P606 - Homework Set 1
Due Tues. Sept 16 by 4:30pm

1) **Dimensionally regularized Green function**
The key formula

$$\nabla^2 \frac{1}{|\mathbf{x} - \mathbf{x}'|} = -4\pi \ \delta^3(\mathbf{x} - \mathbf{x}') \tag{1}$$

is mathematically tricky to prove because of the singularity at the origin. Various ways to soften that singularity to regularize the mathematics are employed. Prove this formula using the form

$$G(\mathbf{x} - \mathbf{x}') = \lim_{\eta \to 1^-} \ \frac{1}{|\mathbf{x} - \mathbf{x}'|^\eta}, \tag{2}$$

with η approaching unity from below.

2) **Potential energy of a spherical ball**
a) Use Gauss' Law to get the electric field inside and outside a spherical ball of charge Q and radius R, with a uniform charge density.
b) Calculate the potential energy that is needed to assemble this configuration by building up the ball bringing infinitesimal spherical shells in from $r = \infty$.
c) Verify that this potential energy is equal to

$$W = \frac{\epsilon_0}{2} \int \ d^3x \ E^2(x) \tag{3}$$

3) **Neutral Hydrogen**
Jackson 1.5. Please explicitly verify that the total charge is zero.

4) **Capacitance**
Jackson 1.7. You do not need to estimate the gauges of wires required. It would be good to review the formulas relating to capacitance that were covered in your introductory physics course.