can reject with impunity the main assumption of each of their arguments. They also have an independent argument *ad hominem* against Lewis's theory of classes. I do not accept Lewis's theory (see Bricker forthcoming a, §2); but in any case, there is a natural fix that avoids the problem. An independent third argument based on the possibility of co-location can also be gleaned from their discussion. I postpone my response until the next section. I am still under the working assumption that all parts of worlds are spatiotemporal parts.

The thesis Size of Worlds is captured by what Hawthorne and Uzquiano call "Indefinite Extensibility," which they express as follows:

> *Indefinite Extensibility.* There could not be so many angels as to exceed each and
> every aleph, but for each α, there could be exactly $\aleph_\alpha$-many angels in
> existence.

Before discussing the arguments they give against Indefinite Extensibility, let me say three things with respect to how it compares with the thesis Size of Worlds that I endorse. First, a terminological point. I prefer to restrict "indefinite extensibility" to the view that some primitive modality is needed to characterize the extent of reality, a view I reject. They are not supposing that the modality in terms of which Indefinite Extensibility is expressed is primitive. Indeed, it is interpreted in terms of quantification over possible worlds when they present their arguments against the modal realist. Second, although the discussion around their introduction of Indefinite Extensibility suggests that they are concerned with how many angels can be co-located at a single point of spacetime, their two main arguments apply no less to how many angels can exist in a world, whether co-located or not. I consider the special problem raised by co-location in the

next section. Third, nothing in their arguments depends on any theological assumptions about angels. I set their "transcendental theology" aside here, and consider only how many (mereological) atoms can exist in a world. Wherever they say 'angels' in their arguments, I say 'atoms'. I turn now to their two main arguments, which I will take to be directed against Size of Worlds.

Their first argument is just that Size of Worlds is incompatible with the *Urelement Set Axiom*, that the urelements form a set; for the second half of Size of Worlds entails that there are many atoms throughout modal reality. I do not contest that the Urelement Set Axiom is part of orthodoxy; it has been presupposed, if not explicitly stated, by the majority of those who accept an iterative conception of impure sets. (Zermelo (1930), however, is an important exception.) But this is only, I think, because prior to serious reflection on principles of modal plenitude, the idea that the urelements fail to form a set was given little serious consideration. In any case, accepting the Urelement Set Axiom is out of the question if, like me, one accepts unrestricted composition and holds that the pure sets (or even just the singleton pure sets) are mereological atoms. For any pure set, there is a distinct fusion of that pure set with the Eiffel Tower; so there are as many such fusions as there are pure sets, too many urelements for them to form a set.

Orthodox or not, I do not know of any good reason to accept the Urelement Set Axiom. As noted in Section 2, the cardinals of pure set theory are all that is needed to determine which pluralities of urelements form a set: a plurality of urelements form a set just in case they are in one-one correspondence with the members of some cardinal. Thus, if a plurality of urelements is few, they form a set; if many, they do not form a set. Given that we have the means to say which pluralities of urelements form sets and which do not, what reason could there be for requiring that the plurality of all urelements form a set?

I am here disputing Lewis's (1986: 104) claim that we "have no notion what could stop any class of individuals—in particular, the class of all worlds—from comprising a set." In support of this, he writes: "the obstacle to sethood is that the members of the class are not yet all present at any rank of the iterative hierarchy. But all the individuals, no matter how many there may be, get in already on the ground floor." I do not see why using the cardinals of pure set theory as an absolute measure of size for pluralities of urelements does not suffice to give us a notion of why some pluralities of urelements do not form a set. Thus there is no need to define an iterative hierarchy over the urelements in order to say which pluralities form a set. But if we

choose, we can apply the iterative conception directly to the urelements and get the same result. Go ahead and put all the urelements (along with the null set) on the ground floor. Then use the hierarchy of pure sets to determine which impure sets to add at subsequent stages. In particular, in forming the next stage of the hierarchy based on the urelements, put in sets whose members are pluralities of sets and urelements from previous levels just when the cardinality of that plurality does not exceed all the cardinalities of the pure sets at that stage of the pure hierarchy. Eventually, all and only those pluralities of sets and urelements that are not too big to form a set will find a place in the hierarchy. Letting all the urelements "get in already on the ground floor" does not require that one ever introduce a set of all the urelements.[35]

The main reason Hawthorne and Uzquiano give for accepting the Urelement Set Axiom is that rejecting it "threatens the universality of mathematics, which is supposed to investigate structures presented by the other sciences": there may be "structures constituted by non-sets" that cannot be mathematically represented (pp. 11-2). But in my view the structures instantiated in worlds are mathematically represented by (pure) mathematical systems that share those structures, by isomorphism. No sets based on urelements are needed for mathematical representation. Granted, such mathematical systems can only represent structures with small domains. But given my structural pluralism—the absolute isolation of worlds—there is no (non-logical) structure that encompasses (large) aggregates of worlds. I conclude that Size of Worlds, and the rejection of the Urelement Set Axiom, is no threat to the universal applicability of mathematics.

Hawthorne and Uzquiano concede that the case for the Urelement Set Axiom is not overwhelming. They put more weight on a second argument to which I now turn. First, they characterize the thesis of Limitation of Size as follows: "a plurality forms a set if and only if they

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[35] This is, in essentials, the method of Zermelo (1930), what he calls the "canonical development" of the cumulative hierarchy. A different response to Lewis's argument comes from Menzel (2014). If one is willing to reject Limitation of Size and weaken the Replacement Axioms, then one can allow that the urelements are greater in size than any aleph and still form a set—a so-called "wide set." This allows one to accept Size of Worlds *and* the Urelement Set Axiom. And it allows one, as is customary, to take that next stage of the iterative hierarchy to contain, for *any* plurality of sets and urelements formed at previous stages, a set with that plurality as its members. I object that, without Limitation of Size, we have no way of saying how wide the set of urelements is. But such agnosticism might instead be taken to be a positive feature, not a bug.

are not in one-one correspondence with the entire universe of all objects." Call this the "axiom" of Limitation of Size. It follows immediately from this that the *only* way for a plurality to fail to form a set is for them to be in one-one correspondence with the universe of all objects, and that therefore there is only one size that pluralities that do not form sets can have. But this conflicts with unrestricted composition, a principle that I am committed to. Their argument is this. First, for any plurality greater than one, there are more subpluralities than members of that plurality.[36] Now consider the plurality of atoms in modal reality. On my view they are barely many, the same size as the cardinals. But every subplurality of the plurality of atoms has a fusion, and different subpluralities have different fusions. So there are more fusions of atoms than there are atoms. But that contradicts the axiom of Limitation of Size, which only allows one way for a plurality to be many.

It would not be fruitful to enter a dispute over what to call "Limitation of Size." Weaker and stronger versions of the doctrine have been put forward from Cantor to the present day.[37] The strong version that Hawthorne and Uzquiano focus on comes from von Neumann. It takes a plurality's failure to be in one-one correspondence with the universe of objects to be a sufficient condition for the plurality forming a set. The weaker version of Limitation of Size that I endorse must reject this sufficient condition, since there are pluralities that neither form a set nor are in one-one correspondence with the universe. Rather, the sufficient condition for forming a set is being in one-one correspondence with some cardinal. The question that matters is: what benefit does the stronger version provide that the weaker version does not? According to Hawthorne and Uzquiano, a significant consequence of von Neumann's axiom of Limitation of Size is this: "The scale of alephs … form a proper foundation for the metaphysics of size by forming a kind of universal ruler, in the sense that the size of a plurality is determined by its relation to the ruler." (p. 13) Their ruler, they go on to say, has a notch for each aleph: if the plurality corresponds with an aleph, that aleph gives its size; if not, then one can deduce that it has the size of the universe. (Here I suppose they intended the ruler to have a notch not just for each aleph, but for each cardinal number, including the natural numbers.) But it seems to me the weaker version of

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[36] They sketch a proof of this claim, which they call "Remark 2," on pp. 60-2.

[37] See Hallett (1984) for a discussion of weaker and stronger versions of the doctrine of Limitation of Size.

Limitation of Size also provides a foundation for the metaphysics of size. Extend the ruler to add a notch for the size of the cardinals and another notch for the size of the fusions of cardinals. (The space between these two notches can be left as a blur.) We can speak of a plurality corresponding to these notches beyond all the alephs no less than to the notches given by the alephs themselves. If a plurality corresponds to the former added notch, it is barely many; if it corresponds to the latter added notch, it is maximally many. We can still say, it seems to me, that "the scale of alephs [cardinals] forms a proper foundation for the metaphysics of size."

If there is a cost to endorsing the weaker version of Limitation of Size, it presumably comes from this: on the weaker version, Global Choice does not follow from Limitation of Size and would need to be posited as an additional axiom. (Global Choice is the principle that there is a function that selects from every plurality exactly one member of that plurality; how to represent the "function" in question will depend on the framework.) That may be a cost in systematization. But it is not a cost in our ability to know the truth. Global Choice, I say, is obviously true. (Indeed, I never met a choice principle applying to sets, or set-like entities, or pluralities that I didn't like.) But perhaps the problem is this. Global Choice implies a global well-ordering of the universe by the usual argument. But if the universe is greater in size than the ordinals, there will be well-orderings with no standard ordinal numbers to represent them. There is a temptation, then, to introduce super-ordinals to measure these large well-orderings. But the introduction of super-ordinals would lead to an argument for super world-structures, say, to be the structure of worlds with super-ordinal time. (See Section 8 above.) And we would be off to the races once again. But, as already noted, such super-ordinals can and must be resisted. We can allow that some pluralities can be well-ordered without introducing set-like entities in reality to serve as measures of those well-orderings.[38]

It is worth noting that Hawthorne and Uzquiano are committed to allowing, in a sense, that there is more than one size beyond the alephs. For they accept that, for any plurality more numerous than one, there are more subpluralities than members. And this suggests a clear sense in which it is legitimate ask how many subpluralities of alephs there are, and to answer: a size greater than the size of all the alephs. There are at least two absolutely infinite sizes. A similar

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[38] See Uzquinao (2015: 15-6) who argues on somewhat different grounds that allowing for more than one size beyond the alephs will push one to introduce what I call "super ordinals."

dialectic then ensues. Global Choice should somehow be extended to apply not only to pluralities, but also to pluralities of pluralities; and it should follow that any plurality of pluralities can be well-ordered. There is the same pressure to introduce super-ordinals, and the same reason to resist that pressure. To deny that we can meaningfully assign sizes to these pluralities of pluralities, whether directly or by coding, would seem to belie the obvious. One must beware, however, as always, that "seeming obvious" has a checkered history when it comes to sets and classes and pluralities; Russell's paradox lies in wait just beyond the corner.

Hawthorne and Uzquiano (2011: 21-2) consider the weaker version of Limitation of Size that I endorse only in connection with Lewis's theory of classes. They rightly notice that Lewis's theory runs into trouble if he rejects the Urelement Set Axiom, and holds that the urelements are many. Their argument is this. According to Lewis, the atoms in reality set the standard of size: there are barely many atoms. Suppose Lewis holds that there are barely many urelements among the atoms (as I do). Then, since Lewis accepts unrestricted composition, there are more fusions of urelement atoms than atoms; that is, the fusions of urelement atoms are many, but not barely many. But Lewis also accepts a fundamental principle called Domain that gives the domain of the singleton function, part of which says, any part of the null set has a singleton. (The null set is the sum of all the urelements.) Then, since any fusion of urelement atoms is a part of the null set, it follows that any fusion of urelement atoms has a singleton. So there are many, but not barely many, singletons. But the singletons are mereological atoms on Lewis's theory. So, there are many, but not barely many atoms. *Contradiction.*

The first thing to say is that this problem is peculiar to Lewis's theory of classes. Since I reject that theory, I do not take the problem to bear on my acceptance of Size of Reality and Size of Worlds. In particular, I endorse no principle like Domain that maps the urelements one-one into the atoms. The second thing to say is that the problem does not afflict Lewis prior to 1991 when *Parts of Classes* was written. At that time, he still believed, as he explicitly endorses in *On the Plurality of Worlds*, that the urelements are few, not many. But by 1991, Lewis was willing to allow that the worlds are many, and that there is no cardinal bound on the size of worlds. (See Bricker 1991 expanded version, n. 17.) That change in view would require some modification to his theory of classes. The most natural fix would be to accept Domain in a weaker version that claims only: any *small* part of the null set has a singleton. That restriction is mysterious, I think, but no more mysterious, and along similar lines, as the restriction on singleton formation that

Lewis is already committed to, namely, that only sets, not proper classes, have singletons. With the restriction in place, it no longer follows that there are many, but not barely many singletons, and the contradiction is blocked.[39]

11. DISCHARGING ASSUMPTIONS AND CONCLUSION

Prior to plunging into the details of the Forrest-Armstrong argument and its numerous variations, I made two simplifying assumptions. I assumed that no worlds have non-spatiotemporal parts (thus ruling out spatiotemporal co-location), and that no worlds contain atomless gunk. In fact, I disbelieve the first assumption, and am (officially) agnostic about the second. (For how I think the possibility of gunk relates to principles of plenitude, see Bricker forthcoming d.) In this final section, I check whether these assumptions might have affected any of my conclusions. The most important thing to note in this regard is that each of these assumptions, if false, had the effect of restricting our attention to only some of the worlds in modal reality, rather than all of the worlds. But adding more worlds does nothing to undermine the force of the Forrest-Armstrong argument. For if Unrestricted Recombination requires that there be a world Giganto that leads to contradiction, it matters not whether Giganto recombines some but not all of the worlds. All that matters is that Unrestricted Recombination when combined with plausible principles of plenitude entails that some world exists that cannot possibly exist. Thus the main conclusion, that the Forrest-Armstrong argument or one of its variations is sound, in no way depended on those two assumptions.

But perhaps rejecting one or both of these assumptions makes trouble in another way, by undermining my defense of Size of Worlds. That defense rested on my account of plenitude of

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[39] When I first read a draft of *Parts of Classes* in 1989, I sent Lewis comments, one of which was aimed at the problem currently under discussion. I wrote: "For *Domain*, do you want: any *small* part of the null set has a singleton? Or do you want another hypothesis: the null set is small? (Am I the only one who worries about this?!)" (Letter of March 22, 1989). Replying to this comment, Lewis wrote: "I don't know whether I want to add either of these things. Do you see any trouble from leaving them off? If not, I want to be guided by the pursuit of orthodoxy. However I'm not clear whether orthodox set theory rules out that there might be so many individuals that the class of them is a proper class. Part of the problem is that axiomatic set theory is so often pure set theory." (Letter of June 13, 1989).

structures, and in particular, an iterative conception of structure according to which world structures are represented by sets. But perhaps once co-location or gunk is allowed, an iterative conception of structure will not be sufficient to defend Size of Worlds. Consider first co-location. In fact, I favor a trope theory that allows multiple tropes to be co-located at the same point of spacetime. (See Bricker forthcoming a, §5 and Bricker 2017.) What I say here about co-location of tropes applies also to other putative cases of co-location involving, say, bosons or angels, whether or not the bosons or angels are themselves, as I think, composed of tropes.) I also allow that throughout modal reality the atomic tropes are many, indeed, the *kinds* of atomic tropes are many, where atomic tropes are of the same kind iff they are duplicates of one another. In virtue of what do I hold that the atomic tropes co-located at a single spacetime point must be few? Even if an iterative conception of structures is accepted, so that the places in any world structure are few, how does that entail that the atoms at any world are few in accordance with Size of Worlds? Why can't a world be large, not in virtue of having a large spacetime, but in virtue of having many tropes (or bosons or angels) co-located at some point of spacetime? Call this *hyper-co-location*. It appears that a separate principle would have to be introduced to prohibit hyper-co-location, a restriction on what recombinations of tropes are possible. But that goes against my claim that the only restriction on recombination comes from the plenitude of structures, from what structures are possible. A separate restriction on recombination, say, restricting how many duplicates can be co-located, might seem unmotivated and *ad hoc*.[40]

I respond as follows. No separate restriction on recombination is needed because co-location is a structural feature; worlds with and without co-location *differ structurally*. There are two ways, however, in which co-location may figure into the structure of a world, and it is important to distinguish them. One way takes co-location to be reflected in the world's "underlying structure"; the other takes co-location to be reflected in the world's "instantial structure." Recall from Section 7 that the underlying structure, which we have been supposing is

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[40] This argument is adapted from Hawthorne and Uzquiano (2011: 18), though they direct it explicitly against Lewis's (1986) view that there is a cardinal bound on the size of worlds. They write: "a restriction on the possible size and shapes of concrete universes do not help us much in a context where angels can be packed into a single point." See also Pruss (2001) who had earlier argued that Lewis's proviso "size and shape permitting" will be ineffective if co-location is possible.

spatiotemporal, is the structure over which the fundamental elements are distributed. Suppose, for purposes of illustration, that these fundamental elements are atomic tropes. The instantial structure then arises from the pattern of instantiation, or occupation, of the atomic tropes, where the pattern reflects which places in the underlying structure are occupied by duplicate tropes (and so instantiate the same fundamental property). Consider first how spatiotemporal co-location may be reflected in the underlying spacetime structure. When atomic tropes are co-located, each of the tropes separately stands in spatiotemporal distance relations to some or all of the other tropes; the co-located tropes themselves are distance zero from one another. On this way of understanding spacetime structure, we only get a standard spacetime (Minkowskian, Euclidean) when we identify the co-located tropes, and take the quotient structure. (If the standard spacetime is a metric space, the underlying structure is a *pseudo*-metric space, allowing distinct elements to be zero distance apart.) Now, the iterative conception of structure requires that the places in the underlying pseudo-spacetime structure are few. And so if spatiotemporal co-location is understood in this way, as being reflected in the underlying structure, it follows that the co-located atomic tropes must be few. No separate restriction on recombination is needed.

But I suppose we might instead understand spatiotemporal co-location to be reflected in the instantial structure. On this approach, the underlying structure is the same, whether there is co-location or not; but the places in the underlying structure take plural arguments. If the distribution of tropes over the underlying structure assigns a plurality of more than one trope to a given place in the structure, then tropes are co-located at that place; if a single trope (a "plurality of one") is assigned to the given place, then there is no co-location at that place. The *facts* of co-location are determined by how the tropes are distributed across spacetime. The *possibilities* of co-location, then, are determined by how tropes can be recombined in a possible spacetime. Since hyper-co-location can occur in an ordinary spacetime, is a special restriction on recombination needed to rule out hyper-co-location? I think not. Spatiotemporal co-location is no longer reflected in the underlying spacetime structure, but it still a structural feature; it has been shifted to instantial structure. For once we allow plural occupation, we must include the *size* of the plurality occupying a given place as part of the instantial structure, the pattern of instantiation. Thus the only restriction needed comes from considerations of what structures are possible. To see how this works, consider (LPR), now extended to allow for the recombination of tropes: for any tropes and any non-overlapping spatiotemporal arrangement of those tropes, there

is a world that recombines those tropes according to that arrangement. As noted in Section 3, (LPR) is a *restricted,* or *qualified,* principle of recombination: some tropes can be recombined only if there is a possible arrangement that recombines them. What arrangements are possible depends in part on what underlying structures are possible, and so is covered by plenitude of structures. But what arrangements are possible also depends on what instantial structure is possible. And that too is covered by plenitude of structures, though not plenitude of *world* (i.e. *underlying*) structures. It is no less governed by the iterative conception of structure, which is the ultimate determiner of what structures are possible, that is, instantiated in some world. In any case, no adjustment to (LPR) is needed, in particular, no separate restriction on how tropes can be recombined. I conclude, then, that allowing spatiotemporal co-location does not undermine the motivation for accepting Size of Worlds.

I turn now to atomless gunk. The assumption that no world contains gunk was made for convenience. It allows one to measure the size of a world—or a part of a world, or modal reality as a whole—by how many atoms it contains. But we can do just as well in a gunky setting by making use of "relative atoms."[41] To measure the size of a world, be it atomic, gunky, or partly gunky, consider all the ways of partitioning the world into non-overlapping parts. Each such partition has associated with it some number: the number of non-overlapping parts. Let the size of the world be the least upper bound of all those associated numbers. If the world is atomic, this method gives the same result as measuring by the number of atoms. If the world is gunky, this method makes the world smaller than a corresponding expanded atomic world which has atoms added, so to speak, at infinity. For example, a world with a standard spacetime continuum (and no co-location) has size $\beth_1$. A world with a gunky spacetime continuum, where each part of spacetime corresponds to a regular open set of the standard continuum, has size $\beth_0$ ($= \aleph_0$). All partitions of that gunky spacetime are countable.

Allowing gunky worlds raises a question analogous to the question raised by allowing co-location. And it gets an analogous answer. The question is: does *hypergunk* exist in any world?[42] An object is *hypergunk* if and only if it is gunk and its parts are many. If some world contains

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[41] Armstrong (1989a: 69) introduced relative atoms to extend his combinatorial account to possibilities (perhaps doxastic only) containing individuals or properties that were not composed of simples.

[42] On hypergunk, see Nolan (2004) and Hazen (2004).

hypergunk, then Size of Worlds is false: hypergunk can be partitioned into κ non-overlapping parts, for any cardinal κ, and so a world with hypergunk is large. But hypergunk, no less than hyper-co-location, is ruled out on an iterative conception of structure. For the mereological structure of a world is part of the world's underlying structure. And no world's underlying structure, on the iterative conception, can accommodate hypergunk. Hypergunk, then, is no threat to Size of Worlds. Note, however, that there is this difference between the case of hyper-co-location and hypergunk. Since hyper-co-location occurs in no world, it is impossible. But if one allows, as I do, that it is possible for more than one world to be actual, indeed, even for every world to be actual, then hypergunk is possible (if gunk is) even though it exists at no world. For suppose that, for any cardinal κ, gunk of size κ or larger exists at some world. It follows that the fusion of all the gunk in all the worlds will be hypergunk. (Hazen 2004 calls this "the Big Blob.") If it is possible for every world to be actual, then hypergunk is possible, even though it exists at no world. But perhaps when Nolan (2004) claims that hypergunk is possible, he has in mind what Hazen (2004) calls *strong hypergunk*. Strong hypergunk is gunk such that the parts of any of its parts are many. If strong hypergunk were to exist in modal reality, then it would exist in any world that it overlaps, violating Size of Worlds. Strong hypergunk, then, is impossible, even if we allow for the possibility of universal actualization. It is impossible not because it is formally inconsistent, but because it conflicts with the principles of the framework.

So much for hyper-co-location and hypergunk. Having discharged assumptions, I can now briefly conclude. The Forrest-Armstrong argument, once fortified, demands that the Unrestricted Principle of Recombination (UPR) be rejected. But the restriction comes entirely from the plenitude of structures, from independently motivated considerations as to what structures are instantiated in possible worlds. No additional restriction on recombination is required. When a modest version of Limitation of Size is added, a simple and attractive picture of modal reality emerges, as captured by Size of Reality and Size of Worlds. In sum: the worlds are many, but the parts of any world are few. Or, in other words: modal reality is large, but each of the worlds included in modal reality is small. That, I think, is a reasonable way for modal reality to be. Indeed, it may just be the *only* reasonable way for modal reality to be.

Bibliography

Armstrong, D. M. (1989), *A Combinatorial Theory of Possibility* (Cambridge University Press).

Bricker, Phillip (1987), "Reducing Possible Worlds to Language," in *Philosophical Studies* 52: 331-55.

Bricker, Phillip (1991), "Plenitude of Possible Structures," in *Journal of Philosophy* 88: 607-19. Expanded version with postscript forthcoming in *Modal Matter: Essays in Metaphysics* (Oxford University Press).

Bricker, Phillip (1996), "Isolation and Unification: The Realist Analysis of Possible Worlds," in *Philosophical Studies* 84: 225-38.

Bricker, Phillip (2001), "Island Universes and the Analysis of Modality," in G. Preyer and F. Siebelt (eds.), *Reality and Humean Supervenience: Essays on the Philosophy of David Lewis* (Rowman and Littlefield), 27-55.

Bricker, Phillip (2008), "Concrete Possible Worlds," in T. Sider, J. Hawthorne, and D. Zimmerman (eds.), *Contemporary Debates in Metaphysics* (Blackwell), 111-34.

Bricker, Phillip (2017), "Is There a Humean Account of Quantities?" in *Philosophical Issues* 27: 26-51.

Bricker, Phillip (forthcoming a), "Introduction: A Sketch of Reality," in *Modal Matters: Essays in Metaphysics* (Oxford University Press).

Bricker, Phillip (forthcoming b), "Principles of Plenitude," in *Modal Matters: Essays in Metaphysics* (Oxford University Press). Available at: http://blogs.umass.edu/bricker/files/2016/06/Principles_of_Plenitude-website.pdf.

Bricker, Phillip (forthcoming c), "Realism Without Parochialism," and Postscript, in *Modal Matters: Essays in Metaphysics* (Oxford University Press). Available at: http://blogs.umass.edu/bricker/files/2019/03/Realism-Without-Parochialism.pdf.

Bricker, Phillip (forthcoming d), "Postscript" to Bricker (1991), in *Modal Matters: Essays in Metaphysics* (Oxford University Press). Available at: http://blogs.umass.edu/bricker/files/2017/06/Postscript-Possible-Structures.pdf.

Cowling, Sam (2012), "Haecceitism for Modal Realists," in *Erkenntnis* 77: 399-414.

Enderton, Herbert (1977), *Elements of Set Theory* (Academic Press).

Forrest, Peter, and Armstrong, D. M. (1984), "An Argument Against David Lewis' Theory of Possible Worlds," in *Australasian Journal of Philosophy* 71: 132-44.

Fritz, Peter (2017), "A Purely Recombinatorial Puzzle," in *Noûs* 51: 547-64.

Hallett, Michael (1984), *Cantorian Set Theory and Limitation of Size* (Oxford University Press).

Hawthorne, John and Russell, Jeffrey Sanford (2018), "Possible Patterns," in K. Bennett and D. Zimmerman (eds.), *Oxford Studies in Metaphysics, vol. 11* (Oxford University Press), 149-92.

Hawthorne, John and Uzquiano, Gabriel (2011), "How Many Angels Can Dance on the Point of a Needle? Transcendental Theology Meets Modal Metaphysics," in *Mind* 120: 53-81.

Hazen, Allen (1996), "Worlds as Complete Novels," in *Analysis* 56: 33-8.

Hazen Allen (2004), "Hypergunk," in *The Monist* 87: 322-38.

Lewis, David (1986), *On the Plurality of Worlds* (Blackwell).

Lewis, David (1991), *Parts of Classes* (Blackwell).

Lewis, David (1993), "Mathematics is Megethology," in *Philosophia Mathematica* 1: 3-23.

Linnebo, Øystein, and Rayo, Agustín (2012), "Hierarchies Ontological and Ideological," in *Mind* 121: 269-308.

McDaniel, Kris (2004), "Modal Realism with Overlap," in *Australasian Journal of Philosophy* 82: 137-52.

Menzel, Chris (2014), "Wide Sets, ZFCU, and the Iterative Conception," in *Journal of Philosophy* 111: 57-83.

Nolan, Daniel (1996), "Recombination Unbound," in *Philosophical Studies* 84: 239-62.

Nolan, Daniel (2004), "Classes, Worlds, and Hypergunk," in *The Monist* 87: 303-21.

Pruss, Alexander (2001), "The Cardinality Objection to David Lewis's Modal Realism," in *Philosophical Studies* 104: 169-78.

Rosenstein, Joseph (1982), *Linear Orderings* (Academic Press).

Uzquiano, Gabriel (2015), "Recombination and Paradox," in *Philosophers' Imprint* 15.

Zermelo, Ernst (1930), "Über Grenzzahlen und Mengenbereiche: Neue Untersuchungen über die Grundlagen der Mengenlehre," in *Fundamenta Mathematicae* 16: 29-47.