EFT and GR - Saalberg 4

Note Title ~~8/29/2013~~ 9/13/13

Old view: GR + QM incompatable

New view: GR + QM = EFT

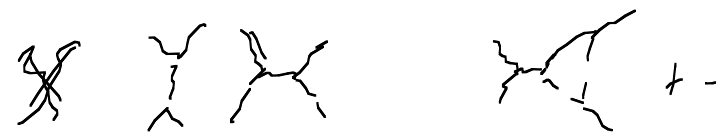
EFT have limits

Quantum + Gravity completion UV

Many interesting questions

Progress:

Graviton-graviton scattering



Dunbar Norridge

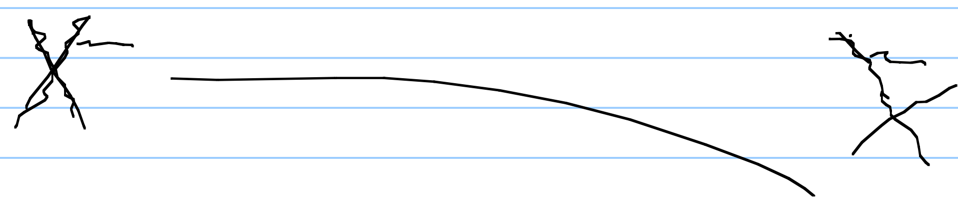
$$\mathcal{A}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}\left(1+\frac{\kappa^2\, s\, t\, u}{4(4\pi)^{2-\varepsilon}}\,\frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\;\times\right.$$

$$\times\left[\frac{2}{\varepsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.$$

IR

$$\left.\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]\right)$$

no C_1, C_2 – pure gravity 1-loop finite



$$d\sigma(GG\to GG) + d\sigma(\cancel{G}G \to GGG_s) = \text{finite}$$

Corrections to Reissner Nordstrom + Kerr Newman Metric (M, Q, S)

– QED

$$T_{\mu\nu}(E) = \begin{pmatrix} \frac{1}{2}E^2 & \\ & -E_\mu E_\nu \end{pmatrix}$$

$$g_{00} = 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\alpha\hbar}{3\pi m r^3}$$

$$g_{ij} = \left(\ldots\ldots \quad \frac{2}{\pi}\frac{G\alpha\hbar}{mr^3} \right)(\vec{S}\times\vec{r})_i$$

Limits of EFT

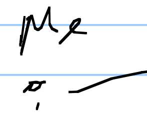
$$\mathcal{M} = GE^2\left(1 + GE^2 + GE^2 \ln E\right)$$

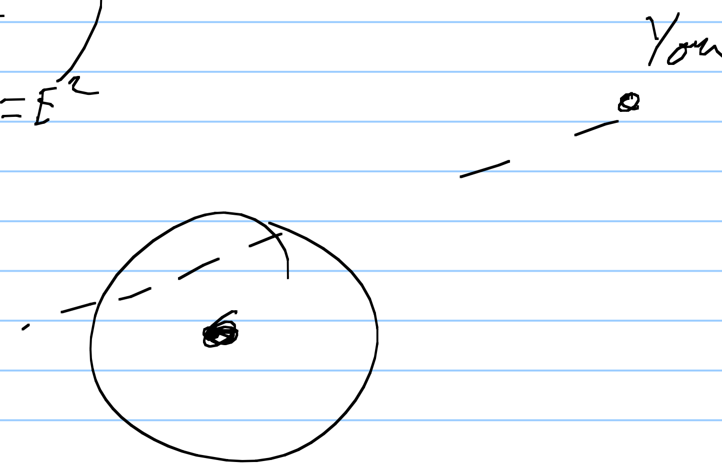
$\leftarrow O(1) = M_p^2 = E^2$

Low E issues

R small

↑ potential long issues



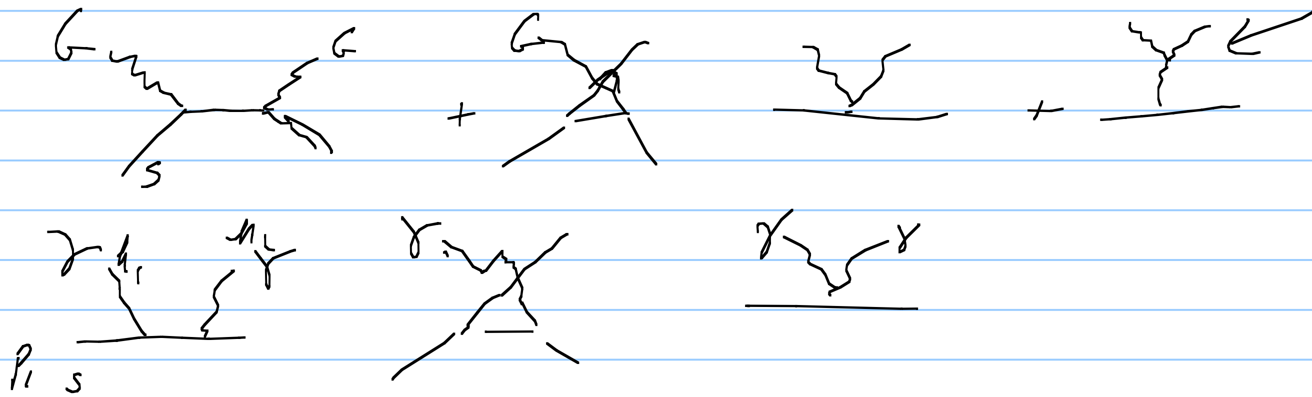


$\swarrow$ R small, Integrated curvature Ry^2

$$g_{\mu\nu} = \eta_{\mu\nu} + R_{\mu\nu\alpha\beta}\, y^\alpha y^\beta + \cdots$$

Gravity = "Square of Gauge Theory"

KLT O C



$$M_{Gs}(\quad) = \frac{K^2}{2e^2}\,\frac{p_1\cdot k_1\; p_1\cdot k_2}{k_1\cdot k_2}\,A_{\gamma s}(\quad)\,A_{s0}(\quad)$$

$$\varepsilon_{\mu\nu}(+2) = \varepsilon_\mu(+1)\,\varepsilon_\nu(+1)$$

Helicity amplitude $\qquad \bar{u}(k_2)\,\gamma\,u(k)$

$$A_0 = \frac{m^2 \ell\,[k_1, k_2]^2}{4(p_1\cdot k_1)(p_1\cdot k_2)}$$

L. Dixon TASI

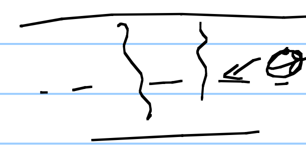
R.K. Ellis

$$\mathcal{M}_G = \frac{\kappa^2}{16}\,\frac{1}{k_1\cdot k_2}\,\frac{[k_1, k_2]^2}{p_1\cdot k_1\; p_2\cdot k_2}$$

On shell methods

Real parts ~ Imag parts

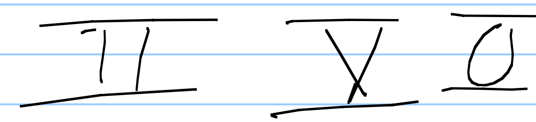
← on shell states



← on shell

1) Dispersion relation $\quad V(q^2) = \frac{1}{\pi}\int dt \, \frac{\rho(t)}{t - q^2}$ ← on shell states only

2) Matching to box, triangle + bubble diagrams



- Passarino - Veltman - reduced to
- identifiable cuts ←
- $\mathcal{M} = N_B \, \mathrm{Box}(p_1, p_2, k_1, k_2) + N_T \, \mathrm{Triang}(\;) + N_{Bub} \, \mathrm{Bubble}$ (Bern)

Bjerrum Bohr , Vanhove , JFD

Cosmology

B. El Menoufi

$$\left(\frac{\dot a}{a}\right)^2_t = \frac{8\pi G}{3} T_{\mu\nu}(t) + \underbrace{G\hbar \left(\frac{\dot a}{a}\right)^2_t \int_{-\infty}^{t} \frac{dt'}{t-t'}}_{\text{Memory of past expansion}} \qquad \left(\frac{\dot a}{a}\right)^2_{t'}$$

$\ln q^2$

QFT + Cosmology

Scattering $\quad S_{fi} = \langle \psi_f(+\infty) | U_I(+\infty, -\infty) | \psi_i(-\infty) \rangle$

$$\mathcal{M} = \langle 0 | T\big(\mathcal{O}(t)\, U_I(+\infty, -\infty)\big) | 0 \rangle$$

"in-in", "closed time path", "Schwinger Keldysh"

$$\mathcal{M} = \langle \Psi(-\infty) | \, U^\dagger(-\infty, t) \; T_{\mu\nu}(t) \; U(t, -\infty) \, | \Psi(-\infty) \rangle$$

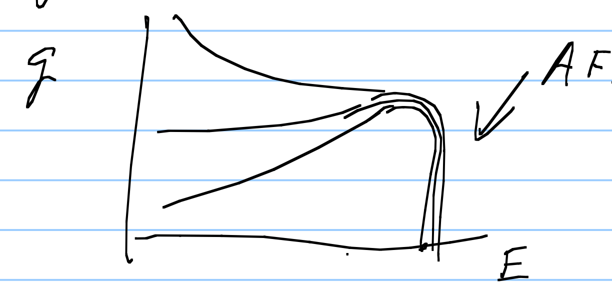
$$= \langle \Psi(-\infty) | \; \underbrace{\underbrace{U^\dagger(-\infty, t) \; U^\dagger(t, \infty)}_{U^\dagger(-\infty,\, \infty)}}_{\text{anti time ordered!}} \; \underbrace{\underbrace{U(\infty, t) \; T_{\mu\nu}(t) \, U(t, -\infty)}_{T(T_{\mu\nu}(t)\, U(\infty,\, -\infty))}}_{\text{usual}} \; | \Psi(-\infty) \rangle$$

<u>Velkovisky</u>

Running couplings + gravity

$\leftarrow g\kappa^2 E^2$

Robinson + Wilczek $\beta(g, E) = b g^3 + c g \kappa^2 E^2$



1) RW wrong g does not run
   — dim reg

2) RW right g does run
   — cutoff reg

EFT to rescue

$$\mathcal{M}_i = a_i e^2 + e^2\left[b_i \kappa^2 q^2 + c_i \kappa^2 q^2 \ln q^2\right] \quad \Leftarrow \text{one process}$$

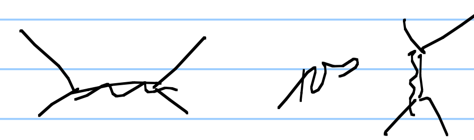
RW renormalize at $q^2 = E^2$

$$= a_i e^2\left[1 + \frac{b_i}{a_i}\kappa^2 E^2\right] + e^2 b_i \kappa^2 \underbrace{(q^2 - E^2)}$$

$$= a_i e^2(E)$$

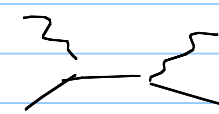
<u>But:</u>

1) Kinematic $q^2 = +E^2$, $q^2 = -E^2$



$e^2(E)$ ↓ or ↑ ⇐ not universal

2) Not universal a_i, b_i, c_i depend on process



Do calculation $\Longrightarrow$ no useful or universal def of $e^2(E^2)$

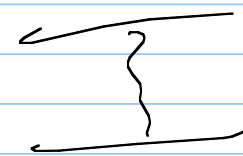
Cutoff: $eA \to A$

$$\mathcal{L} = \frac{1}{4e_0^2} F^2 + \bar{\Psi}(i\not{D} - m)\Psi$$

no elect charge

one loop

$$= \frac{1 + cK^2\Lambda^2}{4e_0^2} F^2 - \ldots$$



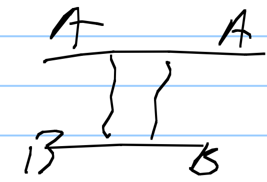
$$\beta(\Lambda) = \Lambda \frac{\partial}{\partial \Lambda}\left(\frac{e^2}{1 + cK^2\Lambda^2}\right) \neq 0 \Longrightarrow \text{running } e$$

$$\frac{e^2}{4\pi} = \frac{e_0^2}{(1+cK^2\Lambda^2)}\frac{1}{4\pi} = \frac{1}{137}$$

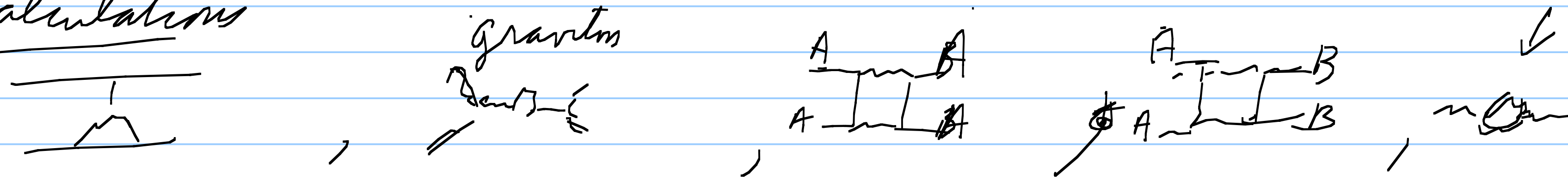
Running e with gravity is no a good concept

Running $G(E)$?

- Λ cutoff wrong



Calculations



massive

6 def of $G(E)$ $\longrightarrow$ Wildly different $\begin{cases}\to \text{sign} \\ \to \text{magnitude}\end{cases}$

$$\left.\begin{array}{l} \text{1) } q^2 = \pm E^2 \text{ - ambiguity} \in \\ \text{2) Non universality} \end{array}\right\} \text{ } G(E) \text{ in Lorentzian spaces is not useful or universal.}$$