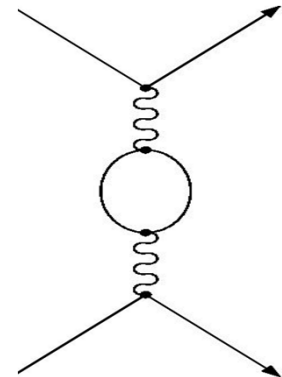
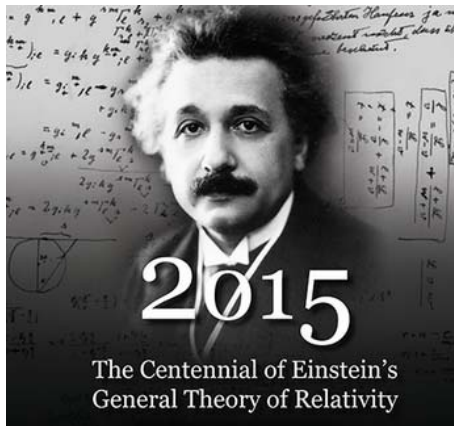


A Modern View of Quantum Gravity

John F. Donoghue
at
Universität Wien
October 13, 2015



AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS

Physics at the interface: Energy, Intensity, and Cosmic frontiers

University of Massachusetts Amherst

Goal: We have made progress!

- 1) Bad quotes – good quotes
- 2) The Effective Field Theory revolution
- 3) A minimal primer for General Relativity
- 4) How to make your head hurt!
- 5) Quantum Gravity from the ground up
- 6) The quantum correction to the Newtonian potential

$$V(r) = -\frac{GMm}{r} \left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi} \frac{G\hbar}{r^2 c^3} \right]$$

- 7) Limits of the EFT

Standard lore: Quantum gravity doesn't exist

I will argue that the standard lore is **wrong** in an important way

General relativity makes a fine quantum theory!

- called “effective field theory”
- valid over all scales that we have tested

Progress in Quantum Gravity!!!

There still are many issues to explore

Need to reshape the way that we think of quantum gravity

But, but, but....

“Quantum mechanics and general relativity are incompatible”

“Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.”

“The biggest single problem of all of physics is how to reconcile gravity and quantum mechanics”

“Of all the fundamental forces of nature, only gravity still stands outside the rubric of the quantum theory.”

“Quantum gravity does not yet exist as a working physical theory.”

“The existence of gravity clashes with our description of the rest of physics by quantum fields”

In more detail:

“The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory”

“From a technical point of view, the problem is that the theory one gets in this way is not renormalizable and therefore cannot be used to make meaningful physical predictions.”

“But for gravity, renormalization theory fails, because of the inherent nonlinearities in general relativity.

“Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.”

Why is this situation unacceptable?

Physics is an experimental science:

- don't know D.O.F. and interactions beyond range of expt.
- OK for modifications at high energy

But QM and GR expected to be applicable over the same range of scales

- is one really wrong at these scales?

Standard lore reflects an **outdated view** of field theory

Crucial rephrasing:

Are QM and GR compatible over accessible scales, where we expect both to be correct?

Answer is **YES**

- this is what effective field theory can do for you

A more modern quote:

“A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.”

Frank Wilczek
Physics Today
2002

A more modern quote:

wichtigtueriesches Gefasel

“A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.”

verdeckt

Frank Wilczek
Physics Today
2002

Another thoughtful quote:

“I also question the assertion that we presently have no quantum field theory of gravitation. It is true that there is no *closed*, internally consistent theory of quantum gravity valid at all distance scales. But such theories are hard to come by, and in any case, are not very relevant in practice. But as an *open* theory, quantum gravity is arguably our *best* quantum field theory, not the worst.

{Here he describes the effective field theory treatment}

From this viewpoint, quantum gravity, when treated –as described above- as an effective field theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances.”

J.D. Bjorken

Even Wikipedia has learned this lesson!

QG as an effective field theory [\[edit\]](#)

Main article: [Effective field theory](#)

In an [effective field theory](#), all but the first few of the infinite set of parameters in a non-renormalizable theory are suppressed by huge energy scales and hence can be neglected when computing low-energy effects. Thus, at least in the low-energy regime, the model is indeed a predictive quantum field theory.^[12] (A very similar situation occurs for the very similar effective field theory of low-energy pions.) Furthermore, many theorists agree that even the Standard Model should really be regarded as an effective field theory as well, with "nonrenormalizable" interactions suppressed by large energy scales and whose effects have consequently not been observed experimentally.

Recent work^[12] has shown that by treating general relativity as an [effective field theory](#), one can actually make legitimate predictions for quantum gravity, at least for low-energy phenomena. An example is the well-known calculation of the tiny first-order quantum-mechanical correction to the classical Newtonian gravitational potential between two masses.

My own phrasing

The quantum theory of general relativity exists at scales below the Planck scale and is described by an effective field theory.

The Effective Field Theory Revolution

“We have learned in recent years to think of our successful quantum field theories, including quantum electrodynamics, as ‘effective field theories’, low energy approximations to a deeper theory”

Steven Weinberg, QFT 1

What is Effective Field Theory?

-clear thinking about energy scales in usual field theory

Effective:

- 1) Low energy limit of a more complete theory
 - only low energy D.O.F. active
- 2) Useful – able to have effect

Why do quantum calculations work?

The problem: QM says to sum over **all** intermediate states

$$\sum_I \frac{\langle f|V|I\rangle\langle I|V|i\rangle}{E_i - E_I}$$

But, physics is an experimental science

-know particles and interactions up to some energy scale

So, how can you sum over all states if you don't know what they are or how they interact??

Some possible solutions:

1) The energy denominator suppresses high energy states

$$\sum_I \frac{1}{E_i - E_I} \rightarrow \int d^3p_I \frac{1}{E_i - \frac{p_I^2}{2m}}$$

2) Perhaps matrix elements are small to high energy states

All observables sensitive to high energy at some order in PT

The solution:

The uncertainty principle

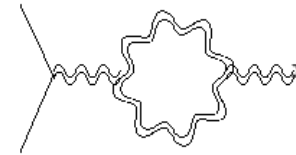
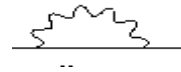
High energy effects look local - very short range

→ Look like some term in a local Hamiltonian/Lagrangian

Mass term or charge coupling

Shift in mass or coupling

We measure **total** mass and coupling



Appelquist Carrazone theorem:

- effects of high energy either absorbed in “coupling constants” or suppressed by powers of the heavy scale

Low energy calculations:

Physics distinct from local Lagrangian effects:

Non-local effects in coordinate space

$$1/r^n$$

Non-analytic effects in momentum space

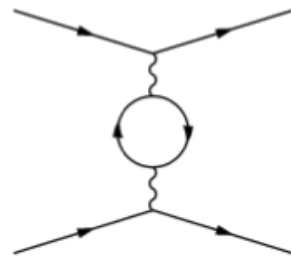
$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

The Era of Renormalizable Theories

High energy sensitivity in small number of parameters



$$\delta m^2$$

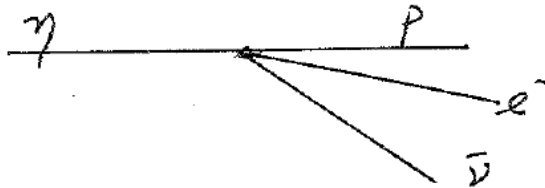


$$\delta e^2$$

After this – closed, self-contained

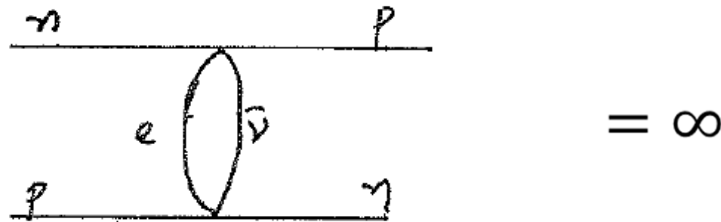
- Long thought to be the only predictive QFTs

A non-renormalizable interaction

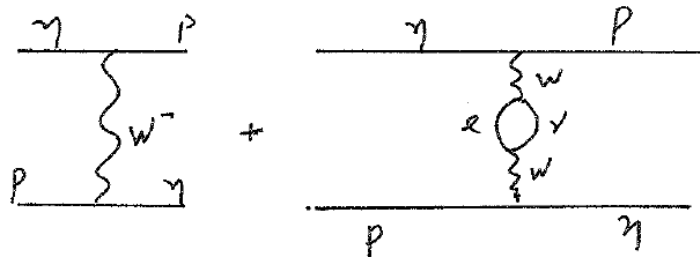


$$\mathcal{L}_\beta = \frac{G_F}{\sqrt{2}} \psi_e^* \psi_\nu \psi_p^* \psi_n$$

Generates new divergences

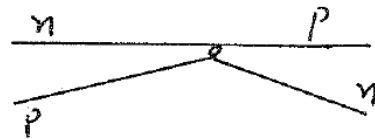


Note – safe in full theory. In the Standard Model:



Need for EFT and EFT logic

Part of the calculation is unreliable

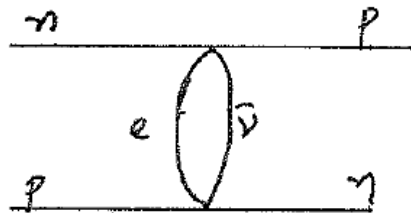


short distance

$$\mathcal{L}_{eff} = c_{eff} \psi_n^* \psi_p \psi_p^* \psi_n$$

Measure, or predict from full theory (SM)

Part of calculation is real



- long distance

$$V_{exc}(r) = -\frac{G_F^2}{8\pi^3} \frac{1}{|x-y|^5}$$

Real physics included in non-renormalizeable interaction

Effective Field Theory in Action:

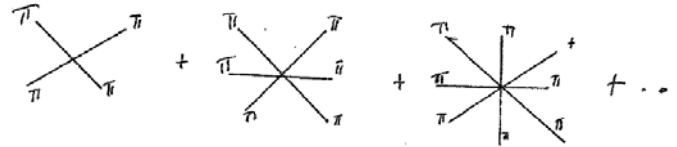
Chiral Perturbation Theory

-QCD at very low energies –pions and photons

Non-linear lagrangian required by symmetry:

$$\mathcal{L} = F_\pi^2 \text{Tr}(D_\mu U D^\mu U^\dagger) + L_1 [\text{Tr}(D_\mu U D^\mu U^\dagger)]^2 + \dots$$

$$U = \exp[i \frac{\tau \cdot \phi}{F_\pi}]$$



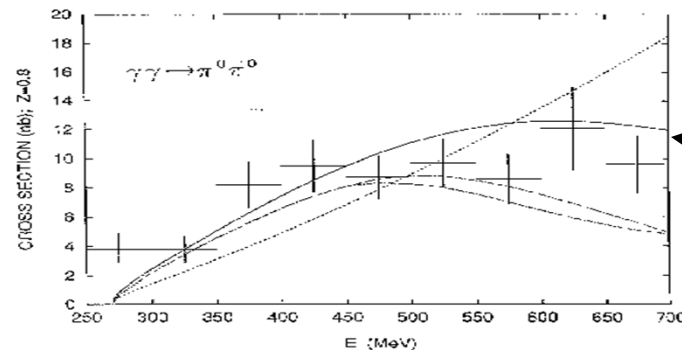
Very well studied: Theory and phenomenology

- energy expansion, loops, symmetry breaking,
experimental constraints, connection to QCD.

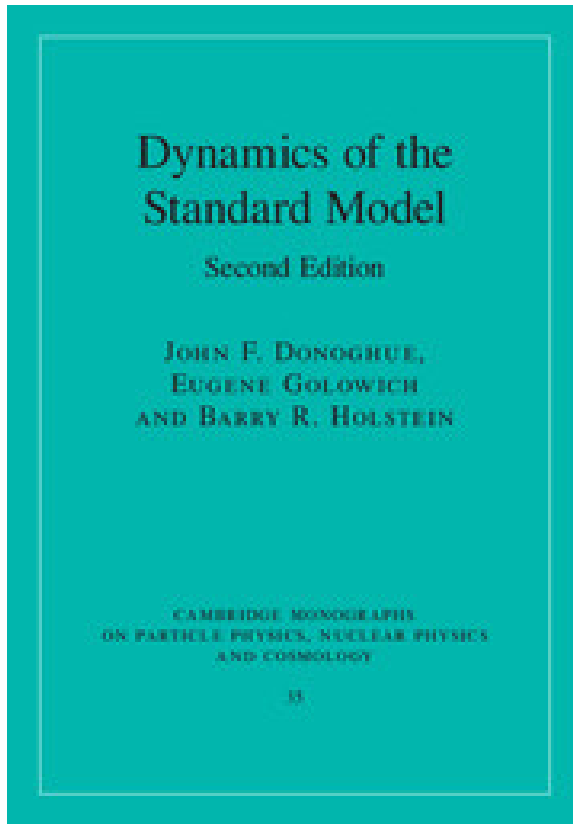
Sample calculation:

- no direct couplings at low energy
- pure loops
- essentially parameter free at low energy

$$\gamma\gamma \rightarrow \pi^0 \pi^0$$



Advertisement: Read all about it!



Second edition



Two minute primer for General Relativity

1) Space-time – time and space treated on equal footing

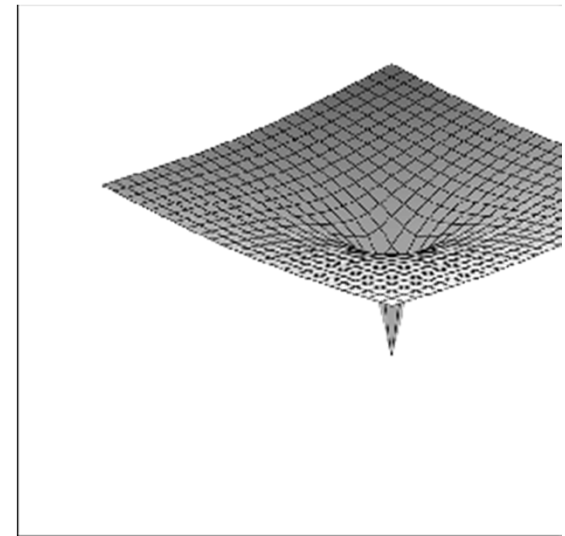
$$x^\mu = (ct, \vec{x}) \quad \mu = 0, 1, 2, 3$$

Special relativity – Lorentz invariance

$$ds^2 = c^2 dt^2 - d\vec{x}^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

2) General relativity

- theory of gravity
- based on curved space-time
- amount of “curving” \sim grav. field



Aspects that we will need

- 1) The gravitational field –deviation from flat space

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

- 2) Symmetry – general coordinate invariance

- restricts Lagrangian to invariant curvature terms

$$\mathcal{L} = \frac{1}{G} \bar{R} \sim \frac{1}{G} [\partial h \partial h + h \partial h \partial h + \dots]$$

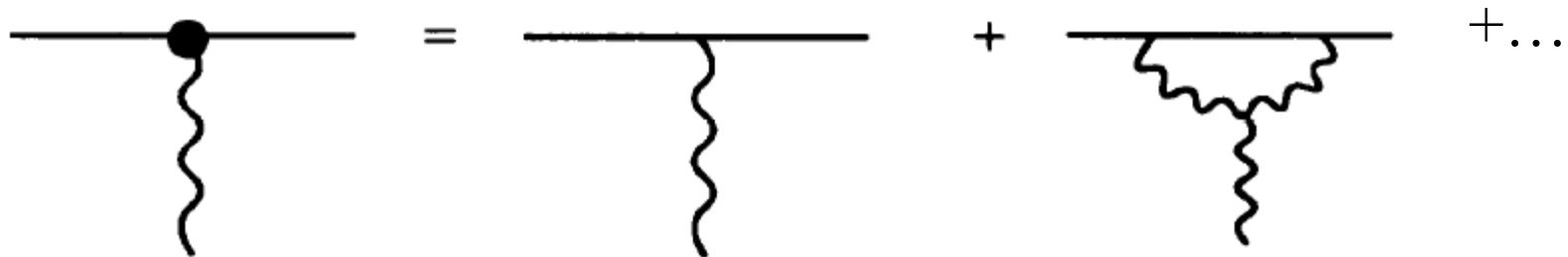
- 3) Gravity couples to energy and to itself –Einstein's equation

$$\nabla^2 h = G [H_{\text{matter}} + \partial h \partial h]$$

Calculating the gravitational field:

$$h(x) = \int \bar{d}y G(x - y) < \bar{H}_{\text{matter}}(y) + \partial h \partial h(y) >$$

In pictures:



Scale of gravity

$$l_{\text{P}} := \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33} \text{ cm},$$

$$t_{\text{P}} := \frac{l_{\text{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.39 \times 10^{-44} \text{ s},$$

$$m_{\text{P}} := \frac{\hbar}{l_{\text{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-5} \text{ g} \approx 1.22 \times 10^{19} \text{ GeV}/c^2.$$

The technical slide

The Einstein action: $S_{grav} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R \right]$

$\kappa^2 = 32\pi G$, $g = \det g_{\mu\nu}$, $g_{\mu\nu}$ is the metric tensor and $R = g^{\mu\nu} R_{\mu\nu}$

$$\begin{aligned} R_{\mu\nu} &= \partial_\nu \Gamma_{\mu\lambda}^\lambda - \partial_\lambda \Gamma_{\mu\nu}^\lambda + \Gamma_{\mu\lambda}^\sigma \Gamma_{\nu\sigma}^\lambda - \Gamma_{\mu\nu}^\sigma \Gamma_{\lambda\sigma}^\lambda \\ \Gamma_{\alpha\beta}^\lambda &= \frac{g^{\lambda\sigma}}{2} (\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}) \end{aligned}$$

This is not the most general lagrangian consistent with general covariance.

Key: R depends on two derivatives of the metric

\Rightarrow Energy expansion – expansion in number of derivatives

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

How to make your head hurt!

“Quantum gravity means quantizing spacetime itself”

- time in QM is a parameter
- time in GR is dynamical!
- what does it mean to have probability as some time, if time itself is fluctuating

If spacetime is fluctuating, is causality only probabilistic?

If spacetime fluctuates, can we fluctuate to a different topology?

Is there a wave function of the Universe?

- an observer to “collapse the wavefunction”?

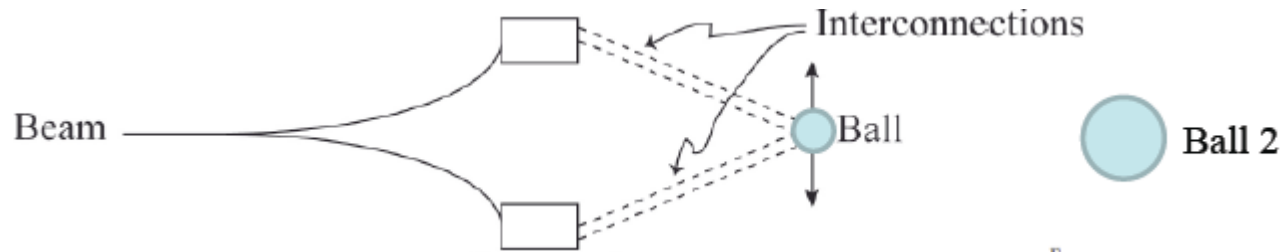
Somewhat more technically:

- 1) If you try “canonical” path to form Hamiltonian of GR
 - get constraint that $H=0$
 - quantum states evolve with e^{-iHt}
 - so nothing evolves – time does not exist
 - problem of time
- 2) If you try “covariant” approach, find “odd” infinities
 - can you ever make predictions?

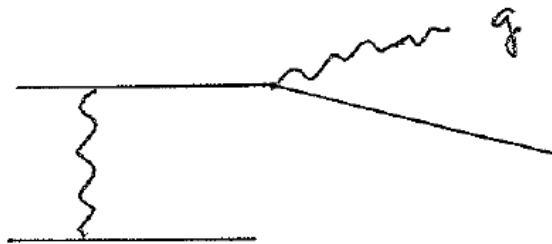
Nevertheless gravity needs to be quantum

- 1) Feynman – matter superposition implies gravity superposition

Stolen from
M.. Aspelmeyer



- 2) Matter quanta implies graviton quanta



Quantum Gravity from the ground up

What does it mean to quantize fields?

-Modern approaches go through the **path integral**

$$\int [d\phi] e^{iS(\phi)}$$

\uparrow
 $S = \int dt d^3x \mathcal{L}$

Lagrangian invariant under “everything”

Minimum of action = classical equations of motion

Quantum fluctuations = **propagators and Feynman rules**

GR is not the only theory that was tough to quantize

- QCD has equally difficult issues

It is not time that fluctuates, but the metric

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$$



We live in a particular spacetime



Quantum fluctuations around
our spacetime

$h_{\mu\nu}$ has wavelike solutions

- gravitational waves
- when quantized – gravitons

Quantizing general relativity

Feynman quantized gravity in the 1960's

QUANTUM THEORY OF GRAVITATION*

BY R. P. FEYNMAN

(Received July 3, 1963)

Quanta = gravitons (massless, spin 2)

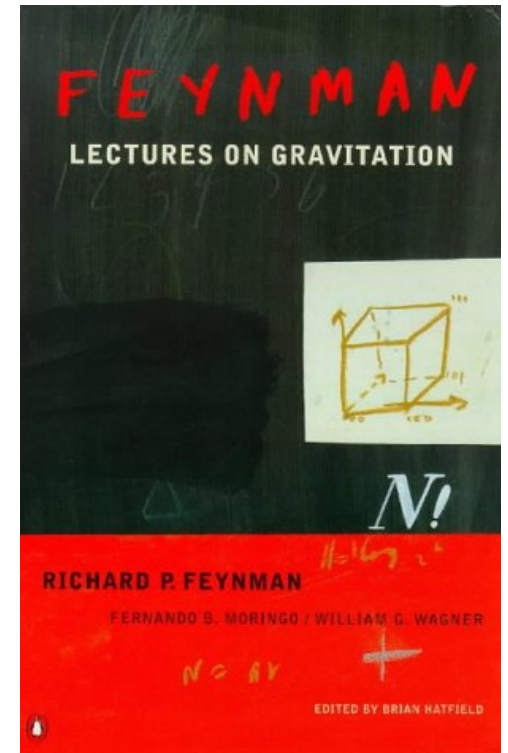
Rules for Feynman diagrams given

Subtle features:

$h_{\mu\nu}$ has 4x4 components – only 2 are physical DOF!
-need to remove effects of unphysical ones

Gauge invariance (general coordinate invariance)

- calculations done in some gauge
- need to maintain symmetry



In the end, the techniques used are very similar to other gauge theories

Feynman rules:

A.1 Scalar propagator


The massive scalar propagator is:



$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:



$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2} \left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta} \right]$$

A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:



$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2} [p^\mu p'^\nu + p'^\nu p^\mu - \eta^{\mu\nu} ((p \cdot p') - m^2)]$$

A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:



$$= \tau_2^{\lambda\rho\sigma}(p, p', m)$$

$$\tau_2^{\lambda\rho\sigma}(p, p') = i\kappa^2 \left\{ \Gamma^{\lambda\alpha\delta} \Gamma^{\rho\sigma\beta}_{\delta} - \frac{1}{4} \left\{ \eta^{\lambda\lambda} \Gamma^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma} \Gamma^{\lambda\alpha\beta} \right\} \right\} (p_\alpha p'_\beta + p'_\alpha p_\beta) - \frac{1}{2} \left\{ \Gamma^{\lambda\rho\sigma} - \frac{1}{2} \eta^{\lambda\lambda} \eta^{\rho\sigma} \right\} [(p \cdot p') - m^2]$$

(61)

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2} (\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]



$$= \tau_3^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q)$$

where

$$\begin{aligned} \tau_3^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q) = & -\frac{i\kappa}{2} \times \left(\mathcal{P}_{\alpha\beta\gamma\delta} \left[k^\mu k^\nu + (k - q)^\mu (k - q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\ & + 2q_\lambda q_\sigma \left[I_{\alpha\beta}^{\sigma\lambda} I_{\gamma\delta}^{\mu\nu} + I_{\gamma\delta}^{\sigma\lambda} I_{\alpha\beta}^{\mu\nu} - I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\lambda} - I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\lambda} \right] \\ & + \left[q_\lambda q^\mu \left(\eta_{\alpha\beta} I_{\gamma\delta}^{\nu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\nu\lambda} \right) + q_\lambda q^\nu \left(\eta_{\alpha\beta} I_{\gamma\delta}^{\mu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\lambda} \right) \right. \\ & \left. - q^2 \left(\eta_{\alpha\beta} I_{\gamma\delta}^{\mu\nu} - \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\nu} \right) - \eta^{\mu\nu} q_\sigma q_\lambda \left(\eta_{\alpha\beta} I_{\gamma\delta}^{\sigma\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\sigma\lambda} \right) \right] \\ & + \left[2q_\lambda \left(I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta}^{\nu\mu} (k - q)^\mu + I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta}^{\mu\nu} (k - q)^\nu - I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta}^{\nu\mu} k^\mu - I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta}^{\mu\nu} k^\nu \right) \right. \\ & \left. + q^2 \left(I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\sigma} + I_{\alpha\beta}^{\nu\sigma} I_{\gamma\delta}^{\mu\sigma} \right) + \eta^{\mu\nu} q_\sigma q_\lambda \left(I_{\alpha\beta}^{\lambda\rho} I_{\gamma\delta}^{\sigma\rho} + I_{\gamma\delta}^{\lambda\rho} I_{\alpha\beta}^{\sigma\rho} \right) \right] \\ & + \left\{ (k^2 + (k - q)^2) \left[I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\sigma} + I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\sigma} - \frac{1}{2} \eta^{\mu\nu} \mathcal{P}_{\alpha\beta\gamma\delta} \right] \right. \\ & \left. - \left(I_{\gamma\delta}^{\mu\nu} \eta_{\alpha\beta} k^2 + I_{\alpha\beta}^{\mu\nu} \eta_{\gamma\delta} (k - q)^2 \right) \right\} \Bigg) \end{aligned}$$

(62)

The quantum correction to the Newtonian potential

- 1) The bad news: Quantum correction is too small to observe
- or is this related to good news?

BUT

- 2) Calculation is fundamental and has interesting features:

$$V(r) = -\frac{GMm}{r} \left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi} \frac{G\hbar}{r^2 c^3} \right]$$

Newtonian realm

large mass or v^2/c^2
(Precession of Mercury)

shorter distances]

- why did this only get done in 1994/2002?

- 3) Reliable calculation of quantum general relativity!

Why power law corrections?

Uncertainty principle:

Massive particles have finite range

$$\Delta r \sim \frac{\hbar}{Mc}$$

Massless particles (eg graviton) have infinite range

→ Only massless particles can give power law correction

Power law effects are more predictable:

- we don't know what massive particles may exist at high energies
- but we do know GR at low energies → massless graviton

Power law terms not influenced by massive particles

→ reliable prediction of GR

Dimensional analysis

GMm/r has units of energy

Mc^2 has units of energy

\hbar has units of energy times distance

→ $\frac{GM}{rc^2}$ is dimensionless

-nonlinear features of GR are expressed as an expansion in this quantity

→ $\frac{G\hbar}{r^2c^3}$ is dimensionless

-unique form linear in both G and \hbar

$$\left(\frac{GMm}{r}\right)\left(\frac{\hbar c}{r}\right)\left(\frac{1}{Mc^2}\right)\left(\frac{1}{mc^2}\right) = \frac{G\hbar}{r^2c^3}$$

What to look for:

The key to calculation

General expansion:

$$V(r) = -\frac{GMm}{r} \left[1 + a \frac{G(M+m)}{rc^2} + b \frac{G\hbar}{r^2 c^3} \right] + cG^2 Mm \delta^3(r)$$

Classical expansion parameter
Quantum expansion parameter
Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2} \left[1 + a' G(M+m) \sqrt{-q^2} + b' G\hbar q^2 \ln(-q^2) + c' Gq^2 \right]$$

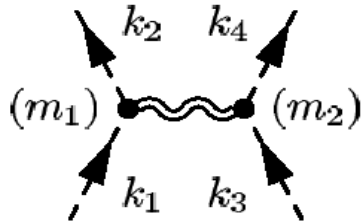
Classical
quantum
short range

Corrections to Newtonian Potential

Technical definition: The scattering potential

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4 \delta^{(4)}(p - p') (\mathcal{M}(q)) \\ &= -(2\pi) \delta(E - E') \langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}\quad \text{with} \quad V(\mathbf{x}) = \frac{1}{2m_1} \frac{1}{2m_2} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \mathcal{M}(\vec{q})$$

Newtonian potential = one graviton exchange



$$iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1, k_2, m_1) \left[\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2} \right] \tau_1^{\alpha\beta}(k_3, k_4, m_2)$$

or

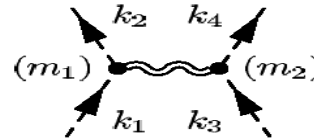
$$\underline{M_{1(a)}(\vec{q})} = -\frac{4\pi G m_1 m_2}{\vec{q}^2}$$

yields

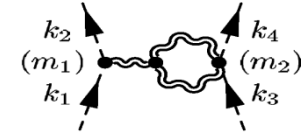
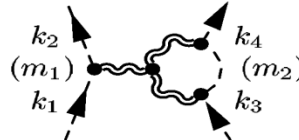
$$V_{1(a)}(r) = -\frac{G m_1 m_2}{r}$$

The calculation:

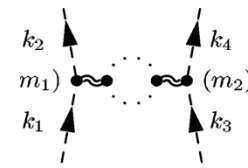
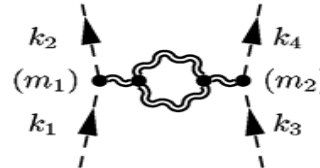
Lowest order:



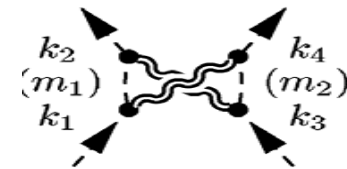
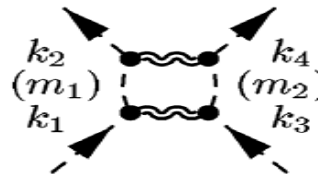
Vertex corrections:



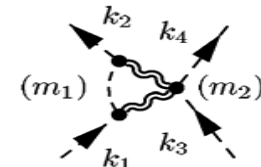
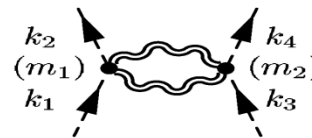
Vacuum polarization:
(Duff 1974)



Box and crossed box



Others:



Results:

Pull out non-analytic terms:

-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2 (m_1 + m_2)}{|\vec{q}|} + \frac{5}{3} \log \vec{q}^2 \right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{GMm}{r} \left[1 + 3 \frac{G(M+m)}{rc^2} + \frac{41}{10\pi} \frac{G\hbar}{r^2 c^3} \right]$$

Classical
correction

Quantum
correction

Comments

1) Quantum correction is unmeasurably small

Expansion scale = Planck Mass $G \sim \left(\frac{1}{10^{19} \text{ GeV}} \right)^2 \sim (10^{-34} \text{ meters})^2$
- if $r = 10^{-15} \text{ m}$, correction is 10^{-39}

2) The best perturbation theory in physics!

- pert. theory work best if correction is small
- very small correction

→ Perturbation theory of gravity is **very** well behaved!

3) No signs of problems of QM and GR

Iwaski
Gupta, Radford

4) Why was this calculation not done 40 years ago?

- classical correction has been done this way (1970's)
- focus was in high energy end – the divergences (unreliable)
- needed to look at low energy end
- reasoning of “effective field theory”

Why are there no divergences?

Decades of worry about divergences in quantum gravity

Yet, this calculation has no infinities!

Reason: Infinites come from high energy part of calculation
high energy part is short range (again uncertainty principle)

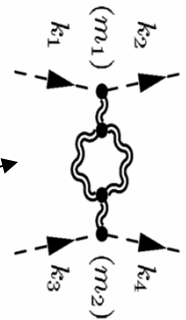
→ Power law terms not sensitive to any infinities

How this works in practice:

$$V_{R^2} \sim G^2 M m \frac{1}{q^2} q^4 \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 M m \delta^3(x)$$

Divergent term

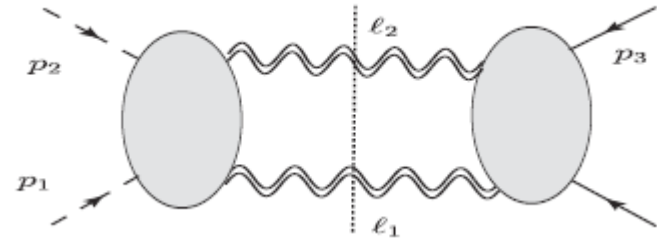
$$\sim q^4$$



Light bending at one loop

Bjerrum-Bohr, JFD, Holstein
Plante, Vanhove

- Another EFT calculation
- Uses remarkable fact:
Gravity Compton amplitude is
square of EM Compton amplitude
- Compare massless spin 0 and photon



$$\theta_\eta \simeq -\frac{b}{\omega} \int_{-\infty}^{+\infty} \frac{V'_\eta(b\sqrt{1+u^2})}{\sqrt{1+u^2}} du$$

$$\simeq \frac{4GM}{b} + \frac{15}{4} \frac{G^2 M^2 \pi}{b^2} + \frac{8bu^\eta + 9 + 48 \log \frac{b}{2r_o} G^2 \hbar M}{\pi b^3}$$

$\leftarrow bu^\varphi = 3/40$ and $bu^\gamma = -161/120$.

bu^i is different
coefficient for
spin 0 and 1

Massless particles deviate from null geodesics

- irreducible tidal effects from loops
- also non-universal – violation of some forms of EP
- perhaps energy dependence

The old problem with quantum GR

The Einstein theory has one parameter G :

$$S = \int d^4x \sqrt{g} \frac{1}{G} R \sim \int d^4x \frac{1}{G} \partial h \partial h + \dots$$

But, divergences and (large) high energy effects
can't be absorbed into G – different structure

How to proceed????

Here effective field theory comes to the rescue:

- divergences look like **some** term in general Lagrangian
- renormalization program can work

How to treat GR as effective theory

Weinberg
JFD

Derivative expansion for the Lagrangian:

$$\begin{aligned} S &= \int d^4x \sqrt{g} \left[\frac{1}{G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right] \\ &\sim \int d^4x \left[\frac{1}{G} \partial h \partial h + c_{1,2} \partial^2 h \partial^2 h + \dots \right] \end{aligned}$$

new terms
suppressed by
powers of M_p

Divergences and high energy effects get absorbed in parameters

- parameters should be measured (but unmeasurably small effects)
- **but this is just “irrelevant” technicality**
- divergences not reliable part of theory

Real quantum effects are low energy effects

- can isolate these in practical calculations
- example – previous calculations

Successes of the effective field theory for GR

Rational basis for quantum GR

Understanding the renormalization properties of GR

Understanding which effects are calculable

Modest number of calculations:

- Newtonian potential
- graviton graviton scattering
- quantum corrections to Schwarzschild, Kerr, Reissner-Nordstrom and Kerr-Newman metrics
- light bending
- scattering of massless particles
- Possible singularity avoidance?

Limitations of the effective field theory

It predicts that it falls apart at high energy:

$$Amp \sim A_0 \left[1 + Gq^2 + Gq^2 \ln q^2 + \dots \right]$$

Predicts uncontrolled expansion at $(\text{Energy})^2 \sim 1/G \sim (10^{19} \text{ GeV})^2$
- this is the “high energy scale” – the Planck scale

Needs to be replaced by more complete theory at that scale

Many interesting questions require full theory at
or beyond Planck scale

Black Holes as a frontier

Does the EFT fall apart with black holes???

- propagating past BH
- horizon – info falls in
- Hawking radiation – information paradox
- firewalls – drama at horizon
- singularity at center and EFT
- remnants?

Reformulate problem of quantum gravity

Old view: GR and Quantum Mechanics incompatible

Shocking!

New view: We need to find the right “high energy”
theory which includes gravity

Less shocking:

- not a conflict of GR and QM
- just incomplete knowledge

THIS IS PROGRESS!

Gravity fits well with our other interactions in Core Theory

$$\begin{array}{c}
 \text{quantum mechanics} \quad \text{spacetime} \quad \text{gravity} \\
 \text{---} \quad \text{---} \quad \text{---} \\
 W = \int_{k < \Lambda} [Dg][DA][D\psi][D\Phi] \exp \left\{ i \int d^4x \sqrt{-g} \left[\frac{m_p^2}{2} R \right. \right. \\
 \left. \left. - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i \bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}_L^i V_{ij} \Phi \psi_R^j + \text{h.c.} \right) - |D_\mu \Phi|^2 - V(\Phi) \right] \right\} \\
 \text{---} \quad \text{---} \quad \text{---} \\
 \text{other forces} \quad \text{matter} \quad \text{Higgs}
 \end{array}$$

Slide due to
Sean Carroll

Summary

We have a quantum theory of general relativity

-quantization, renormalization, perturbative expansion

It is an effective field theory

-valid well below the Planck scale

-corrections are very well behaved

$$V(r) = -\frac{GMm}{r} \left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi} \frac{G\hbar}{r^2c^3} \right]$$

Need full theory at or before Planck scale

-not conflict between QM and GR, but lack of knowledge
about fundamental high energy theory

-effective theory may be full quantum content of GR