Constraints on the variability of quark masses from nuclear binding

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Abstract

Based on recent work on nuclear binding, we update and extend the anthropic constraints on the light quark masses, with results that are more tightly constrained than previously obtained. We find that nuclei would fall apart (because the attractive nuclear central potential becomes too weak) if the sum of the light quark masses $m_u + m_d$ would exceed their physical values by 64%, (at 95% confidence level). We summarize the anthropic constraints that follow from the existence of atoms. With the additional assumption that the quark Yukawa couplings do not vary, these constraints provide a remarkably tight anthropic window for the Higgs vacuum expectation value: $0.39 < v/v_{\text{physical}} < 1.64$.

1 Introduction

To a first approximation, the fundamental parameters that describe our world appear to be uniform in space and constant in time. However, it is possible that their apparent constancy is illusory because of our limited ability to make observations across space and time. There are mechanisms, such as chaotic inflation and the string landscape, that can lead to a multiverse in which regions far outside of our visible horizon have different parameters from those that we see. Similarly it is possible (if there exists some nearly massless "moduli field") that the parameters could have been different in the early universe, and there are even experimental hints for this option. While these possibilities may or may not bear fruit in future studies, it is important to explore these options as carefully as possible.

The possibility of variable parameters changes the way that we approach the open questions of fundamental physics [1, 2, 3, 4]. For example, the existence of a multiverse with different parameters in different domains would modify the way that we approach the issue of using those parameters as a test of the underlying theory. Rather than looking for a unique set of parameters to emerge from a fundamental theory, we would expect them to be distributed in some typical range. However, for some parameters there is a further restriction in that there are combinations of parameters that would lead to a domain that could not support life. While there is some fuzziness in the constraints for the existence of life, certain clear physical properties can be used to delineate the extreme limits of the possible ranges. For example, *atoms must exist* and this restricts the ranges of the quark and lepton masses and possibly the Higgs vacuum expectation value (vev) [3]. This "atomic constraint" is particularly significant for the Higgs vev because the small value of this parameter (on the GUT, or Planck, scale) is one of the great fine-tuning problems of the Standard Model and consequently it is one of the greatest motivations for new physics. Alternatively, taking for granted this fine-tuning changes the way one can approach the need for new physics (and notably supersymmetry [4]).

The work of Agrawal et al (ABDS) [3] has used this *atomic principle* (as it was called in [4]), i.e. the need for the existence of atoms, to provide secure anthropic constraints on quark masses and the Higgs vev. In order to translate from the direct bound on the quark masses, this work assumes that the other parameters of the Standard Model remain fixed while the Higgs vev is allowed to vary. In a realistic theory, if the Higgs vev is able to take on different values, then the other parameters may also vary. However, the expected range of the Higgs vev is far larger in the absence of other new physics - this is why this vev is viewed as a great fine-tuning problem. As a consequence this key anthropic constraint may still have a robust meaning, even if other parameters are allowed to vary. Within the Standard Model the quark masses follow from the weak interaction, and are proportional to that scale. The Higgs vev sets the scale of the weak interactions. In contrast, the major contributions to nuclear masses are determined by the strong interactions. The general constraint then is that the effects of the scale of the weak interaction must overlap the scale of the strong interactions. It is the interplay of these two very different interactions that allows the existence of atoms. There is then a narrow volume of parameter space that produces nuclei and atoms.

A temporal variation of parameters could have yet different implications. A continuous variation of some quantity implies that this quantity is a field, i.e it carries a space-time dependence. For this variation to occur over cosmological time scales the field must be nearly massless. This then suggests that such a field coupled to matter would lead to violations of the equivalence principle, for example, or to other observable consequences.

In this paper we use recent work on nuclear binding to address some of these issues. In particular, we refine the understanding of the viable range of quark masses which follows from the existence of nuclei. Other atomic constraints bound the possible masses of the electron. We briefly discuss the constraint on the Higgs vev if the Yukawa couplings are held fixed. We reserve to a companion paper the issue of implications for tests of the equivalence principle [5].

Our paper is organized as follows: In Section 2 we use recent work on effective field theory to estimate which variation in quark masses would "unbind" heavy nuclei. In Section 3 we provide more physical insight into the sensitivity of nuclear binding to scalar interactions between nucleons by considering a simple model for homogeneous nuclear matter. Finally, Section 4 displays the anthropically allowed range of the masses of the first generation of quarks and leptons: m_u, m_d, m_e.

2 Anthropic constraints from the existence of nuclei using effective field theory

To the extent that we understand how the Standard Model leads to the physical world that we observe, we should be able to understand how that world would change if we modify the parameters of the theory to take on values in the neighborhood of their physical values. While we feel that we do understand the overall phenomenology of the Standard Model, the precision that we claim in these calculations continues to advance at a relatively modest pace, especially for what concerns the link between the fundamental Lagrangian and nuclear physics. However, the recent advances in nuclear physics have been impressive, largely through the application of the *effective field theory approach*[6]. Since the energies in nuclear processes are low, the effective field theory framework parameterizes the key ingredients in terms of a relatively small number of low energy constants. This method has been applied extensively to nuclear binding and has put traditional nuclear phenomenology on a more solid basis.

We consider a relatively simple but reasonably model-independent description of the parameters that influence nuclear binding, limiting ourselves to those that appear most important. For all but the lightest of nuclei, the key aspect of binding comes from a central potential that is isospin symmetric and which does not involve the spin of the nucleons. These will be parameterized by a small number of contact interactions[7, 8, 9]. While the other components of the nuclear force are important for the detailed descriptions of nuclear states, the main contributions to the binding energy comes from this spin-singlet and isospin-singlet central potential.

In addition to this model-independent framework, we employ the results of recent work on the variation of the dominant coupling constants with a changing quark mass [10]. While there are clearly some uncertainties in this calculation, it is easy to argue that the dominant effects are kinematic. The coupling constants are calculated using a dispersive representation[11, 12], with the threshold of the dispersion integral appearing at the physical threshold of $4m_\pi^2$. Raising the threshold is seen to lead to a kinematic suppression of the coupling strength. While our estimate is much more sophisticated than this, nevertheless the dominant effect is that of the kinematic threshold.

In effective field theory, the propagation of the very light degrees of freedom must be treated dynamically because these particle can propagate long distances. However, at low energies, the more massive degrees of freedom cannot propagate far and can be represented by contact interactions - i.e. delta function potentials and derivatives of delta functions. This has the effect of simplifying the contributions of various possible particle exchanges, with various spatial potentials, into a few low energy constants describing the strength of the interactions. In nuclear processes, it is useful to treat the direct effects of one-pion exchange dynamically, but to treat the other components of the nuclear force by contact interactions. For the spin singlet and isospin singlet central potential responsible for nuclear binding there are then two possible contact interactions, called *scalar* and *vector*

$$H_{\text{contact}} = G_S \bar{N} N \bar{N} N + G_V \bar{N} \gamma_\mu N \bar{N} \gamma^\mu N + ... \tag{1}$$

where G_S is negative (i.e. attractive), while G_V is positive (i.e. repulsive). In traditional meson exchange models, the scalar component corresponds to the exchange of the $\sigma(600)$ meson and the vector component corresponds to the exchange of the $\omega(783)$ meson.

Our first task is to understand the primary ingredients of nuclear binding in this framework. Fortunately the dominant ingredients in the binding of heavy nuclei have been elucidated in a set of papers by Furnstahl, Serot and co-workers [7, 8, 9]. For heavy nuclei, one-pion-exchange is not very important because pion exchange is proportional to the spin and isospin operators and the spins and isospins of most nucleons average to a total that is close to zero. Instead the isoscalar and spin independent contributions sum over all nucleons and are dominant once one is away from the few-nucleon cases. This is in accord with the standard wisdom that the nuclear central interaction ($J = 0$ and $I = 0$) is responsible for nuclear binding. The results for heavy nuclei can be extracted from Fig. 1 and Fig. 2 of [7]. As expected, the dominant effects are the scalar and vector contributions described above. Other interactions play reduced roles, although for a complete understanding of the binding about a half-dozen contact interactions are required. Here we will focus our attention on the dominant isoscalar-scalar and isoscalar-vector interactions.

Using Ref. [7], one can quantify these contributions to nuclear binding. We parameterize the results in terms of the strengths of the contact interactions, normalized to their physical values, defining

$$\begin{aligned} \eta_S &\equiv \frac{G_S}{G_S|_{\text{physical}}} \\ \eta_V &\equiv \frac{G_V}{G_V|_{\text{physical}}} \end{aligned} \tag{2}$$

The contributions to the binding energy (B.E) for ^{16}O (in MeV)[1] are

$$\frac{\text{B.E.}}{A} \simeq -82\eta_S + 44\eta_V + 30 \tag{3}$$

where A denotes the total baryon number. The first two terms are the effects of the scalar and vector isoscalar interactions. The third term is the sum

[1]Though we shall use here for convenience the usual physical units MeV (or GeV), one should think of these (when considering variations of the quark masses) as being defined as some pure number times the chiral limit of the QCD confinement scale, say $\Lambda^{(0)}_{\text{QCD}}$.

of all other smaller contributions to the binding energy and kinetic energy contributions. There is in addition the Coulomb energy and a small center of mass correction. For ^{208}Pb, the result is

$$\frac{\text{B.E.}}{A} \simeq -104\eta_S + 57\eta_V + 36 \tag{4}$$

The results of these calculations can be generalized to other nuclei by a parameterization that resembles the semi-empirical mass formula. For local interactions, because the nuclear density is nearly constant in the central region one expects that the binding energy will have a dependence on the volume, which in turn is proportional to the number of particles, $r^3 \sim A$, and that interactions that occur near the nuclear surface would have a modified result proportional to the number of nucleons near the surface, $r^2 \sim A^{2/3}$. This suggests that binding effects can be parameterized in terms of behavior in A and in $A^{2/3}$. Using the results for nuclear matter and for specific nuclei, we find a good fit of the form

$$\frac{\text{B.E.}}{A} = -(120 - \frac{97}{A^{1/3}})\eta_S + (67 - \frac{57}{A^{1/3}})\eta_V + \text{residual terms} \tag{5}$$

The primary difficulty in applying these ingredients to anthropic constraints is the need to connect the contact interactions to the fundamental parameters of QCD. However, there is two decades worth of work exploring the ingredients in this connection. The framework used below follows [10, 12] in employing dispersion relations, which can be used to express the desired couplings as integrals over reactions involving physical intermediate states. The low energy portions of these reactions can be well predicted by chiral perturbation theory, in which one has reasonable control over the quark mass dependence.

In general, an effective field theory prediction would be expected to have the following structure. The high energy end of a dispersion integral would be expected to depend on the quark masses only weakly. This is known from the dependence of hadron masses and couplings on the quark mass parameters. For example, if the $u,\ d$ masses were doubled (keeping $\Lambda^{(0)}_{\text{QCD}}$ fixed) the nucleon mass would increase by about a half a percent. However, the low energy portions of a dispersion integral can have a much greater change. For example, the doubling of the $u,\ d$ masses would raise the energy threshold in the dispersion relation by 40% and would forbid any contributions below this new threshold. In this case, a reasonable first approximation to an effective field theory calculation would be to treat the high energy portion of the dispersive integral as being independent of the masses and to calculate the low energy effects using chiral perturbation theory. Any large dependence on the light quark masses should should come from the low energy end. This is the result of our work.

The reasoning above suggests that the most important effect is in the scalar channel. This is the only portion of the central force that receives large effects from low energy, as two pion exchange is the most important contribution[2]. This channel has been explored in great depth within the context of chiral perturbation theory, including studies very similar to the approach used in this paper[13]. One of the authors has recently extended this work to include the constraints of unitarity [10, 12]. The result is a description of two-pion exchange that carries the main properties needed for the scalar central potential. We will employ this work in our analysis below. In this work, we use chiral perturbation theory at low energies and also attempt to extend the description to high energy. The low energy part is then model-independent while the high energy portion is less rigorous. However the high energy portion plays little role in our answer, since it conforms with the expectation that it should be largely independent of the quark mass. Moreover, we should note here that the primary ingredient is independent of the details of this calculation. The general trend is inescapable - as the pion mass gets larger, the effect of two pion exchange must get smaller. In the chiral framework, the connection of the quark masses to the two pion threshold is well defined, and as noted above, most of the effect found in Ref. [10] is kinematic.

Let us summarize the results of [10] and extend them to larger values of the pion mass. First, it was found that the pion mass dependence of omega exchange (corresponding to the *vector* channel) is of "normal" size, i.e. $O(m_\pi^2/(1\mathrm{GeV})^2)$. Such a "normal" sensitivity to m_π^2 (and therefore to quark masses) leads to sub-leading corrections compared to the effects linked to the m_π^2 sensitivity of the scalar channel. Indeed, because of the dependence on the two pion threshold, the scalar contact interaction is much more sensitive to the pion mass. In full generality, one has the sum rule

$$G_S = \frac{2}{\pi} \int_{2\,m_\pi}^{\infty} \frac{d\mu}{\mu}\ \rho_S(\mu) \tag{6}$$

where $\rho_S(\mu)$ is the spectral function that describes the physical two pion intermediate state at energy μ. The dependence of this spectral function

[2]The vector channel has also been explored in [10] but has little low energy effect and only a very small mass dependence. We will include it in our numerics below, while focussing, in the text, on the dominant scalar-channel effects.

on the quark masses is explored in detail in Ref. [10]. The rise of the amplitudes from the threshold value of $\mu = 2\,m_\pi = 270$ MeV is tempered at higher energy by unitarity effects such that the main contributions come from energies near $\mu = 500-600$ MeV. When changing the quark masses, all ingredients change to some extent. However the key effect is the threshold behavior. In lowest order, the pion mass-squared is proportional to the light up and down quark masses

$$m_\pi^2 = B_0(m_u + m_d) \tag{7}$$

where B_0 is a constant[3] (proportional to $\Lambda_{\rm QCD}^{(0)}$). The evidence is that this relation holds throughout the region of interest to us here [14]. The higher threshold then cuts off the effect of two pion exchange as the pion mass increases.

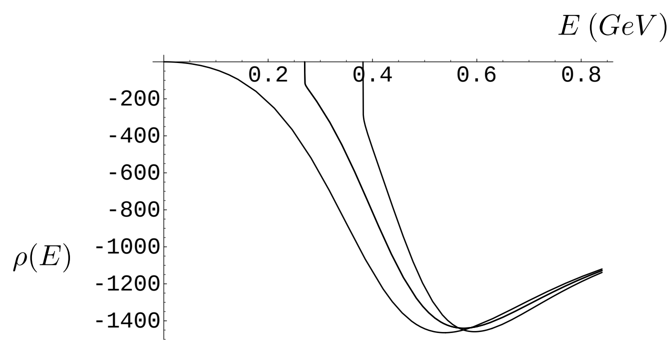

Figure 1: The scalar spectral function for three values of the pion mass, $m_\pi = 0, m_{\rm phys}$, and $\sqrt{2}m_{\rm phys}$, with thresholds starting at $\mu = 2\,m_\pi$.

In detail, the framework of Ref. [10] includes all variations in the parameters governing two-pion exchange, including g_A, F_π and the $\pi\pi$ rescattering amplitude. While that work was focussed on the situations where the pion mass was lighter than its physical value, the framework also extends to larger values of the pion mass. For example, the comparison of the result at the physical mass to the case where the pion mass is 40% larger than the physical value is shown in Fig. 1. The spectral integral will clearly show a *decrease* when the pion mass is *increased.*

[3] The precise value of B_0 depends on the renormalization scale used to specify the quark masses.

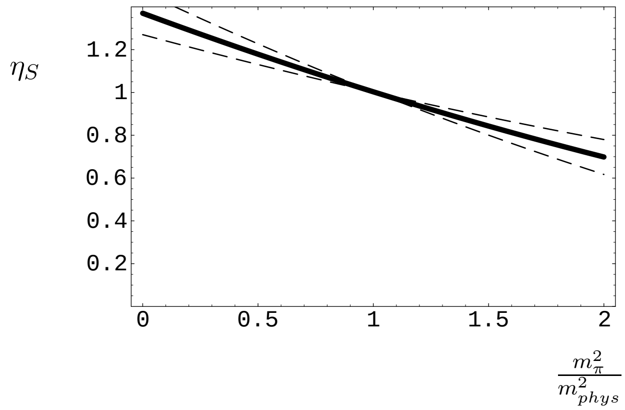

Figure 2: The value of the scalar strength η_S as a function of the pion mass.

In [10] it was found that the scalar strength G_S reached, in the chiral limit, the larger[4] value

$$\frac{G_S|_{\text{chiral}}}{G_S|_{\text{physical}}} = 1.37 \pm 0.10 \tag{8}$$

The error bar comes from the limitation of our understanding of the dependence of various couplings on the pion mass. This result could be used by itself to reasonably extrapolate to larger values of the pion mass since the extrapolation is almost linear in m_π^2. However, there are some non-linear features. In practice, a more detailed calculation, including required non-analytic contributions yields the result shown in Fig. 2 for η_S, i.e. the value of G_S normalized to the physical value, as a function of the pion mass. The estimates of these uncertainties are also shown in Fig. 2. The error bars come from from our lack of understanding of the dependence of some of the pion and nucleon parameters on the value of m_π. The largest source of uncertainty is the mass dependence of the axial coupling g_A. These uncertainties are discussed in more detail in [10].

In this calculation we have calculated the spectral integral up to an energy of 850 MeV. This includes some energies above the scale where the chiral perturbation theory description is valid - the upper end of this integral is modeled by using the continuation of the unitarized chiral amplitudes above the region where they are known to be correct. However, because very little mass variation is seen in the upper energy region, there is an

[4] In absolute value; let us indeed recall that G_S is *negative*.

alternate procedure which does not make this model-dependent assumption yet which yields essentially the same result. In this procedure, one calculates the spectral integral only in the region where the chiral expansion is valid, for example up to an energy of 600 MeV, and includes a short distance contact interaction to account for the effects of higher energy. (This rationale is described in more detail in [10].) If one assumes that the mass dependence of the short distance effect is of normal size (i.e. of order $m_\pi^2/(1\mathrm{GeV})^2$), then essentially all the mass variation comes from the low energy end, reproducing the result quoted above within error bars.

Because the effect of the scalar interaction is attractive ($G_S < 0$) while the effect of the vector interaction is repulsive ($G_V > 0$), there is a substantial cancelation between these two effects (see next Section for an analytical discussion exhibiting this cancelation). The mass dependence of the vector interaction has also been estimated in [10]. It has no significant threshold dependence because the dominant feature - the ω meson - is a narrow pole with only a small dependence on the quark masses. We have taken this into account in our numerics, but do not discuss it further here.

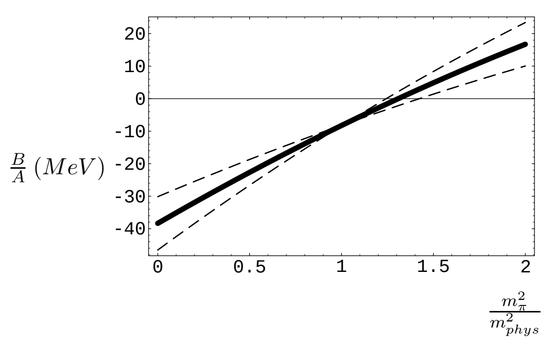

Figure 3: The binding energy per nucleon in ^{16}O as a function of the pion mass. The corresponding result in ^{210}Pb is very similar.

Because the scalar strength has significant variation while the vector is more modest in size, the cancelation between the two has an even larger percentage variation. In particular, as the *attractive* scalar interaction becomes weaker, it no longer dominates over the *repulsive* vector interaction, and the binding energy can change sign (from “binding” to “unbinding”) as m_π^2 increases above its physical value. Using the results (3), (4) quoted above, we see that the binding energy vanishes for a scalar strength only

10% smaller than the physical values

$$\begin{aligned} \eta_S|_{\text{critical}} &= 0.90 \qquad \text{for } {}^{16}O \\ \eta_S|_{\text{critical}} &= 0.89 \qquad \text{for } {}^{208}Pb \end{aligned} \tag{9}$$

Study of the general formula shows that these values are typical of the whole range in A. As we have seen above, increasing the pion mass will lead to a decrease in the scalar strength. In Fig. 3 we show the resulting nuclear binding for ${}^{16}O$ as a function of the pion mass, including the estimated error bar. In producing this figure we have assumed that the other small contributions to the binding formula do not have significant variations. We see that this element becomes unbound when the pion mass-squared is $36 \pm 14\%$ larger than the physical value. This critical value is almost independent of the value of A.

The anthropic constraint on quark masses can be inferred from these results. Using the basic relation (7) between the pion mass and the quark masses, one obtains the constraint

$$\frac{m_u + m_d}{(m_u + m_d)_{\text{phys}}} < 1.36 \pm 0.14 \quad , \tag{10}$$

from the requirement that nuclear binding exist at all. To the best of our present understanding of pion physics from chiral studies and from lattice simulations, the corrections to the basic relation between pion and quark masses are negligible compared to the other uncertainties in the calculation. If we had used the binding of ${}^{208}Pb$ we would have obtained essentially the same constraint on the pion mass. The use of the semi-empirical mass formula described above says that this constraint is roughly independent of the value of A. If we include the error bar and convert to a 95% confidence level upper bound we conclude that

$$\frac{m_u + m_d}{(m_u + m_d)_{\text{phys}}} < 1.64 \quad . \tag{11}$$

We will use this as our final "atomic bound".

3 Constraints using a model for nuclear matter

In this section we use a simple model for nuclear matter to provide more physical insight into the sensitivity of nuclear binding to the scalar strength and to reinforce the results of the previous section. The model is a variant of the description of nuclear matter discussed in Ref. [9] using nucleonic and mesonic fields. It reproduces the dominant contact interactions used above and also includes higher order dependencies on the scalar couplings and the kinetic energy. We will see that these higher order dependencies increase the sensitivity to G_S and hence to the quark masses.

The starting Lagrangian is

$$L = \bar{\psi}\left[i\gamma^\mu\partial_\mu - g_V V_0\gamma_0 - (M - g_S\phi)\right]\psi + \frac{1}{2}m_V^2 V_0^2 - \frac{1}{2}m_S^2\phi^2 \tag{12}$$

where ψ is the nucleon field, ϕ is a scalar, isoscalar field ("the sigma") and V_μ is an isoscalar vector field ("the omega").

We now consider the effect of this Lagrangian in an infinite nuclear medium. The nucleon field fills the available states up to the Fermi energy. The density of nucleons is given by

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k = \frac{\gamma k_F^3}{6\pi^2} \tag{13}$$

where γ is the number of degrees of freedom ($\gamma = 4$ for isoscalar nuclear matter, which we will use in our numerical work) and k_F is the Fermi momentum. The nucleon field acts as the source of the scalar and vector fields. Solving for the energy density of this uniform distribution, one finds

$$\epsilon = \frac{1}{2}\frac{g_V^2}{m_V^2}\rho_B^2 + \frac{1}{2}\frac{g_S^2}{m_S^2}\rho_S^2 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\ E^*(k) \tag{14}$$

where the scalar density is

$$\rho_S = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k \frac{M_*}{E^*} \tag{15}$$

and

$$\begin{aligned} E^* &= \sqrt{k^2 + M_*^2} \\ M_* &= M - g_s\rho_S \end{aligned} \tag{16}$$

We approximate this by non-relativistic kinematics, which is a reasonable approximation for nuclear matter. If we then solve for the energy per baryon (which is ϵ/ρ_B) we find

$$\frac{E}{B} - M = \frac{\gamma}{12\pi^2}k_F^3\left(\frac{g_V^2}{M_V^2} - \frac{g_S^2}{M_S^2}\right) + \frac{3}{10}\frac{k_F^2}{M\left(1 - \frac{\gamma g_S^2 k_F^3}{6\pi^2 m_S^2 M}\right)} \tag{17}$$

Here in the first term we see the effects of the scalar and vector contact interactions (with $G_S = -g_S^2/m_S^2$ and $G_V = +g_V^2/m_V^2$), while the second term is the kinetic energy of the nucleons propagating in the nuclear medium. In the language of Ref. [7], these latter terms would be described as higher order contributions in the kinetic energy term.

If the couplings are chosen appropriately, one reproduces the existence of nuclear matter. As the density ($\propto k_F^3$) increases, the kinetic energy initially gives a positive contribution which is eventually overcome by the potential energy (if $g_V^2/M_V^2 - g_S^2/M_S^2 = G_V + G_S$ is sufficiently negative), with nuclear saturation seen in the existence of a minimum in the potential energy function. Using appropriate values ($G_S = -362$ GeV^{-2} and $G_V = 270$ GeV^{-2}) an energy function very similar to that of [9] is shown as the bottom curve in Fig 4, reproducing the correct binding energy and Fermi momentum.

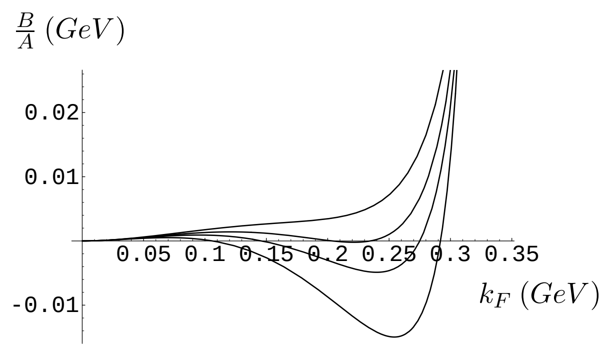

Figure 4: The binding energy per nucleon in nuclear matter as a function of the Fermi momentum for various values of the scalar strength G_S. From bottom to top in the figure, the values of G_S are (362, 340, 328, 315) GeV^{-2}

Now let us consider variations in the scalar coupling. Various other values of G_S are also shown in Fig. 4. We observe that the binding is highly sensitive to the scalar coupling. In particular, nuclear matter disappears for a critical value of G_S only 10% smaller than the physical value

$$\eta_S|_{\rm crit} = \frac{G_S|_{\rm crit}}{G_S|_{\rm phys}} = 0.904 \tag{18}$$

This confirms the sensitivity to this parameter found in finite nuclei by Ref. [7]. In fact, we can see from both the formula and the numerics that higher order effects G_S have the effect of making the sensitivity greater.

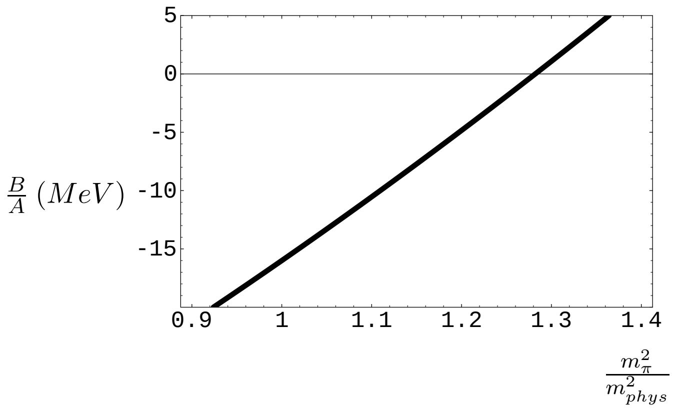

Figure 5: The binding energy per nucleon in nuclear matter as a function of the pion mass.

Finally let us translate this into a constraint on the quark masses. Using the calculation of G_S as our guide, the binding energy of nuclear matter as a function of the pion mass is shown in Fig. 5. We see that the central value of the constraint satisfies

$$\frac{m_u + m_d}{(m_u + m_d)_{\rm phys}} < 1.28 \pm 0.14 \tag{19}$$

This is completely consistent with, and slightly stronger than, the bound quoted in the previous section. Because the two constraints overlap, to be conservative we will use the upper bound of the previous section as our final constraint.

4 Summary of quark and lepton mass constraints

In this section, we display the anthropically allowed range of the masses of the first generation of quarks and leptons, m_u, m_d, m_e, updating Ref.[15]. There are two primary constraints. One is a bound on the sum of quark masses $m_u + m_d$ derived above. If this combination becomes too large, all nuclei fall apart because the attractive central potential becomes too weak. The other bound follows from the constraint that if the neutron mass is lighter than the sum of the masses of the proton and electron, hydrogen will be unstable through the capture of electrons $e^- + p \to n + \nu$, such that hydrogen atoms will decay into neutrons[5]. In practice, these two constraints suffice to provide tight bounds on these three masses.

For the first constraint due to our bound following from the binding of nuclei, we need to express this in terms of absolute masses. While our constraint, Eq. (11), involves the ratio of masses, which is scale independent, the absolute masses depend on the scale that they are specified at. The most canonical values of the quark masses $m_d \sim 7$ MeV and $m_u \sim 4$ MeV are typically taken to apply at a scale of 1 GeV, and we will use this prescription. In this case, the bound on the ratio, Eq. (11), implies that

$$m_u + m_d \leq 18 \text{ MeV} \quad . \tag{20}$$

The second constraint - that hydrogen exists - involves a bound on the physical masses

$$m_P + m_e \leq m_N \quad . \tag{21}$$

If this relation is violated, the electron in the hydrogen atom will be captured by the proton[6] To convert this relation to the quark level we need to estimate both the quark contribution to the neutron-proton mass difference and the electromagnetic contributions. Let us parameterize these by

$$m_N - m_P = Z_0(m_d - m_u) - \epsilon_{EM} \tag{22}$$

Here the first term on the right hand side is the contribution due to the differences in quark masses, while the second part is the electromagnetic contribution to the mass difference. Since the quark masses are scale dependent, so also is Z_0, such that the product is scale independent. Both potential models [17] and bag models [18] yield remarkably similar values for the electromagnetic contribution, $\epsilon_{EM} \sim 0.5$ MeV. The use of the canonical values of the masses at a scale of 1 GeV then implies that $Z_0 = 0.6$ in order to obtain the correct neutron-proton mass difference. This is a very reasonable value and we will adopt it in our numerics. Using these values, we find that the difference in quark masses is also bounded

$$m_d - m_u \geq \frac{m_e + \epsilon_{EM}}{Z_0} \tag{23}$$

[5]We assume throughout that the neutrino mass, if it is indeed allowed to also vary, remains negligibly small. There is, moreover, an anthropic constraint that ensures this result[16].

[6]The violation of this relation will also cause important modifications in heavy nuclei. However, bound protons can still exist in heavy atoms if they are sufficiently more deeply bound than neutrons, such that the Pauli principle blocks the proton to neutron conversion. We do not attempt to analyze this situation in detail, using instead the simpler constraint on hydrogen as the main anthropic bound.

or

$$m_d - m_u - 1.67 m_e \geq 0.83 \text{ MeV} \quad . \tag{24}$$

The right hand side of this latter constraint is evaluated at the physical value of the fine structure constant and the QCD scale. It is linear in both of these quantities.

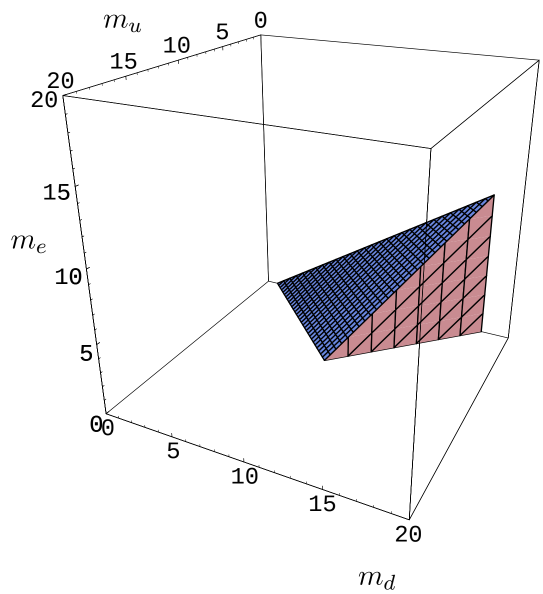

Figure 6: The anthropic constraints on m_d, m_u, m_e in MeV units.

These constraints are plotted in Figs. 6, which shows a 3D plot listing the allowed values of each combination of mass. The important point is that the two constraints manage to *provide bounds on all three of the masses.* Note that m_u and m_e have no lower anthropic bounds, while m_d is constrained to be non-zero.

We can also take projections into various two-dimensional subsections. The constraints on various combinations of the masses are shown in Fig 7. In each case, the outer range is shown allowing the third mass parameter to take on any allowed value. Also marked by a dashed line on these plots is the overall range of the masses when the third mass parameter takes on its physical value.

We see that quite small changes in the quark masses would lead to unlivable conditions.

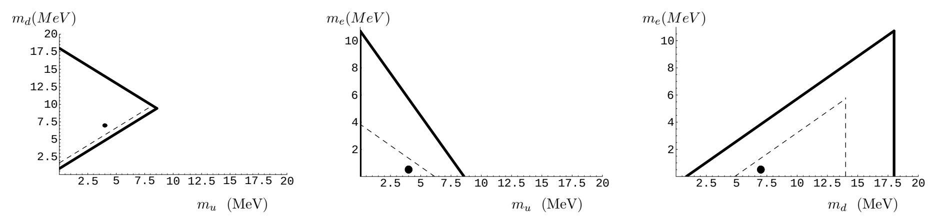

Figure 7: The projection of the anthropic constraints of Fig. 6 into the planes of each pair of masses. The solid lines denote the total allowed region, while the dashed line shows the remaining area if third mass takes on its physical value. The dot shows the physical values of the masses.

These ranges for the masses can be converted to an allowed range for the Higgs vacuum expectation value, under the additional assumption that the other parameters of the Standard Model (Yukawa and gauge couplings) are held fixed. This extra assumption could possibly occur in grand unified theories, where there are many attempts to predict gauge and Yukawa couplings. In this situation, the Yukawa couplings could be fixed by the symmetries of the grand unified theory, and our quark mass constraints translate directly into constraints on the Higgs vev. From our work above on the binding of nuclei we would then find

$$\frac{v}{v_{\mathrm{phys}}} < 1.64 \tag{25}$$

at 95% confidence. This constraint is both stronger than and independent from the final result of ABDS [3]. The latter was based on the fact that as the quark masses increases, at fixed Yukawa couplings, the neutron-proton mass difference increases until eventually all bound neutrons decay and only protons exist. Thus, that bound constrains $m_d - m_u$, while ours constrains $m_d + m_u$. Moreover, our present bound is tight enough that it supersedes the bound on the mass difference, because $m_d - m_u$ can never be greater than $m_d + m_u$. The lower constraint comes from the other process discussed in this section - the stability of hydrogen atoms against the reaction $p + e \to n + \nu$. If the Higgs vev becomes too small, the proton becomes heavier than the neutron due to electromagnetic interactions and this reaction occurs. Since the up, down and electron masses are all proportional to v, one finds that this constraint is

$$\frac{v}{v_{\mathrm{phys}}} \geq 0.39 \tag{26}$$

When combined one finds a very restricted range for the vev, under the stated assumptions.

$$0.39 \leq \frac{v}{v_{\rm phys}} \leq 1.64 \tag{27}$$

which is especially tight if one considers it in the the context of Grand Unification, where the natural range for the vev could extend up to the GUT scale.

Of course, it is also possible that the extra assumption about the constancy of the Yukawa couplings is not correct. In the discussions of the string landscape, there are so many possible vacua that others with different values of the Yukawa couplings should be possible. However, our quark mass constraints should still be relevant for describing the likely values of the Higgs vev[19]. Even though extreme cases with disparate scales may be possible[20], it is plausible that the need for light quarks makes it likely that the Higgs vev is close to the scale of the strong interactions[19]. Moreover, in theories such as supersymmetry which use dynamics to stabilize the fine tuning problem, the anthropic constraint could be an explanation of the overall scale of supersymmetry breaking.

It may be possible to provide tighter bounds on the masses by considering more specific constraints. One that has been discussed in the literature is the bound following from the stability of deuterium [3, 21]. The deuteron is very weakly bound and small changes in the masses will suffice to unbind it[7]. This happens for more modest changes than is required for the unbinding of the rest of the elements. Since deuterium is involved in the standard mechanisms of nucleosynthesis in the early universe and in stars, the lack of a stable deuteron could be the obstacle to providing the elements needed for life. However, this bound is less robust that considered above. On the one hand, there may be alternate pathways to the production of enough elements needed for life. In addition, Weinberg estimates that even an unstable deuteron could live long enough to generate the elements [1]. Moreover, there are extra subtleties in estimating the quark-mass sensitivity of the various two-nucleon systems [23, 24, 25]. For all these reasons, we consider only the most robust of constraints, as discussed above. These strong constraints already provide very strong bounds on the masses, as summarized above.

[7]For example Eugene Golowich (private communication)[22] has estimated that if the scalar coupling is decreased by 5.2%, the deuteron will be unbound. This is half the variation that we showed was needed to unbind the heavy elements, and would lead to a tighter bound of 1.33 for the ratio of Eq. 11.

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