Calculating 2

Note Title 3/4/2010

Identical particles

$$\int \frac{d^3p_3}{(2\pi)^3}\,\frac{d^3p_4}{(2\pi)^3}\,\delta^4(p_1+p_2-p_3-p_4)$$ final states

– overcounts final states if identical

3: $\vec{p}_3 = 10\hat{z}$, 4: $\vec{p} = -10\hat{z}$ 3: $\vec{p} = -10\hat{z}$, 4: $p = +10\hat{z}$ same state

$\Rightarrow$ rules $\frac{1}{S}\int d^3p_3\, d^3p_4 \,- - -$

$\leftarrow S = 2$ (n identical particles $n!$)

Final generalization

$$A \to P_1 + P_2 + P_3 + \dots$$

$$A + B \to C + D + E + F + \dots$$

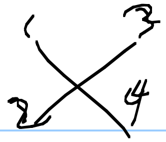
$$\Gamma = \frac{1}{2M_A}\,\frac{1}{S}\int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2\omega_1}\,\frac{d^3p_2}{(2\pi)^3}\frac{1}{2\omega_2}\,\frac{d^3p_3}{(2\pi)^3}\frac{1}{2\omega_3}\,\frac{d^3p_4}{(2\pi)^3}\frac{1}{2\omega_4}\cdots(2\pi)^4\delta^4(p_A - p_1 - p_2 - p_3 - p_4)\,|M|^2$$

$$\sigma = \frac{1}{2\omega_A\, 2\omega_B\, |\vec{v}_A - v_B|}\,\frac{1}{S}\int \text{ - ------ '' - }$$

$$\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega} = LIPS \text{ (Lorentz inv. phase space)}$$

$$= \int \frac{d^4p}{(2\pi)^4}\, 2\pi\,\delta(p_0^2 - \vec{p}^{\,2} - m^2)\,\Theta(p^0)$$ ← also Lorentz inv ✓

with $\int dE\,\delta(E^2 - p^2 - m^2) = \frac{1}{2E}$

Example  scattering $-i\mathcal{M} = -6i\lambda$

Work in center of mass (transform afterwards)

$$E_1 + E_2 = E_{cm} = E_3 + E_4$$

$$2\omega_1 = 2\omega_2 = 2\omega_3 = 2\omega_4 = E_{cm} = \sqrt{s}$$

Variables

$s = (p_1 + p_2)^2 = E_{cm}^2$

$t = (p_1 - p_3)^2$

$u = (p_1 - p_4)^2$

$$d\sigma = \frac{1}{2\omega_1}\frac{1}{2\omega_2}\frac{1}{|\vec{v}_1 - \vec{v}_2|}\underbrace{\frac{1}{2}}_{\text{symmetry}}\int \frac{d^3p_3}{(2\pi)^3}\frac{1}{2\omega_3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2\omega_4}(2\pi)^4\delta^4(\underbrace{p_1+p_2}_{E_1+E_2=\sqrt{s}}-p_3-p_4)\,|6i\lambda|^2$$

$$= \frac{1}{s}\,\frac{1}{2|\vec{v}|}\,\frac{1}{2}\int\frac{d^3p_3}{(2\pi)^3}\,\frac{1}{s}\,(2\pi)\,\delta(\sqrt{s} - 2\omega_3)\,|6\lambda|^2$$

$|\vec{v}| = p/E$, $\quad \omega_3 = E$

$p^2\,dp\,d\Omega = pE\,dE\,d\Omega$

$E^2 = p^2 + m^2$

$2E\,dE = 2p\,dp$

$$= \frac{1}{4s^2}\,\frac{E}{p}\,\frac{1}{(2\pi)^2}\,\frac{pE\,d\Omega}{2}\,|6\lambda|^2$$

$$= \frac{1}{128\pi^2 s}(6\lambda)^2$$

$$\int dE\,\delta(f(E)) = \frac{1}{|f'(E)|}$$

Comments

1) Good in any frame $\quad S = (p_1 + p_2)^2$

2) No angle dependence in cm $\quad \sigma$

3) Non rel limit $\quad s = (m_1 + m_2)^2 = 4m^2$

4) Dimensions $\quad \lambda$ dimensionless $\quad \lambda \phi^4 \sim (E)^4$

$$\frac{1}{s} \sim \frac{1}{(\text{Energy})^2} \sim (\text{length})^2$$

$$\sigma \sim (\text{length})^2 \checkmark$$

$\mathcal{L} = E^4$

$S = \int d^4x \, \mathcal{L}$, $\quad d^4x \sim 1/E^4$

$\Delta p \, \Delta x \sim 1$

Correspondence

Old QM

$\swarrow (\overrightarrow{p_1 - p_3}) \sim$ angles

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \left| \frac{-m}{2\pi} \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \, V(r) \right|^2$$

a) If $V(r) = g\,\delta^3(r)$

$$\frac{d\sigma}{d\Omega} = \frac{m^2 g^2}{4\pi^2} \quad \leftarrow \text{isotropic}$$

b) If $\vec{q} = (\overrightarrow{p_1 - p_3}) \to 0$ very low E

$$g = \int d^3r \, V(r) \qquad \text{if finite range} \quad g < \infty$$

$$\frac{d\sigma}{d\Omega} = \frac{m^2 g^2}{4\pi^2}$$

$\lambda \phi^4$ is model for $\delta^3(x)$ interaction (short range) (Think $\psi^*(x)\psi(x)\,\delta^3(x-y)\,\psi^*\psi(y)$ $\sim \phi^4$)

or <u>any</u> finite range interaction at low energy

Short distance with propagators

Propagator with m "large"

$$i D_F(x-y) = \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2 - m^2} e^{iq\cdot(x-y)}$$

$$= \int \frac{d^4q}{(2\pi)^4} -i \left[\frac{1}{m^2} + \frac{q^2}{m^4} + \cdots \right] e^{iq\cdot(x-y)}$$

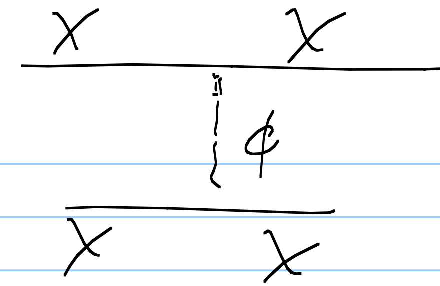
$$q^2 \to -\Box = -\partial_\mu \partial^\mu$$

$$= -i \left(\frac{1}{m^2} - \frac{\Box}{m^4} + \cdots \right) \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot(x-y)}$$

$$= -i \left(\frac{1}{m^2} - \frac{\Box}{m^4} \right) \delta^4(x-y)$$

Exchange

$\mathcal{L} = -g\,\chi^{*}\chi\,\phi$



At second order

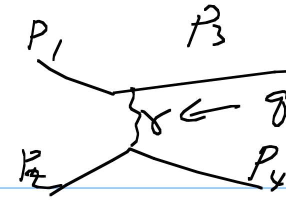
$$\frac{1}{2} T \int d^4x\, d^4y \left(\underbrace{-ig\,\chi^\dagger(x)\chi(x)\phi(x) \quad -ig\,\chi^\dagger(y)\chi(y)\phi(y)}_{} \right)$$

$$iD_F(x-y) = \frac{-i}{m^2}\,\delta^4(x-y)$$

$$i\,\frac{g^2}{2m^2}\int d^4x \Big[\underbrace{\left(\chi^\dagger(x)\chi(x)\right)^2}_{\text{like } \lambda\phi^4} - \underbrace{\chi^\dagger(x)\chi(x)\,\frac{\Box}{m^2}\,\chi^\dagger(x)\chi(x)}_{\text{momentum dependence}} + \dots \Big]$$

$\sim p^2/m^2 \leftarrow$ external state

Electromagnetic scattering



$q = p_1 - p_3 = p_4 - p_2$

$$-i\mathcal{M} = -i g_1 (p_1+p_3)^\mu \qquad \frac{-i g_{\mu\nu}}{q^2+i\epsilon} \qquad -i g_2 (p_2+p_4)^\nu$$

Tricks $m_1 \neq m_2$

C.M. $(p_1+p_2)^\mu = (E_c, \vec{0}) = (\sqrt{s}, \vec{0})$ $\qquad E_1 = \sqrt{p_{cm}^2+m_1^2} \neq E_2 = \sqrt{p_{cm}^2+m_2^2}$

$\Phi)$ $\displaystyle\int \frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2\omega_3}\frac{1}{2\omega_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$

$\displaystyle\frac{1}{(2\pi)^2}\frac{1}{2\omega_3 2\omega_4}\int d^3p\, \delta\left(\sqrt{s} - \sqrt{p^2+m_1^2} - \sqrt{p^2+m_2^2}\right)$ $\qquad$ Use $\displaystyle\int dx\, \delta(f(x)) = \frac{1}{|f'(x)|}$

$\displaystyle p^2 d\Omega \frac{1}{\frac{p}{E_3}+\frac{p}{E_4}} = \frac{1}{(2\pi)^2}\frac{1}{2E_3}\frac{1}{2E_4}\, p\, \frac{E_3E_4}{\underbrace{E_3+E_4}_{E_3+E_4=\sqrt{s}}}$ $\qquad \displaystyle\frac{d}{dp}\sqrt{p^2+m^2} = \frac{p}{\sqrt{p^2+m^2}}$

$$= \frac{1}{16\pi^2} \frac{p}{\sqrt{s}} d\Omega$$

b) Flux $\frac{1}{2E_1 2E_2 |v_1 - v_2|} = \frac{1}{2E_1 2E_2 \left[\frac{p}{E_1} + \frac{p}{E_2}\right]} = \frac{1}{4p\sqrt{s}}$

$$d\sigma = \underbrace{\frac{1}{2E_1 2E_2} \frac{1}{|v_1 - v_2|}}\ \underbrace{\int \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} \frac{1}{2E_3} \frac{1}{2E_4} (2\pi)^4 \delta^4(\quad)}\ |M|^2$$

$$= \frac{1}{4p\sqrt{s}} \frac{1}{16\pi^2} \frac{p}{\sqrt{s}} d\Omega \left[\frac{(p_1+p_3)\cdot(p_2+p_4)}{(p_1-p_3)^2 - m^2}\right]^2 g_1^2 g_2^2$$

$$\frac{d\sigma}{d\Omega} = \frac{g_1^2 g_2^2}{64\pi^2 s} \left[\frac{(p_1+p_3)\cdot(p_2+p_4)}{q^2 - m^2}\right]^2$$

$m = 0$