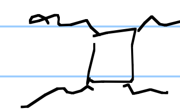
EFT and GR #3

9/12/13

Note Title 8/29/2013

Other EFT



$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$

1) QED

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha}{60\pi m^2} F_{\mu\nu} \Box F^{\mu\nu} + \frac{\alpha^2}{90 m^4}\left[(F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{10} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right] + \cdots$$

2) N.R. EFT

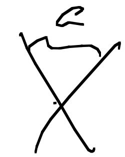
$$\psi = \begin{pmatrix} u(x,t) \\ \ell(x,t) \end{pmatrix} e^{-imt} \xrightarrow{\text{remove } \ell} \mathcal{L} = \mathcal{L}(u)$$

$\leftarrow$ antiparticle $\sim \ell \sim \frac{1}{2m} u$

3) Integrating out high freq. parts of single field

$\sim$ Wilson ($\Lambda \sim$ cutoff

$$\mathcal{H}_W = \int d^4x \, D_F(x, M_W)\, J_\mu(x)\, J^\mu(0)$$



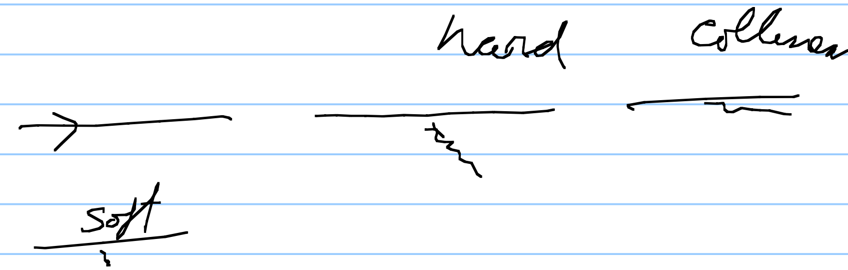
$$= \sum_n C_n(\Lambda)\, O_n(0)$$

wilson coeff. — all beyond scale Λ — operators

$$\langle \mathcal{H} \rangle = C_\mu \langle O \rangle_\Lambda \quad \leftarrow \text{"matching"}$$

In practice dim reg not Λ

4) Soft Collinear ET SCET



5) <u>New Physics</u>   higher dim operators

GR as EFT

$$D_\mu A^\lambda = \partial_\mu A^\lambda + \Gamma^\lambda_{\mu\nu} A^\nu$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left[\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right]$$

$$[D_\mu , D_\nu] A_\alpha = R^\beta{}_{\mu\nu\alpha} A_\beta$$

$$R^\beta{}_{\alpha\mu\nu} = \partial_\mu \Gamma^\beta_{\alpha\nu} - \partial_\nu \Gamma^\beta_{\alpha\mu} + \Gamma^\lambda_{\alpha\nu} \Gamma^\beta_{\lambda\mu} - \Gamma^\lambda_{\alpha\mu} \Gamma^\beta_{\lambda\nu}$$

$$R_{\alpha\mu} = R^\lambda{}_{\alpha\mu\lambda} \quad \Leftarrow$$

$$R = g^{\alpha\mu} R_{\alpha\mu} \quad \Leftarrow \text{ 2 deriv.}$$

$$S = \int d^4x \sqrt{g} \left[\frac{2}{\kappa^2} R + \mathcal{L}_m \right] \qquad \kappa^2 = 32\pi G$$

$$\sqrt{g}\, T_{\mu\nu} = -2 \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{g}\, \mathcal{L}_m \right)$$

Quantization 60's Feynman DeWitt

70's 't Hooft Veltman ← background field

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \qquad \kappa \sim \frac{1}{M_{P}}$$

Expand

$$\sqrt{g} = \sqrt{\det g_{\mu\nu}} = e^{\frac{1}{2} \mathrm{Tr} \ln (\bar{g}_{\mu\nu} + \kappa h_{\mu\nu})} = \sqrt{\bar{g}} \left[1 + \kappa h^\lambda_{\ \lambda} - \frac{\kappa^2}{4} h^{\alpha\beta} h_{\alpha\beta} + \frac{\kappa^2}{8} (h^\lambda_{\ \lambda})^2 \right]$$

$$\frac{2}{\kappa^2}\sqrt{g}R = \sqrt{\bar{g}}\left[\frac{2}{\kappa^2}\bar{R} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}\right]$$

$$\mathcal{L}^{1} = \frac{h_{\mu\nu}}{\kappa}\left[\bar{g}^{\mu\nu}\bar{R} - 2R^{\mu\nu}\right]$$

$$\mathcal{L}^{(2)} = \frac{1}{2}D_\alpha h_{\mu\nu}D^\alpha h^{\mu\nu} - \frac{1}{2}D_\alpha h D^\alpha h + D_\alpha h D_\beta h^{\alpha\beta} -$$

$$- D_\alpha h_{\mu\beta}D^\beta h^{\mu\alpha} + \bar{R}\left(\frac{1}{2}h^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu}\right)$$

$$+ \bar{R}^{\mu\nu}\left(2h^\alpha_\mu h_{\nu\alpha} - h h_{\mu\nu}\right)$$

Gauge fixing $\quad Z = \int dh_{\mu\nu}\, \delta(G_\alpha(h)) \det \left| \frac{\delta G_\alpha}{\delta \varepsilon_\beta} \right| e^{iS}$

$\det M = \int d\bar{\eta}\, d\eta\, e^{i\int d^4x\, \bar{\eta} M \eta}$

Harmonic gauge $\quad \partial^\mu h_{\mu\nu} - \frac{1}{2}\partial^\nu h = 0$

$$G^\alpha = \sqrt[4]{g}\,\left(D^\mu h_{\mu\nu} - \frac{1}{2} D_\nu h^\lambda{}_\lambda \right) t^{\nu\alpha}$$

$$\eta_{\alpha\beta}\, t^{\mu\alpha} t^{\nu\beta} = \bar{g}^{\mu\nu}$$

$$\mathcal{L}_{gf} = \sqrt{g}\,\left(\qquad\qquad \right)^2$$

$$\mathcal{L}_{ghost} = \sqrt{g}\; \eta^{*\mu} \left(D_\lambda D^\lambda \bar{g}_{\mu\nu} - R_{\mu\nu} \right) \eta^\nu$$

ghosts

$$S = \int d^4x \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta,\gamma\delta} h_{\gamma\delta} + \eta^{*\mu} (\quad) \eta^{\nu} \right\}$$

$$Z = \int dh_{\mu\nu} \, e^{iS}$$

heat kernal $- a_2(x)$

$$= \det D^{\alpha\beta,\gamma\delta} = e^{\mathrm{Tr} \ln D^{\alpha\beta,\gamma\delta}}$$

$$S_{1\,loop} = \frac{1}{8\pi^2} \left[\frac{1}{d-4} + \cdots \right] \int d^4x \sqrt{\bar{g}} \left\{ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right\}$$

4 deriv

Pure gravity is one loop finite $\bar{R}_{\mu\nu} = 0$ (real world - divergent)

$$S_{2\,loop} = (\quad) \frac{1}{d-4} R_{\alpha\beta\gamma\delta} R^{\gamma\delta\lambda\sigma} R_{\lambda\sigma}{}^{\alpha\beta}$$ ← does not vanish

EFT

$$S = \int d^4x \sqrt{g} \left[\overset{M^4}{\Lambda} + \overset{M^2}{\frac{2}{K^2} R} + \overset{M^0}{c_1 R^2} + \overset{\partial^4}{c_2 R_{\mu\nu} R^{\mu\nu}} + \mathcal{O}(\partial^6) \right]$$

1) Λ = Cosm. const $= (10^{-3}\,eV)^4$ ← drop

2) $K^2 = 32\pi G$

3) $c_1, c_2 \lessgtr 10^{74}$ Stelle

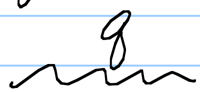
Renormalized $\quad c_1^r = c_1 + \frac{1}{8\pi}\left[\frac{1}{d-4} + \underset{\ln \mu^2}{\sim}\right] \times \frac{1}{120}$

$\quad c_2^r = c_2 +$ " $\frac{7}{20}$

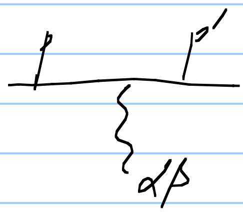
What are the predictions?

- Not divergences
- Not parameters
- low energy propagation – nonanalytic $\ln -q^2$, $\sqrt{-q^2}$

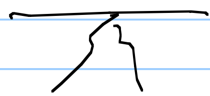
Feynman rules   Minkowski

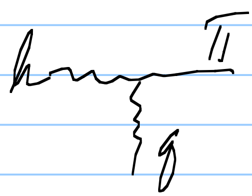


$$= \frac{i}{q^2} P_{\mu\nu\alpha\beta} \qquad P_{\mu\nu\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$



$$= \tau_{\alpha\beta} = -i\frac{\kappa}{2}\left(p_\alpha p'_\beta + p'_\alpha p_\beta - g_{\alpha\beta}\left[p\cdot p' - m^2\right]\right)$$





$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}\right)$$

$$
\begin{aligned}
\tau 3^{\mu\nu}_{\alpha\beta\gamma\delta}(k,q) = -\frac{i\kappa}{2} \times \Bigg(& \mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + \pi^\mu \pi^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu} q^2\right] \\
& + 2q_\lambda q_\sigma \left[I_{\alpha\beta}{}^{\sigma\lambda} I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda} I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma} I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma} I_{\alpha\beta}{}^{\nu\lambda} \right] \\
& + \left[q_\lambda q^\mu \left(\eta_{\alpha\beta} I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}{}^{\nu\lambda} \right) + q_\lambda q^\nu \left(\eta_{\alpha\beta} I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}{}^{\mu\lambda} \right) \right. \\
& \left. - q^2 \left(\eta_{\alpha\beta} I_{\gamma\delta}{}^{\mu\nu} + \eta_{\gamma\delta} I_{\alpha\beta}{}^{\mu\nu} \right) - \eta^{\mu\nu} q_\sigma q_\lambda \left(\eta_{\alpha\beta} I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}{}^{\sigma\lambda} \right) \right] \\
& + \left[2q_\lambda \left(I_{\alpha\beta}{}^{\lambda\sigma} I_{\gamma\delta\sigma}{}^{\nu} \pi^\mu + I_{\alpha\beta}{}^{\lambda\sigma} I_{\gamma\delta\sigma}{}^{\mu} \pi^\nu + I_{\gamma\delta}{}^{\lambda\sigma} I_{\alpha\beta\sigma}{}^{\nu} k^\mu + I_{\gamma\delta}{}^{\lambda\sigma} I_{\alpha\beta\sigma}{}^{\mu} k^\nu \right) \right. \\
& \left. + q^2 \left(I_{\alpha\beta\sigma}{}^{\mu} I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma} I_{\gamma\delta\sigma}{}^{\mu} \right) + \eta^{\mu\nu} q_\sigma q_\lambda \left(I_{\alpha\beta}{}^{\lambda\rho} I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho} I_{\alpha\beta\rho}{}^{\sigma} \right) \right] \\
& + \Bigg\{ (k^2 + \pi^2) \left[\mathcal{P}_{\alpha\beta}{}^{\mu\sigma} \mathcal{P}_{\gamma\delta,\sigma}{}^{\nu} + \mathcal{P}_{\gamma\delta}{}^{\mu\sigma} \mathcal{P}_{\alpha\beta,\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu} (\mathcal{P}_{\alpha\beta,\gamma\delta} - \eta_{\alpha\beta}\eta_{\gamma\delta}) \right] \\
& + \left(\mathcal{P}_{\gamma\delta}{}^{\mu\nu} \eta_{\alpha\beta} \pi^2 + \mathcal{P}_{\alpha\beta}{}^{\mu\nu} \eta_{\gamma\delta} k^2 \right) \Bigg\} \Bigg)
\end{aligned}
$$

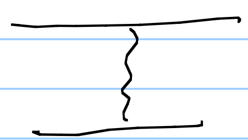
Gravitational potential – scattering

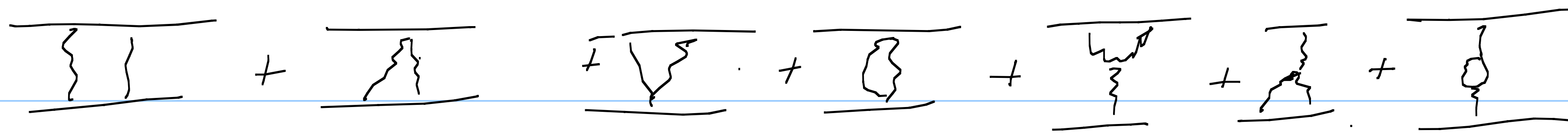
$$\langle p_3 p_4 | T | p_1 p_2 \rangle \sim \mathcal{M}(q)$$

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} e^{i q \cdot r} \mathcal{M}(q)$$

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

*

 $= \tau_1^{\mu\nu} \dfrac{i}{q^2} P_{\mu\nu\alpha\beta}\, \tau_2^{\alpha\beta} \longrightarrow \dfrac{G m m}{r}$



$$V = \frac{Gmm}{r}\left[1 + \frac{Gm}{r} + \frac{G\hbar}{r^2}\right] + G^2\delta^3(x)$$

$$\mathcal{V} = V(q^2) = \frac{1}{q^2}\left[1 + Gm\sqrt{-q^2} + Gq^2\ln q^2 + Gq^2\right]$$

$$\int e^{i\vec{q}\cdot\vec{r}}\frac{d^3q}{(2\pi)^3}\frac{1}{|q|} = \frac{1}{2\pi^2 r^2}$$

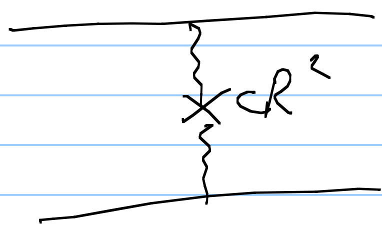
$$\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot r}\ln q^2 = \frac{-1}{2\pi^2 r^3}$$

$P_1, P_2 \to M$

Finite result.

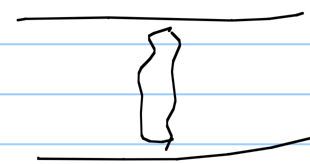
– divergences analytic not $\ln q^2$

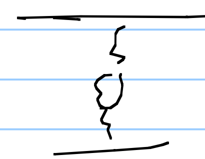
– independent of c_1, c_2



$\sim \frac{1}{q^2}\, q^4\, \frac{1}{q^2} \sim \text{const}$

$\sim \sqrt{q^2}$





not.

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + 3\,\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\,\frac{G}{r^2}\right] + \delta^4(x)$$

no $\hbar$

Classical results from loops?

$$\hbar \int d^4x \cdots \left[\phi D^2 \phi + \frac{m^2}{\hbar^2}\phi^2\right]$$

$\leftarrow$ 70' Gupta + Radford

Small!