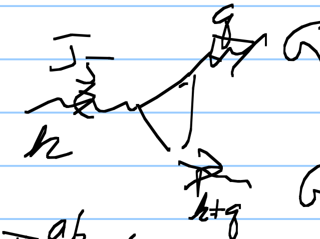
Anomalies 3

Note Title 11/10/2009

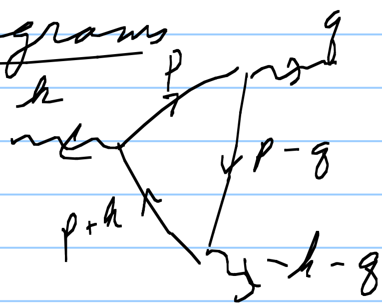
$$J^{(0)}_{5\mu} = \bar u \gamma_\mu \gamma_5 u + \bar d \gamma_\mu \gamma_5 d + \bar s \gamma_\mu \gamma_5 s$$

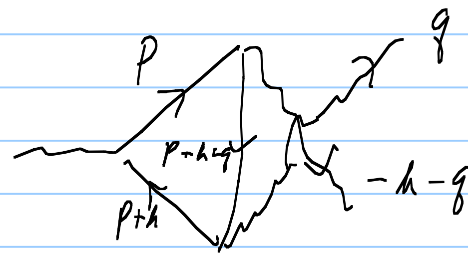


$$\langle G_1^a(q) \quad G_2^b(-k-q) \mid J^0_{5\mu} \mid 0 \rangle = i g^2 \varepsilon_1^{*\alpha}(q) \varepsilon_2^{*\beta} \, T^{ab}_{\mu\alpha\beta}(k,q)$$

$$k^\mu T^{ab}_{\mu\alpha\beta} \overset{?}{=} 0$$

Diagrams





$\leftarrow (\not k + \not p) - \not p$

Test $k^\mu T_{\mu\nu\beta} = \mathrm{Tr}(\not k \gamma_5$ - - - -

Turns into

$$k^\mu T^a_{\mu\nu b} = -6i\,\delta^{ab}\,\epsilon_{\alpha\beta\rho\sigma}\int\frac{d^4p}{(2\pi)^4}\left[\frac{(p+k+q)^\rho p^\sigma}{(p+k+q)^2\,p^2} - \frac{(p+k)^\rho\,(p-q)}{(p+k)\,(p-q)^2}\right.$$

$$\left. + 2 \text{ more}\right]$$

If we could shift $p \to p-q \Rightarrow$ cancels.

Can we do shift?

$$\int_{-\infty}^{\infty} dx\,\left[f(x+a) - f(x)\right] = \int_{-\infty}^{\infty} dx\; a f'(x) = a\left[f(\infty) - f(-\infty)\right]$$

OK if $f(\infty)=0$

$$\int_{-\infty}^{\infty} dx\; x = 0 = \int_{-\infty}^{\infty} dy\,(y+a) \qquad x = y+a$$

$$= a\,\infty$$

$$F(\vec{x}-\vec{x}_0) = F(x) + \vec{x}_0 \cdot \vec{\nabla} F\Big|_{x_0=0} \cdots$$

4 d generalization

$$\int d^4p \left[F(p) - F(p-\ell) \right] = \int d^4p \, \ell^\mu \frac{\partial F}{\partial p^\mu} = \ell^\mu \int dS_\mu \, F(p) \Big|_{p\to\infty}$$

$$I_\nu = \int \frac{d^4\ell}{(2\pi)^4} \left[\frac{p_\nu}{p^4} - \frac{(p-\ell)_\nu}{(p-\ell)^4} \right] = \ell^\mu \int d^3S_\mu \frac{p_\nu}{p^4} \quad \hat{n}$$

$$d^3S \, \frac{p_\mu}{p} \qquad \text{like } \int d\vec{A} = \int dA \, \hat{n}$$

$$p_\mu p_\nu \to \frac{1}{4} g_{\mu\nu} p^2$$

Wick rotate

$$I_\nu = \frac{\ell_\nu}{4} \int d^3S \, \frac{1}{p^3} = \frac{\ell_\nu}{4} \, i \, 2\pi^2 \frac{p^3}{p^3} = \frac{i\pi^2}{2} \ell_\nu$$

$$k^\mu T^{ab}_{\mu\alpha\beta} = \frac{3}{8\pi^2} \delta^{ab} \, \varepsilon_{\mu\beta\rho\sigma} k^\rho q^\sigma$$

Complications

- routing not unique
- gauge current $\quad \epsilon^{\alpha} T^{ab}_{\mu\alpha\beta} \neq 0 \quad *$
- if gauge current are conserved $\Longrightarrow$ unique answer for $k^{\mu} T_{\mu\alpha\beta} = \cdots$

$$\partial^{\mu} J^{(0)}_{5\mu} = \frac{3\alpha_s}{8\pi} F^{a}_{\mu\nu} \tilde{F}^{a\mu\nu} \quad *$$

$\pi^0 \to \gamma\gamma$ story

1) Almost conserved axial current

$$\partial^\mu J^3_{5\mu} = \partial^\mu \left[\frac{1}{2}\left(\bar{u}\,\gamma_\mu \gamma_5\, u - \bar{d}\,\gamma_\mu\gamma_5\, d\right)\right]$$

add EM

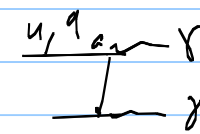
$$= \frac{1}{2}\left[\bar{u}\left(\overleftarrow{\not\partial}\,\gamma_5 + \underbrace{\overrightarrow{\not\partial}\,\gamma_5}_{-\gamma_5 \not D}\right) u + d \;\cdots\right)$$

$$\not D \qquad \not D$$

$$= i m_u\, \bar{u}\gamma_5 u - i m_d\, \bar{d}\gamma_5 d \quad \to 0 \text{ if } m \to 0$$

2) Easy to write eff $\mathcal{L}$ for $\pi^0 \to \gamma\gamma$

$$\mathcal{L} = \frac{F^2}{4}\,\mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right) + \frac{F^2 B_0}{4}\,\mathrm{Tr}\left(m U + U^\dagger m\right) - C_A\, e^2\, \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} A_\nu\, \mathrm{Tr}\left(Q^2\left(U \partial_\alpha U^\dagger + U^\dagger \partial_\alpha U\right)\right)$$



3) These clash $\leftarrow$ noether! Thus

$$J^3_{L\,\mu} = F^2\, Tr\left(\frac{\tilde{\tau}^3}{2}\left(U\,\partial_\mu U^+ - U^+\partial_\mu U\right)\right) - C_A\, e^2\, \varepsilon\, F\, A\; Tr\left(Q^2\left(U\,\tilde{\tau}_3\,U^+ - U^+\tilde{\tau}_3\,U\right)\right)$$

$\checkmark$ $\qquad$ X

$$\partial^\mu J^3_{5;\mu} \neq 0 \qquad \text{as } m \to 0$$

4) Then $C_A \sim O(m_q) \sim O\left(\frac{m_\pi^2}{\Lambda^2}\right)$

$\Rightarrow$ Suppression of $\pi^0 \to \gamma\gamma$ $\quad O\left(\frac{m_\pi^4}{\Lambda^4}\right) \sim 10^{-3}$

$\Rightarrow$ not seen

Answer is anomaly

$$\partial^\mu J^{(3)}_{A\,\mu} = \frac{\alpha_s\, N_c}{12\pi}\, F_{\mu\nu}\,\tilde{F}^{\mu\nu} \qquad + \text{ mass term}$$

γ

$\leftarrow$ fixes coeff in $\mathcal{L}_{eff}$ $\quad \checkmark$

Anomalies & Gauge currents

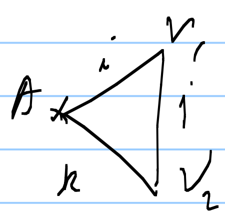
Global symmetries $e^{i\alpha}$ $\Rightarrow$ conservation laws

Gauge symmetries $e^{i\alpha(x)}$ $\Rightarrow$ gauge bosons & dynamics

Anomaly in global is OK $\Rightarrow$ not a symmetry

" " gauge is NOT OK $\Rightarrow$ inconsistent

$\Longrightarrow$ No anomaly in any gauge current



$$\Longrightarrow \sum_{ijk} Q^{A}_{ki}\, Q^{V_1}_{ij}\, Q^{V_2}_{jk} = 0$$

$$\Longrightarrow \mathrm{Tr}\left(Q^{A} Q^{V_1} Q^{V_2}\right) = 0 \quad \star$$

Ex $Z^0 \to \gamma\gamma \Rightarrow Tr(\tau^3 Q Q) = 0$

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}$

$$= +1 \times 0 + \underbrace{(-1)(-1)^2 + (1) \times \left(\tfrac{2}{3}\right)^2 \times 3}_{Q + L \text{ cancel}} + (-1)\left(\tfrac{-1}{3}\right)^2 \times 3 = 0$$