# Calculating in Field Theory 1

Note Title 3/2/2010

**Vertices**

$$\langle -i\mathcal{H}_I \rangle = \langle +i\mathcal{L}_I \rangle$$

drop external stuff

$$e^{-ip\cdot x}\ ,\ \frac{1}{\sqrt{2\omega}}\ ,\ \underbrace{\varepsilon^\mu\ ,\ u(p)}$$

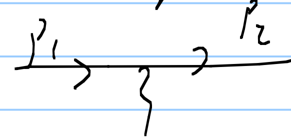
add back in for external state

$$T_{if} = (2\pi)^4\delta^4(p_i - p_f)\frac{1}{\sqrt{2\omega_i \cdots}}\, -i\mathcal{M}$$

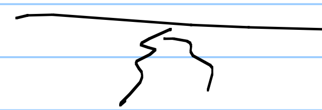
# Compton scattering $\quad \gamma + T \to \gamma' + T'$

i) Boson $\quad \mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi + \dots \qquad \ni g^{\mu\nu} A_\mu A_\nu$

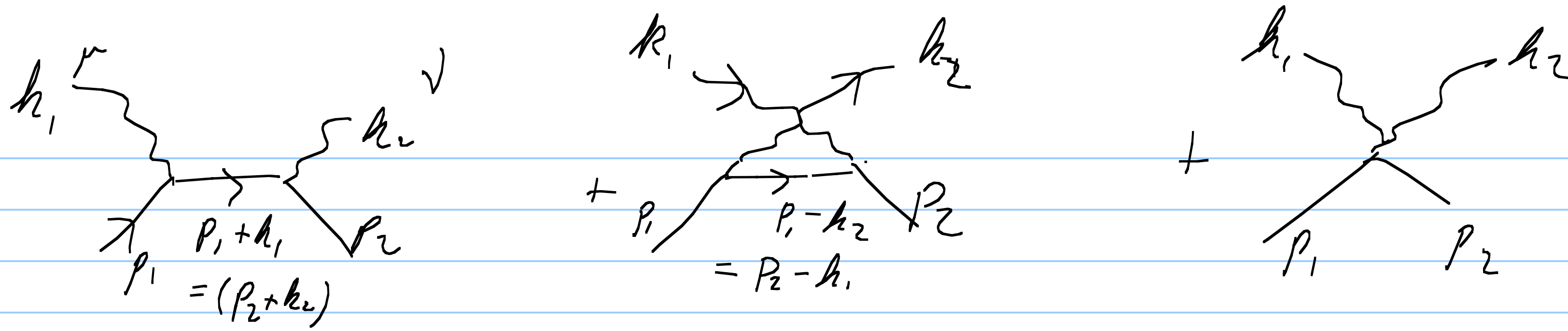
$$= \left[\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi\right] \underbrace{- e A_\mu \big( \underbrace{\phi^* i \overleftrightarrow{\partial}^\mu \phi}_{J_\mu} \big)} + e^2 A_\mu A^\mu \phi^* \phi$$



$= -ie(p_1 + p_2)^\mu$



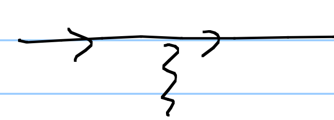
$= 2ie^2 g_{\mu\nu}$



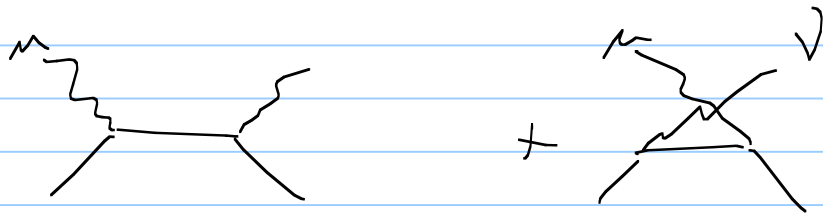

$$-i\mathcal{M} = \Big[(-ie)(2p_2+k_2)_\nu \frac{i}{(p_1+k_1)^2-m^2}(-ie)(2p_1+k_1)_\mu + (-ie)(2p_2-k_1)_\mu \frac{i}{(p_1-k_2)^2-m^2}(-ie)(2p_1-k_2)_\nu + 2i g_{\mu\nu} e^2\Big]\epsilon^{*\mu}(k_1)\epsilon^{*\nu}(k_2)$$

- order is not important here

2) Fermions $\quad \mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi = \bar{\Psi}(i\not{\partial} - m)\Psi - e A_\mu \bar{\Psi}\gamma^\mu\Psi$



$= -ie\gamma_\mu$




(no 3rd diagrams!)

$$-i\mathcal{M} = \bar{u}(p_2)\left[(-ie\gamma_\nu)\frac{i}{\not{p}_1 + \not{k}_1 - m}(-ie\gamma_\mu) + (-ie\gamma_\mu)\frac{i}{\not{p}_1 - \not{k}_2 - m}(-ie\gamma_\nu)\right]u(p_1)\,\epsilon_1^\mu \epsilon_2^{*\nu}$$

– order is crucial – matrices

Non-rel – Both are the same!

## Calculating

- Decay rates
- Cross sections
- identifying symmetries & currents
- Ground state energies
- excitation energies

# Decay rates + cross sections

$$\Gamma(A \to BC) = \frac{1}{2M_A} \int \underbrace{\frac{d^3p_B}{(2\pi)^3} \frac{1}{2\omega_B} \frac{d^3p_C}{(2\pi)^3} \frac{1}{2\omega_C}}_{\text{"Lorentz invariant phase space"}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_A - p_B - p_C)$$

$$\sigma(A + B \to C + D) = \underbrace{\frac{1}{2\omega_A 2\omega_B |\vec{v}_A - \vec{v}_B|}}_{\text{Flux factor}} \int \frac{d^3p_C}{(2\pi)^3} \frac{1}{2\omega_C} \frac{d^3p_D}{(2\pi)^3} \frac{1}{2\omega_D} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D)$$

Fermions $\frac{1}{2\omega}$ or $\frac{1}{2M} \to 1$

Fermi's Golden Rule

# Quick Derivation of FGR – T.D.P.T.

$$V(t) = V_0 e^{i\omega t}$$

$$T_{fi} = \int_{t_0}^{t} dt' \langle f | V_0 e^{i\omega t} | i \rangle e^{i\omega_{fi} t} \qquad \omega_{fi} = \frac{E_f - E_i}{\hbar}$$

$$\xrightarrow[t\to\infty]{} \; = \langle f | V_0 | i \rangle \, 2\pi \, \delta(E_f - E_i - \hbar\omega)$$

$$\text{Prob} = |\text{Amp}_{fi}|^2 = |\langle V \rangle|^2 \left[ 2\pi\, \delta(E_f - E_i - \hbar\omega) \right]^2$$

To understand $\delta^2(E)$

$$2\pi\,\delta(\Delta E \to 0) = \int dt\, e^{-i\Delta E t} = \int dt = T \qquad \Delta E \to 0$$

$$\text{Rate} = \frac{\text{Prob}}{T} = 2\pi\,\delta(E_f - E_i)\, |V|^2$$

F. G. R   Two forms

$$\text{Rate}(i \to f) = 2\pi\, \delta(E_i - E_f)\, |\langle f | V | i \rangle|^2$$

$$= 2\pi\, |\langle V \rangle|^2 \rho(E_f) \leftarrow \text{density of final states}$$

Relation between these

$$\rho(E_f) \equiv \sum_f \delta(E_i - E_f) = (V) \int \frac{d^3 p_f}{(2\pi\hbar)^3}\, \delta(E_i - E_f)$$

$$= \int \frac{d\Omega\, p_f^2\, dp}{(2\pi\hbar)^3}\, \delta(E_i - E_f)$$

$$E^2 = p^2 + m^2$$
$$2E\,dE = 2p\,dp$$

$$= \int \frac{d\Omega\, p_f E_f\, dE_f}{(2\pi\hbar)^3}\, \delta(E_i - E_f) = \frac{\overset{4\pi}{d\Omega}\, p_f E_f}{(2\pi\hbar)^3} \rightarrow \frac{p^2}{2\pi^2\hbar^3}, \quad \rightarrow \frac{p\, mc^2}{2\pi^2\hbar^3}$$

$A^* \to A\gamma$

$\gamma A \to A' + e$

Really

$$\text{Rate} = \int \frac{d^3p_f}{(2\pi)^3} |V|^2 2\pi\, \delta(E_i - E_f) \qquad \Rightarrow \text{FGR}$$

Generalize

– Apply to $A_1^* \to A_2 + \gamma$

$$V_I = e\frac{\vec{p}\cdot\vec{A}}{m} \qquad \text{from } \frac{(\vec{p}-e\vec{A})^2}{2m}$$

$$\Gamma(A^* \to A + \gamma) = \int \frac{d^3p_\gamma}{(2\pi)^3} \left|\langle A\gamma | e\frac{\vec{p}\cdot\vec{A}}{m} | A^*\rangle\right|^2 (2\pi)\, \delta(E_{A^*} - E_{A'} - E_\gamma)$$

Asymmetry: why $d^3p_\gamma$ and not $d^3p_A$

A fixed by momentum conservation

Restore symmetry

$$\Gamma = \int \underbrace{\frac{d^3p_\gamma}{(2\pi)^3}\,\frac{d^3p_A}{(2\pi)^3}}\,\underbrace{(2\pi)^4\,\delta^4(p_{A^*}-p_A-p_\gamma)}\,|\langle V\rangle|^2$$

For us $\langle V\rangle = \dfrac{1}{\sqrt{2\omega_A \cdot 2\omega_B\, 2\omega_\gamma}}\,\mathcal{M}$

$\sqrt{\dfrac{1}{2\omega_{A^*}}} = \dfrac{1}{2M_{A^*}}$ our choice

$$\Gamma = \frac{1}{2M_{A^*}} \int \frac{d^3p_\gamma}{(2\pi)^3}\,\frac{1}{2\omega_\gamma}\,\frac{d^3p_A}{(2\pi)^3}\,\frac{1}{2\omega_A}\,(2\pi)^4\,\delta^4(p_A-p_B-p_\gamma)\,|\mathcal{M}|^2 \quad ✓$$

# Cross section

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \left| -\frac{m}{2\pi}\int d^3r\, e^{i\overbrace{(\vec{p}-\vec{p}')}^{\vec{q}}\cdot\vec{r}}\, V(r)\right|^2$$

1) "Waves on pond"

2) FGR

$$\psi_i = \frac{e^{i\vec{p}_i\cdot\vec{r}}}{\sqrt{V}}, \qquad \psi_f = \frac{e^{i\vec{p}_f\cdot\vec{r}}}{\sqrt{V}}$$

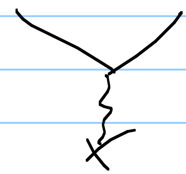
Incoming flux $\frac{|\vec{p}|}{m}\,\frac{1}{V}$

$$d\sigma = \frac{\text{Rate}}{\text{Flux in}} = \frac{1}{p/m}\int \underbrace{\frac{d^3p_f}{(2\pi)^3}}\, |\langle f|V|i\rangle|^2 (2\pi)\,\delta(E_f - E_i) \quad *$$

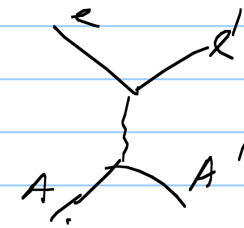
$$= \frac{1}{p_i/m}\cdot\frac{p\,m\,d\Omega}{(2\pi)^2}\left|\int d^3x\, \psi_f^*\, V(r)\, \psi_i\right|^2$$

$$= \left| \frac{m}{2\pi} \int d^3x \, e^{i(p_i - p_f)\cdot x} \, V(r) \right|^2 \qquad \checkmark$$

**Generalize**



is really




$$d\sigma = \frac{1}{|\vec{v}_A - \vec{v}_B|} \int \frac{d^3p_c}{(2\pi)^3} \frac{d^3p_D}{(2\pi)^3} (2\pi)^4 \, \delta^4(p_i - p_f) \left| \langle V \rangle \right|^2$$

$$\langle V \rangle = \frac{1}{\sqrt{2\omega_i \cdots}} \mathcal{M}$$

$$d\sigma = \frac{1}{2\omega_A 2\omega_B} \frac{1}{|\vec{v}_A - \vec{v}_B|} \int \frac{d^3p_D}{(2\pi)^3} \frac{1}{2\omega_D} \frac{d^3p_C}{(2\pi)^3} \frac{1}{2\omega_C} (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \left| \mathcal{M} \right|^2$$