# Effective Field Theory and General Relativity I

Note Title ~~8/29/2013~~ 9/4/13

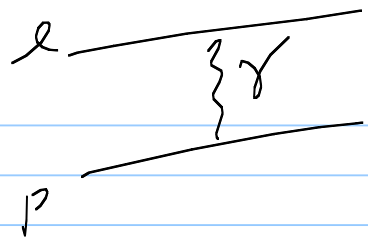
Why do quantum calculations work?

$$\mathcal{M}_{fi} = \sum_{I} \frac{\langle f | V | I \rangle \langle I | V | i \rangle}{E - E_I}$$

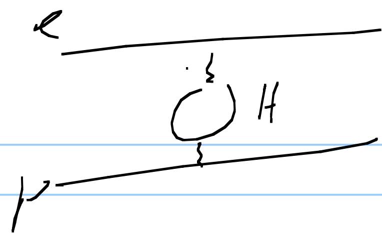
$\int d^3p$

$I = ?$

Uncertainty princ + Appelquist Carrazone



$\neq$



$$J_\mu \frac{e^2\left(1+\Pi(q)\right)}{q^2} J^\mu \qquad \Pi(q^2) = \frac{e^2}{12\pi^2}\left[\underbrace{\frac{1}{\varepsilon}+\dots}_{} - \ln\frac{M_H^2}{\mu^2} + \frac{q^2}{5 M_H^2}\right]$$

Renorm

$$\frac{e_R^2}{4\pi} = \frac{1}{137} \quad \leftarrow \text{include } M_H^2$$

$$\text{model} \sim c\, F_{\mu\nu} \frac{\Box}{M_H^2} F^{\mu\nu}$$

Heavy → local "unc. princ."

↑ $q^2$ or $F \Box F$

Appelquist Carrazone – Heavy particles → renorm of coupling constants

→ suppressed by $q^2/M_H^2$

How to treat $\frac{F \Box F}{M_H}$

– loops $\frac{\Lambda^2}{M_H^2} \sim O(1)$

# Eff Field Theory

- scales of problem
- low E degrees of Freedom

70' Triumph of Renorm QFT
80' EFT developed

→ standard QFT

Outline

Generalities of EFT

Applications

GR as EFT

Unitarity techniques

Cosmology

Running couplings + GR

Exchange of heavy particle

$$D_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot(x-y)}}{k^2 - M^2 + i\epsilon} \sim \frac{-i}{M^2}\,\delta^4(x-y)$$

$\uparrow$ small

#1) Locality

#2) Energy Expansion $\left(\frac{E}{M_H}\right)^n$

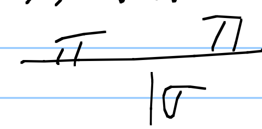
## Linear Sigma model $(\sigma, \vec{\pi})$

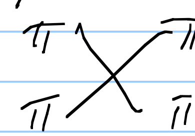
$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} + \frac{1}{2}\mu^2(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2$$

$$\langle\sigma\rangle = v \quad , \quad \sigma = v + \tilde{\sigma}$$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu\tilde{\sigma}\,\partial^\mu\tilde{\sigma} - 2\mu^2\tilde{\sigma}^2\right] + \frac{1}{2}\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - \lambda v\,\tilde{\sigma}(\tilde{\sigma}^2+\vec{\pi}^2) - \frac{\lambda}{4}(\tilde{\sigma}^2+\vec{\pi}^2)^2$$

$m_\sigma^2 = 2\mu^2$



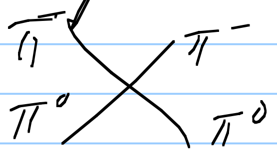



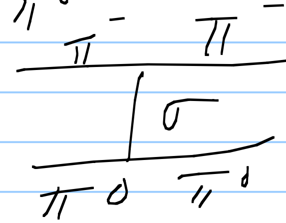

**But**

$$\mathcal{L}_{eff} = \frac{v^2}{4}\,\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) \qquad U = e^{i\frac{\vec{\tau}\cdot\vec{\pi}}{v}}$$

$$= \frac{1}{2}\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} + \frac{1}{6v^2}\left[(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi})\right] + \cdots$$

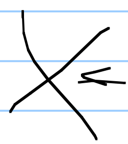
Example: $\pi^-\pi^0 \to \pi^-\pi^0$

1) 


$+$ 


$$iM = -2i\lambda + (-2i\lambda v)^2 \frac{i}{q^2 - m_\sigma^2} = -2i\lambda\left[1 + \frac{2\lambda v^2}{q^2 - 2\lambda v^2}\right]$$

$$= i\frac{q^2}{v^2} + \frac{q^4}{v^2 m_\sigma^2}$$

2)  $\Leftarrow \mathcal{L}_{eff} = i\frac{q^2}{v^2}$

Getting $\mathcal{L}_{eff}$

$$\Sigma = \sigma + i\vec{\tau}\cdot\vec{\pi} \quad = v + \tilde{\sigma} + i\vec{\tau}\cdot\vec{\pi}$$

$$\mathcal{L}_\sigma = \frac{1}{4}\,\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{\mu^2}{4}\,\mathrm{Tr}(\Sigma^\dagger\Sigma) - \frac{\lambda}{16}(\mathrm{Tr}\,\Sigma^\dagger\Sigma)^2$$

$$\Sigma \to L\,\Sigma\,R^\dagger \qquad SU(2)_L \otimes SU(2)_R$$

$$L \leftarrow e^{i\,\vec{\tau}\cdot\vec{\alpha}_L}$$

Change of names $\quad \Sigma = (v + s)\,U \qquad U = \exp\left(i\,\frac{\vec{\tau}\cdot\vec{\pi}}{v}\right)$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu s\,\partial^\mu s - 2\mu^2 s^2\right] + \frac{(v+s)^2}{4}\,\underline{\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)}$$

$$-\lambda v s^3 - \frac{\lambda}{4} s^4$$

$\times \leftarrow \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger$

$|\, S$

$\times \leftarrow \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

$$\sim \frac{q^4}{M_\sigma^2} \leftarrow \text{suppressed}$$

$U \to L U R^\dagger$

$$\mathcal{L} = \underline{\frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)} + \underline{\text{Terms with } S}$$

Haag's Theorem : "Names don't matter"

$$\left.\begin{aligned} \tilde{\sigma} &= S + \cdots \\ \pi' &= \pi + \cdots \end{aligned}\right\} \text{first order Kinetic E same}$$

Correspondence

$$Z[J_H] = \int dS\, d\pi\; e^{i\int d^4x \left[\mathcal{L}(S,\pi) + \vec{J}_\pi \cdot \vec{\pi}\right]}$$

$$= \int [d\pi]\; e^{i\int d^4x \left[\mathcal{L}_{eff}(\pi) + \vec{J}_\pi \cdot \vec{\pi}\right]}$$

low E

unc. princ $\Rightarrow \mathcal{L}_{eff}$ = local

Tree level:

$$\mathcal{L} = (\partial_\mu H \partial^\mu H \ldots) + JH$$

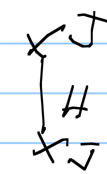
$$\int d^4x\, \mathcal{L}(H,J) = \int d^4x \left[ -\frac{1}{2} H \overset{\Box + M_H^2}{D} H + J H \right]$$

$$= -\frac{1}{2} \int d^4x \left[ \underbrace{(H - D^{-1}J)}_{H'} D \underbrace{(H - D^{-1}J)}_{H'} - J D^{-1} J \right]$$

$$Z = \int dH\, e^{i \int d^4x \left[ -\frac{1}{2} H D H + J H \right]}$$

$$= \int dH'\, e^{i \int d^4x \left[ H' D H' - J D^{-1} J \right]} \qquad dH = dH'$$

$$= Z[0]\, e^{\frac{i}{2} \int d^4x\, \underbrace{J D^{-1} J}}$$




$$W_{eff} = -\frac{1}{2} \int d^4x\, J(x)\, D_F(x-y)\, J(y)$$

$$D_F(x-y) = -\frac{1}{m_H^2}\,\delta^4(x-y) \qquad \leftarrow \text{low } E$$

$$W_{eff} = \int d^4x \; \frac{1}{2m_H^2}\, J(x)\,J(x)$$

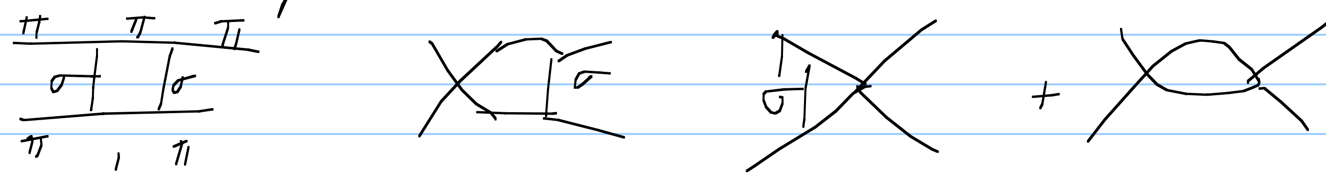
$$= \frac{v^2}{8m_\sigma^2}\left[\mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right)\right]^2 \qquad \leftarrow \sigma \text{ model}$$

$$\mathcal{L}_{eff} = \frac{v^2}{4}\mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right) + \frac{v^2}{8m_\sigma^2}\left[\mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right)\right]^2 + \cdots$$

$$I + X = i\frac{q^2}{v^2} + \frac{q^4}{m_\sigma^2 v^2}$$

## Matching at one loop

Full theory

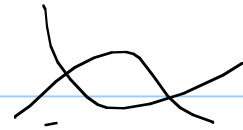



$$\mathcal{M} = \frac{g^2}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2 \qquad t = g^2$$

$$- \frac{1}{144\pi^2 v^2}\left[ s(s-u) + u(u-s)\right]$$

$$- \frac{1}{96\pi^2 v^2}\left[3t^2 \ln \frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]$$

$$+ \mathcal{O}(s^3)$$

Eff theory 

$$= \frac{t}{v^2} + \left[ 8 l_1^r + 2 l_2^r + \frac{5}{192\pi^2} \right] \frac{t^2}{v^2}$$ ← another

$$- \frac{1}{96\pi^2 v^2} \left[ 3t^2 \ln \frac{-t}{\mu^2} + \dots \right]$$

$$\mathcal{L}_{eff} = \frac{v^2}{4} \operatorname{Tr} \partial_\mu U \partial^\mu U^\dagger + l_1 \left[ \operatorname{Tr}( \partial_\mu U \partial^\mu U^\dagger) \right]^2$$

$$+ l_2 \operatorname{Tr}(\partial_\mu U \partial_\nu U^\dagger) \operatorname{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

$$l_1 = \frac{v^2}{8 m_\sigma^2} + \frac{1}{38\pi^2}\left[\ln \frac{m_\sigma^2}{\mu^2} - \frac{35}{6}\right]$$

$$l_2 = \frac{1}{192\pi^2}\left[\ln \frac{m_\sigma^2}{\mu^2} - \frac{11}{6}\right]$$

are exactly equal to $O(E^4)$

$\Longrightarrow$ Matching