EFT + GR #2

9/10/13

Note Title

Review of end of lecture #1:

$$\mathcal{L}_{eff} = \frac{v^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \ell_1 [\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)]^2 + \ell_2\,\mathrm{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right)\,\mathrm{Tr}\left(\partial^\mu U \partial^\nu U^\dagger\right)$$

$$U = \exp\left(i\frac{\vec{\tau}\cdot\vec{\pi}}{v}\right)$$

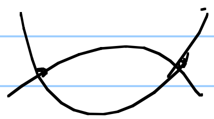
$$\mathcal{M}_{full} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2 - \frac{1}{144\pi^2 v^4}[s(s-u) + u(u-s)] - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right] \quad (3.7)$$

Matching

e effective theory result [Le72, GaL84]has a very similar form but does t know about the existence of the σ,

$$\mathcal{M}_{eff} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4} + \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4 - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right] \quad (3.8)$$

Comments:

1)  $l_1^r = l_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} + \cdots\right]$ renormalized E^4

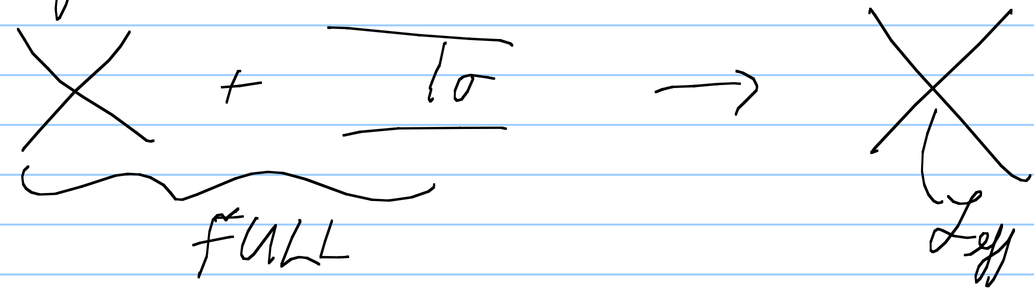
2) Structure same

3) Analytic term s^2, t^2 ... come with $l_1 = \frac{v^2}{8M_\sigma^2}$

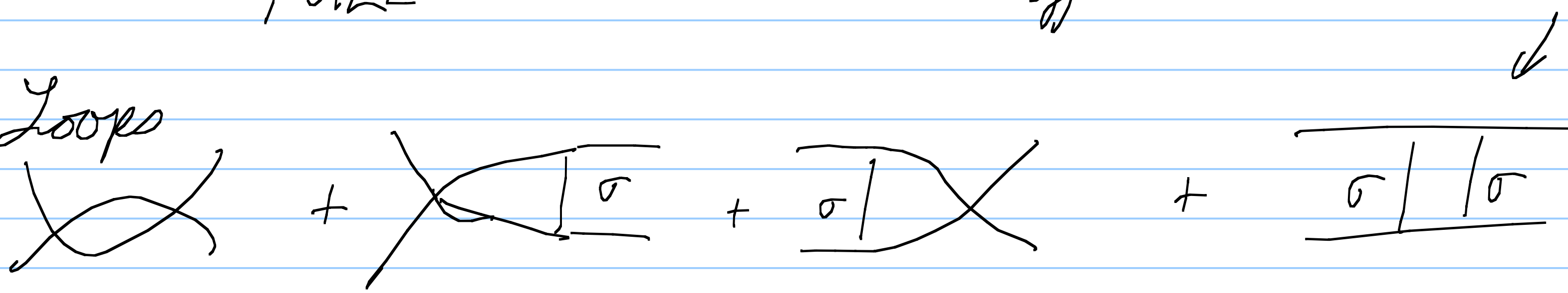
4) Nonanalytic $\ln t$ – no coeff. long distance

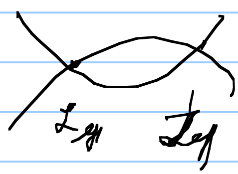
5) Loops E^4

Why does it work?



Loops





→ Low loop momentum — same by construction

→ High loop momenta — goes local $\mathcal{L}_1$

Power counting

$$\mathcal{M} = \frac{t}{v^2} \sim O(E^2)$$

Loops $\sim O(E^4)$ $\qquad \mathcal{L}_4 \sim O(E^4)$

$$I_{loop} = \int \frac{d^4 l}{(2\pi)^4} \frac{E^2}{v^2} \quad \text{Propagators } \frac{E^2}{v^2}$$

$$= \frac{E^4}{v^4}$$ Regularize dimensionally

powers of v $\longrightarrow$

Weinberg's power counting thm

$E^2 \rightarrow$ Tree level

$E^4 \rightarrow \mathcal{L}_4 + 1$ loop

$E^6 \rightarrow \mathcal{L}_6 + 2$ loop $\mathcal{L}_2 + 1$ loop $(\mathcal{L}_2 + \mathcal{L}_4)$

Limits of EFT

light σ ↙ ↙ loop

$$\mathcal{M} = \frac{t}{\Lambda^2} + l_1 \frac{t^2}{\Lambda^2} + \frac{t^2}{96\pi^2\Lambda^4} \ln t$$

"Rules:"

- Most general $\mathcal{L}$ consistent with symmetries
- Order by energy expansion
- Start with lowest order $\mathcal{L}$
- Loop
- renormalize parameters → match to full theory
  → measure
- predictions

Heat kernal

Parameter ↙

$$H(x,\tau) = \langle x | e^{-\tau D} | x \rangle$$

$$D = d_\mu d^\mu + m^2 + \sigma^2 \qquad d_\mu = \partial_\mu + \Gamma_\mu$$

$$H(x,\tau) = \frac{i}{(4\pi)^{d/2}} \frac{e^{-\tau m^2}}{\tau^{d/2}} \left[a_0(x) + a_1(x)\tau + a_2(x)\tau^2 + \cdots \right]$$

$$\det D = e^{\mathrm{tr} \ln D} = e^{\int d^4x \, \mathrm{Tr} \langle x | \ln D | x \rangle}$$

$$\ln b/a = \int_0^\infty \frac{dx}{x} \left(e^{-ax} - e^{-bx} \right)$$

$$a_2 \sim \Gamma(2 - d/2)$$

$$\langle x | \ln D | x \rangle = \frac{-i}{(4\pi)^{d/2}} \sum_{n=0}^{\infty} m^{d-2n} a_n(x) \, \Gamma(n - d/2)$$

$$H(x, \tau) = \int \frac{d^d p}{(2\pi)^d} e^{-ip \cdot x} e^{-\tau D} e^{ip \cdot x} \qquad \langle x | p \rangle = e^{ipx}$$

$$d_\mu e^{ip \cdot x} = e^{ipx} (ip_\mu + d_\mu)$$

$$a_0(x) = 1 \quad , \quad a_1(x) = -\sigma$$

$$a_2(x) = \frac{1}{2}\sigma^2 + \frac{1}{12}[d_\mu, d_\nu][d^\mu, d^\nu] + \frac{1}{6}[d_\mu, [d^\mu, \sigma]]$$

Background field:

$$U = \bar{U} + \delta U \qquad U^\dagger U = 1$$

$$U = \bar{U} e^{i\Delta} \qquad \Delta = \tau^i \Delta^i$$

Eq of M

$$\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) = \mathrm{Tr}(\partial_\mu \bar{U} \partial^\mu \bar{U}^\dagger) - 2i\,\mathrm{Tr}(\bar{U}^\dagger \partial_\mu \bar{U} \partial^\mu \Delta)$$

$$+ \mathrm{Tr}(\partial^\mu \Delta \partial_\mu \Delta + \bar{U} \partial^\mu \bar{U} (\Delta \partial^\mu \Delta - (\partial_\mu \Delta)\Delta))$$

$$\mathcal{L} = \mathcal{L}(\bar{U}) - \frac{v^2}{2} \Delta^i (d_\mu d^\mu + \sigma)^{ij} \Delta^j$$

$$\Gamma_\mu^{ij} = -\frac{1}{4} \mathrm{Tr}([\tau^i, \tau^j] \bar{U}^\dagger \partial_\mu U), \quad \sigma = \mathrm{Tr}([\tau^i, \bar{U}^\dagger \partial_\mu \bar{U}][\tau^j, \bar{U}^\dagger \partial^\mu \bar{U}])$$

+ Heat Kernal

$$l_1^R = l_1 + \frac{1}{384\pi^2}\left[\frac{2}{d-4} + \cdots\right]$$

$$\delta S^{1\,loop} = \frac{1}{32\pi^2}\left[\frac{2}{d-4} + \cdots\right] \int d^4x \left[\frac{1}{12}\left[Tr\left(\partial_\mu \bar{u}^+ \partial^\mu \bar{u}\right)\right]^2 + \frac{1}{6} Tr\left(\partial_\mu \bar{u}^+ \partial_\nu \bar{u}\right) \times Tr\left(\partial_\mu \bar{u}^+ \partial_\nu \bar{u}\right) \right]$$

$\Rightarrow$ get divergences + renormalize

QCD

full theory ψ, G, eff FT (π, K, η)

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi \longrightarrow \psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow V\psi \rightarrow SU(3)$$

$$\mathcal{L} = \bar\psi i \not{D} \psi = \bar\psi_L i \not{D} \psi_L + \bar\psi_R i \not{D} \psi_R$$

$$\psi_L \to L\psi_L, \quad \psi_R \to R\psi_R \qquad SU(3)_L \times SU(3)_R$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^2 + \bar\psi i \not{D} \psi - \bar\psi_L \gamma_\mu l_\mu \psi_L - \bar\psi_R \gamma^\mu r_\mu \psi_R$$

$$- \bar\psi_L (s + ip)\psi_R - \bar\psi_R (s - ip)\psi_R$$

Real pure QCD $l_\mu = r_\mu = p = 0$, $s = M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

+ QED $\quad l_\mu = r_\mu = e Q A_\mu$

$\rightarrow \quad l_\mu = -\frac{g}{2} \vec{\tau} \cdot \vec{W}_\mu \quad , \quad r_\mu = 0$

Invariance $\quad \psi_L \to L(x) \psi_L \quad , \quad \psi_R \to R(x) \psi_R$

$$l_\mu \to L\, l_\mu L^\dagger + (\partial_\mu L) L^\dagger$$

$$(s + ip) \to L (s + ip) R^\dagger$$

$$\mathcal{L} = \frac{F^2}{4} \mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{F^2 B_0}{2} \mathrm{Tr}((s+ip) U^\dagger + (s-ip) U)$$

$$D_\mu U = \partial_\mu U + i l_\mu U - i U r_\mu$$

$$U = \exp i \frac{\lambda^a \phi^a}{F}$$ → π, K, η^8 $\Rightarrow \vec{\tau} \cdot \vec{\pi}$

$F \to$ $\pi \to W \to e \nu$ $\quad F = F_\pi \simeq 93 \text{ MeV}$

$$m_\pi^2 = B_0 (m_u + m_d)$$

$$m_K^2 = B_0 (m_s + m_u)$$

$$\frac{m_u + m_d}{m_s + m_u} = \frac{m_\pi^2}{m_K^2} \sim \frac{1}{25}$$

$m_\pi = 135 \text{ MeV}, \quad m_K = 495 \text{ MeV}$

$O(E^4)$ $\mathcal{L}$ — Gasser + Leutwyler 1984

$L_i \sim$ few $\times 10^{-3}$

$L_1 = 1.12 \pm 0.20 \times 10^{-3}$

⋮

l_1, l_2 from σ model fails

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i O_i$$

$$= l_1 \left[\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)\right]^2 + L_2 \,\mathrm{Tr}\left(D_\mu U D_\nu U^\dagger\right) \cdot \mathrm{Tr}\left(D^\mu U D^\nu U^\dagger\right)$$

$$+ L_3 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\right)$$

$$+ L_4 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)$$

$$+ L_5 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \left(\chi U^\dagger + U \chi^\dagger\right)\right) + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2$$

$$+ L_7 \left[\mathrm{Tr}\left(\chi^\dagger U - U \chi^\dagger\right)\right]^2 + L_8 \,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U \chi^\dagger U \chi^\dagger\right)$$

$$+ i L_9 \,\mathrm{Tr}\left(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U\right) + L_{10} \,\mathrm{Tr}\left(L_{\mu\nu} U R^{\mu\nu} U^\dagger\right)$$

$\sigma(\gamma\gamma \to \pi^0\pi^0)$

all loop