

# EFT and GR #3

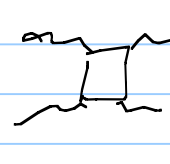
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## Other EFT

1) QED

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha}{60\pi m^2} F_{\mu\nu} \mathbb{B} F^{\mu\nu} + \frac{\alpha^2}{90m^4} \left[ (F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{16} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right] + \dots$$



$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

2) N.R. EFT

$$\psi = \begin{pmatrix} u(x,t) \\ \ell(x,t) \end{pmatrix} e^{-imt} \xrightarrow{\text{remove } \ell} \mathcal{L} = \mathcal{L}(u)$$

↖ antiparticle  $\sim \ell \sim \frac{1}{2m} u$

3) Integrating out high freq. parts of single field

- Wilson  $\Lambda \sim$  cutoff

$$H_w = \int d^4x D_F(x, M_w) J(x) J^\dagger(0)$$



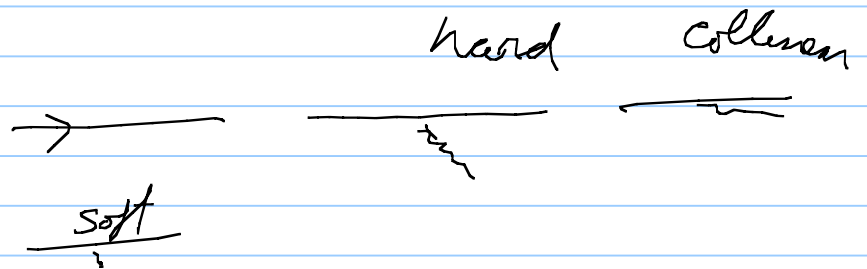
$$= \sum_n C_n(\Lambda) O_n(0)$$

$\uparrow$  Wilson coeff.       $\uparrow$  operators  
 all beyond scale  $\Lambda$

$$\langle H \rangle = c \langle O \rangle \quad \leftarrow \text{"matching"}$$

In practice dim reg not  $\Lambda$

4) Soft Collinear EFT SCET



5) New Physics higher dim operators

## GR as EFT

$$D_\mu A^\lambda = \partial_\mu A^\lambda + \Gamma_{\mu\nu}^\lambda A^\nu$$

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} [\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}]$$

$$[D_\mu, D_\nu] A_\alpha = R_{\mu\nu\alpha}^\beta A_\beta$$

$$R_{\alpha\mu\nu}^\beta = \partial_\mu \Gamma_{\alpha\nu}^\beta - \partial_\nu \Gamma_{\alpha\mu}^\beta + \Gamma_{\alpha\lambda}^\lambda \Gamma_{\nu\mu}^\beta - \Gamma_{\alpha\lambda}^\lambda \Gamma_{\mu\nu}^\beta$$

$$R_{\alpha\mu} = R_{\nu\alpha\mu}^\nu \quad \leftarrow$$

$$R = g^{\alpha\mu} R_{\alpha\mu} \quad \leftarrow 2 \text{ deriv.}$$

$$S = \int d^4x \sqrt{g} \left[ \frac{2}{\kappa^2} R + \mathcal{L}_m \right] \quad \kappa^2 = 32\pi G$$

$$\sqrt{g} T_{\mu\nu} = -2 \frac{\partial}{\partial g^{\mu\nu}} (\sqrt{g} \mathcal{L}_m)$$

Quantization 60's Feynman DeWitt

70's 't Hooft Veltman ← background field

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\kappa \sim \frac{1}{M_{\text{Pl}}}$$

Expand

$$\sqrt{g} = \sqrt{\det g_{\mu\nu}} = e^{\frac{\kappa}{2} \text{Tr} \ln(\bar{g}_{\mu\nu} + \kappa h_{\mu\nu})} = \sqrt{\bar{g}} \left( 1 + \kappa h^\lambda{}_\lambda - \frac{\kappa^2}{4} h^\lambda{}_\lambda h^\rho{}_\rho + \frac{\kappa^2}{8} (h^\lambda{}_\lambda)^2 \right)$$

$$\frac{2}{\kappa^2} \sqrt{g} R = \sqrt{g} \left[ \frac{2}{\kappa^2} \bar{R} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \right]$$

$$\mathcal{L}^{(1)} = \frac{h_{\mu\nu}}{\kappa} \left[ \bar{g}^{\mu\nu} \bar{R} - 2 R^{\mu\nu} \right]$$

$$\begin{aligned} \mathcal{L}^{(2)} = & \frac{1}{2} D_\alpha h_{\mu\nu} D^\alpha h^{\mu\nu} - \frac{1}{2} D_\alpha h D^\alpha h + D_\alpha h D_\beta h^{\alpha\beta} - \\ & - D_\alpha h_{\mu\rho} D^\rho h^{\mu\alpha} + \bar{R} \left( \frac{1}{2} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) \\ & + \bar{R}^{\mu\nu} \left( 2 h_{\mu\alpha} h_{\nu\alpha} - h h_{\mu\nu} \right) \end{aligned}$$

Gauge fixing  $Z = \int dh_{\mu\nu} \delta(G(h)) \det \left| \frac{\delta G_\alpha}{\delta \epsilon_\beta} \right| e^{iS}$

Harmonic gauge  $\partial^\mu h_{\mu\nu} - \frac{1}{2} \partial_\nu h = 0$

$\det M = \int d^4x \det M$   
 $\rightarrow e^{i \int d^4x \bar{\eta} M \eta}$

$G^\alpha = \sqrt{g} \left( D^\mu h_{\mu\nu} - \frac{1}{2} D_\nu h^\lambda{}_\lambda \right) t^{\nu\alpha}$

$\eta_{\alpha\beta} t^{\mu\alpha} t^{\nu\beta} = \bar{g}^{\mu\nu}$

$\mathcal{L}_{GF} = \sqrt{g} \left( \quad \right)^2$

$\mathcal{L}_{ghost} = \sqrt{g} \eta^{*\mu} \left( D_\lambda D^\lambda \bar{\eta}_{\mu\nu} - R_{\mu\nu} \right) \eta^\nu$   
 $\uparrow$  ghosts

$$S = \int d^4x \sqrt{g} \left\{ \frac{2}{\kappa^2} \bar{R} - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta, \gamma\delta} h_{\gamma\delta} + \underbrace{\mathcal{H}^\mu(\cdot) \eta^\nu}_{\text{ghosts}} \right\}$$

$$Z = \int dh_{\mu\nu} e^{iS}$$

$$= \det D^{\alpha\beta, \gamma\delta} = e^{\text{Tr} \ln D^{\alpha\beta, \gamma\delta}}$$

heat kernel -  $a_2(x)$

$$S_{1\text{loop}} = \frac{1}{8\pi^2} \left[ \frac{1}{d-4} + \dots \right] \int d^4x \sqrt{g} \left\{ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right\}$$

Pure gravity is one loop finite  $\bar{R}_{\mu\nu} = 0$  (real world - divergent)

$$S_{2\text{loop}} = \left( \cdot \right) \frac{1}{d-4} R_{\alpha\beta\gamma\delta} R^{\delta\gamma\alpha\beta} R_{\alpha\beta}{}^{\alpha\beta} \leftarrow \text{does not vanish}$$



EFT

$$S = \int d^4x \sqrt{g} \left[ \overset{M^4}{\downarrow} \Lambda + \overset{M^2}{\downarrow} \frac{2}{k^2} R + \overset{m^0}{\downarrow} c_1 R^2 + \overset{d^4}{\downarrow} c_2 R_{\mu\nu} R^{\mu\nu} + \mathcal{O}(\partial^6) \right]$$

1)  $\Lambda = \text{Cosm. const} = (10^{-3} \text{ eV})^4 \quad \leftarrow \text{drop}$

2)  $k^2 = 32\pi G$

3)  $c_1, c_2 \ll 10^{74}$

Renormalized

$$c_1^{\text{ren}} = c_1 + \frac{1}{8\pi} \left[ \frac{1}{d-4} + \dots \right] \times \frac{1}{120}$$

$$c_2^{\text{ren}} = c_2 + \dots \times \frac{7}{20}$$

Stelle  $\ln \mu^2$

What are the predictions?

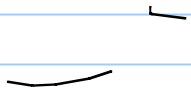
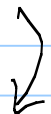
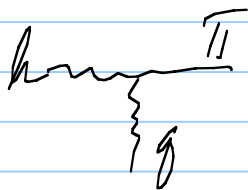
- Not divergences
- Not parameters
- low energy propagation - nonanalytic  $\ln -g^2$ ,  $\sqrt{-g^2}$

# Feynman rules Minkowski

$$\text{spin } g = \frac{i}{g^2} P_{\mu\nu\alpha\beta}$$

$$P_{\mu\nu\alpha\beta} = \frac{1}{2} [\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}]$$

$$\frac{p \quad p'}{\text{wavy } d\beta} = \text{wavy } d\beta = -i \frac{k}{2} (p_\alpha p'_\beta + p'_\alpha p_\beta - g_{\alpha\beta} [p \cdot p' - m^2]) \leftarrow$$



$$\begin{aligned}
\tau_{3\alpha\beta\gamma\delta}^{\mu\nu}(k, q) = & -\frac{i\kappa}{2} \times \left( \mathcal{P}_{\alpha\beta\gamma\delta} \left[ k^\mu k^\nu + \pi^\mu \pi^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\
& + 2q_\lambda q_\sigma \left[ I_{\alpha\beta}^{\sigma\lambda} I_{\gamma\delta}^{\mu\nu} + I_{\gamma\delta}^{\sigma\lambda} I_{\alpha\beta}^{\mu\nu} - I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\lambda} - I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\lambda} \right] \\
& + \left[ q_\lambda q^\mu \left( \eta_{\alpha\beta} I_{\gamma\delta}^{\nu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\nu\lambda} \right) + q_\lambda q^\nu \left( \eta_{\alpha\beta} I_{\gamma\delta}^{\mu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\lambda} \right) \right. \\
& \left. - q^2 \left( \eta_{\alpha\beta} I_{\gamma\delta}^{\mu\nu} + \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\nu} \right) - \eta^{\mu\nu} q_\sigma q_\lambda \left( \eta_{\alpha\beta} I_{\gamma\delta}^{\sigma\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\sigma\lambda} \right) \right] \\
& + \left[ 2q_\lambda \left( I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta\sigma}^{\nu\mu} + I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta\sigma}^{\mu\nu} + I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta\sigma}^{\nu\mu} + I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta\sigma}^{\mu\nu} \right) \right. \\
& \left. + q^2 \left( I_{\alpha\beta\sigma}^{\mu\nu} I_{\gamma\delta}^{\nu\sigma} + I_{\alpha\beta}^{\nu\sigma} I_{\gamma\delta\sigma}^{\mu\nu} \right) + \eta^{\mu\nu} q_\sigma q_\lambda \left( I_{\alpha\beta}^{\lambda\rho} I_{\gamma\delta\rho}^{\sigma} + I_{\gamma\delta}^{\lambda\rho} I_{\alpha\beta\rho}^{\sigma} \right) \right] \\
& + \left\{ (k^2 + \pi^2) \left[ \mathcal{P}_{\alpha\beta}^{\mu\sigma} \mathcal{P}_{\gamma\delta,\sigma}^{\nu} + \mathcal{P}_{\gamma\delta}^{\mu\sigma} \mathcal{P}_{\alpha\beta,\sigma}^{\nu} - \frac{1}{2} \eta^{\mu\nu} (\mathcal{P}_{\alpha\beta,\gamma\delta} - \eta_{\alpha\beta} \eta_{\gamma\delta}) \right] \right. \\
& \left. + (\mathcal{P}_{\gamma\delta}^{\mu\nu} \eta_{\alpha\beta} \pi^2 + \mathcal{P}_{\alpha\beta}^{\mu\nu} \eta_{\gamma\delta} k^2) \right\}
\end{aligned}$$

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2} \left( \eta_{\alpha\gamma} \eta_{\beta\delta} + \eta_{\beta\gamma} \eta_{\alpha\delta} \right)$$

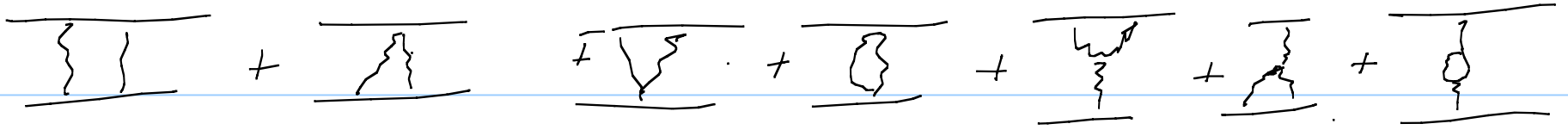
# Gravitational potential - scattering

$$\langle p_3 p_4 | T | p_1 p_2 \rangle \sim \mathcal{M}(q)$$

$$\uparrow V(r) = \int \frac{d^3 q}{(2\pi)^3} e^{i q \cdot r} \mathcal{M}(q)$$

$$V(r) = - \frac{G M_1 M_2}{r} \left[ 1 + 3 \frac{G(M_1 + M_2)}{r} + \frac{41}{15\pi} \frac{G^2}{r^2} \right]$$

$$\text{---} \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} = \underbrace{\quad}_{1}^{M_1} \frac{i}{q^2} P_{\mu\nu\alpha\beta} \underbrace{\quad}_{2}^{\alpha\beta} \longrightarrow \frac{G M_1 M_2}{r}$$



$$V = \frac{G_{mm}}{r} \left[ 1 + \frac{Gm}{r} + \frac{G\hbar}{r^2} \right] + G^2 \int^3 \psi(x)$$

$$\mathcal{M} = V(g^2) = \frac{1}{g^2} \left[ 1 + G \sqrt{-g^2} + G g^2 \ln g^2 + G g^2 \right]$$

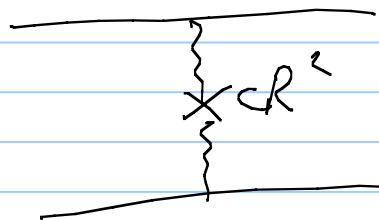
$$\int \frac{e^{i\vec{q}\cdot\vec{r}} d^3\epsilon}{(2\pi)^3 |\epsilon|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3g}{(2\pi)^3} e^{i\vec{g}\cdot\vec{r}} \ln g^2 = \frac{-1}{2\pi^2 r^3}$$

$P_1, P_2 \rightarrow M$   
 $\frac{1}{g}$

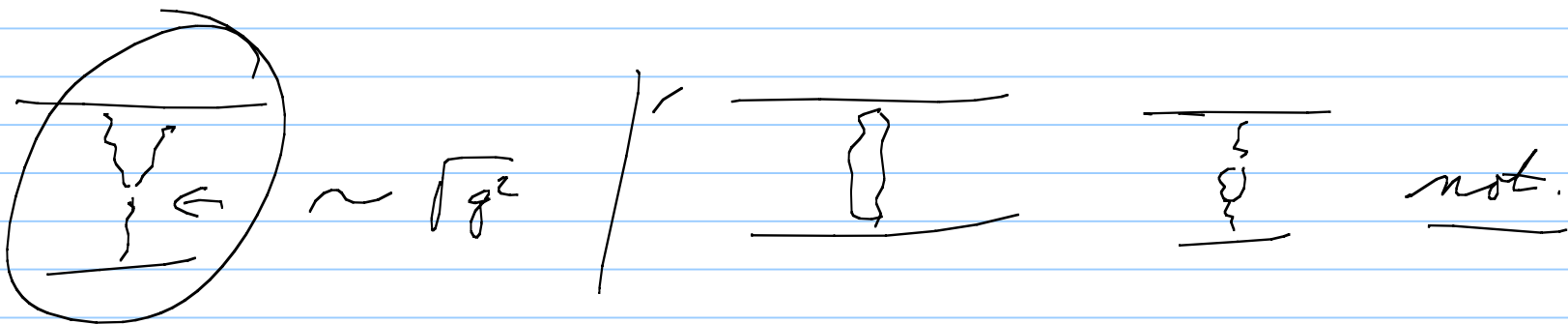
Finite result:

- divergence analytic not  $\ln g^2$
- independent of  $C_1, C_2$



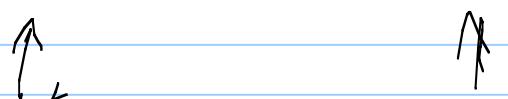
A Feynman diagram showing a horizontal line with a vertical wavy line (tadpole) attached to it. The wavy line has a cross through it, indicating a ghost loop. The diagram is labeled with  $\chi R^2$ .

$$\sim \frac{1}{g^2} g^4 \frac{1}{g^2} \sim \text{const}$$



Three Feynman diagrams are shown, separated by a vertical line. The first diagram is a tadpole with a ghost loop (wavy line with a cross) and is circled. The second diagram is a tadpole with a fermion loop (solid line with arrows). The third diagram is a tadpole with a scalar loop (dashed line). The first diagram is labeled  $\sim \sqrt{g^2}$ . The second and third diagrams are labeled not.

$$V(r) \Rightarrow -\frac{G m_1 m_2}{r} \left[ 1 + 3 \frac{G(m_1 + m_2)}{r} + \frac{4\pi}{10\pi} \frac{G}{r^2} \right] + \mathcal{O}(r^4)$$



Classical results from loops?

$\frac{m v}{\hbar}$

$$\frac{1}{\hbar} \int d^4x \dots \left[ \phi D^2 \phi + \frac{m^2}{\hbar^2} \phi^2 \right]$$



← 70' Gupta + Radford

Small!