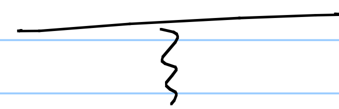
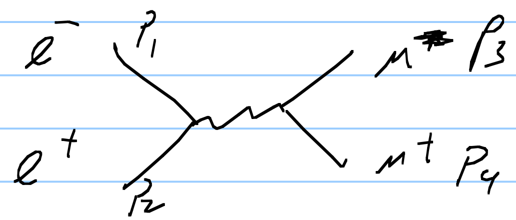
Calculating 3

Note Title 3/9/2010

Feynman rules

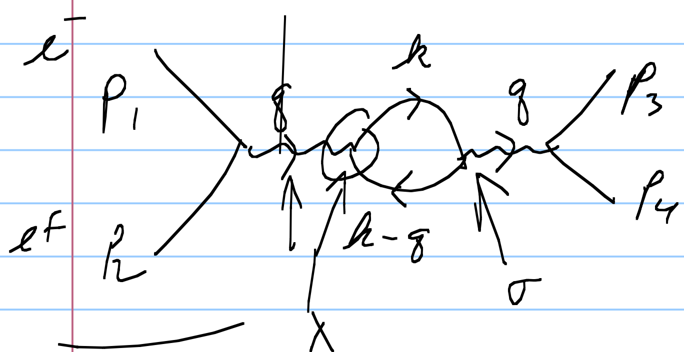
 $= -ie\gamma_\mu$



$$\bar{u}(p_3)\, -ie\gamma^\nu\, v(p_4) \quad \frac{-i g_{\mu\nu}}{q^2+i\epsilon} \quad \bar{v}(p_2)\, \underbrace{-ie\gamma_\mu}\, u(p_1)$$

$q \leftarrow p_1+p_2$

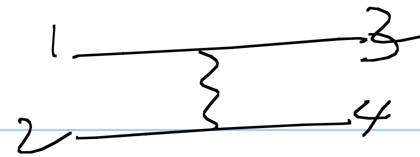
$\bar{v}(p_2)$: $\underbrace{(- - - -)}_{\text{row}}$ $\quad -ie\gamma_\mu$: $\underbrace{\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}}$ $\quad u(p_1)$: $\underbrace{\begin{pmatrix} \vdots \end{pmatrix}}_{\text{column}}$



$$[\bar{u}\, -ie\gamma^\nu\, v] \int \frac{d^4k}{(2\pi)^4} \quad \frac{-i g_{\nu\sigma}}{q^2+i\epsilon} \quad \underbrace{\Pi^{\sigma\chi}(q)}_{\text{Vacuum polarization}} \quad \frac{-i g_{\mu\chi}}{q^2+i\epsilon} \quad [\bar{v}\, -ie\gamma_\mu\, u]$$

$$\Pi_{\sigma\lambda} = \mathrm{Tr}\left[-ie\gamma_\lambda \frac{i}{\not{k} - m + i\epsilon} \; -ie\gamma_\sigma \frac{i}{\not{k} - \not{q} - m} \right]$$

From last time



$$\frac{d\sigma}{d\Omega} = \frac{q_1^2 q_2^2}{64\pi^2}\,\frac{1}{s}\,\frac{\left[(p_1+p_3)\cdot(p_2+p_4)\right]^2}{\left[(p_1-p_3)^2 + i\epsilon\right]^2}$$

Nonrel limit, static $\quad m_2 \gg m_1$

$$s = (p_1+p_2)^2 = (m_1+m_2)^2 = m_2^2$$

$$(p_1+p_3)\cdot(p_2+p_4) = 4 m_1 m_2$$

$$(p_1-p_3)^2 = \underbrace{(E_1-E_3)^2}_{0} - (\vec{p}_1-\vec{p}_3)^2 = -\vec{q}^{\,2} = -2p^2(1-\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{q_1^2}{4\pi}\right)\left(\frac{q_2^2}{4\pi}\right)\frac{4m_1^2}{\vec{q}^{\,4}} \qquad (m_2 \text{ drops out})$$

In QM

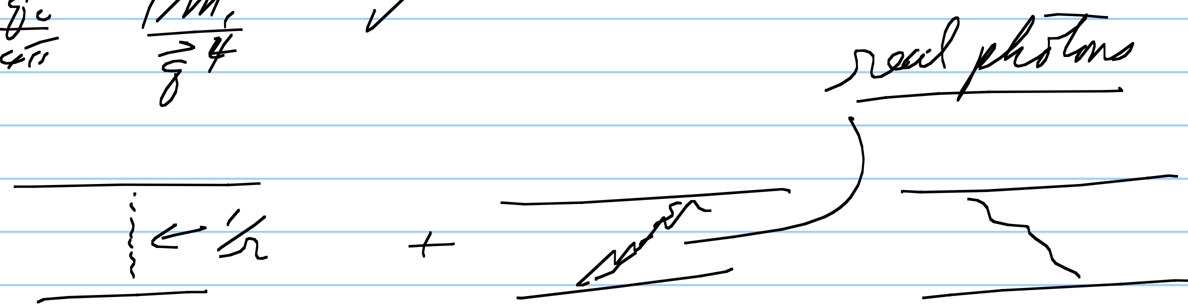
$$\frac{d\sigma}{d\Omega} = \left| \frac{-m}{2\pi} \int d^3x \, e^{i\vec{q}\cdot\vec{x}} \, V(r) \right|^2$$

$$V(r) = \frac{g_1 g_2}{4\pi} \frac{1}{r}$$

$$\int d^3x \, e^{i\vec{q}\cdot\vec{x}} \, V(r) = \frac{g_1 g_2}{4\pi} \, \frac{4\pi}{\vec{q}^{\,2}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{g_1^2}{4\pi}\right) \frac{g_2^2}{4\pi} \, \frac{4M_1^2}{\vec{q}^{\,4}} \quad \checkmark$$

QFT result



$= QM + \underline{\text{more}}$

Symmetries, Ground states, symmetry breaking, masses---

Currents & Charges

Noether's thm Continuous symmetry of $\mathcal{L}$

$\Longrightarrow$ conserved charge $\frac{d}{dt}Q = 0$

$\Longrightarrow$ conserved current $\partial_\mu J^\mu = 0$

These are connected

$$Q = \int d^3x\, J^0 \Rightarrow \frac{dQ}{dt} = \int d^3x\, \frac{\partial}{\partial t} J^0(x,t) = \underbrace{\int d^3x\, -\vec{\nabla}\cdot\vec{J}}_{\text{Total deriv}} = 0$$

Surface at ∞

$$= \oint d\vec{A}\cdot\vec{J}$$

Internal symmetries

- leave x, t alone
- fields ϕ_i $\quad i = 1 - - -$
- transformation $\quad \phi_i \Rightarrow \phi_i' = f_{ij}(\alpha)\, \phi_j$ $\quad$ ← parameters
- invariance $\quad \mathcal{L}(\phi_i', \partial_\mu \phi') = \mathcal{L}(\phi_i, \partial_\mu \phi_i)$

Plan:

- Let $\alpha_k \rightarrow \delta\alpha_k(x)$ $\quad$ infinitessimal + function of $\vec{x}, t$
- $\delta S = 0$ for any $\delta\alpha_k(x)$ (using eq of motions)
- calculate $\delta S = \int d^4x \left[\underbrace{\partial_\mu J^{k\mu}}_{=0} \right] \delta\alpha_k(x)$

Let $\quad \alpha_k \Rightarrow \delta\alpha_k(x)$

$$\phi_i' = f_{ij}(\delta\alpha)\,\phi_j$$

$$\partial_\mu \phi_i' = \partial_\mu\left[f_{ij}(\delta\alpha(x))\,\phi_j(x)\right] = f_{ij}(\delta\alpha)\,\partial_\mu\phi_j + \left[\partial_\mu\,\delta\alpha_k(x)\right] f^{k}_{ij}\,\phi_i$$

$$\delta S = \int d^4x \left[\mathcal{L}'(\phi_i', \partial_\mu\phi_i') - \mathcal{L}(\phi_i, \partial_\mu\phi_i)\right]$$

$$= \int d^4x \left[\frac{\partial \mathcal{L}'}{\partial\,\partial_\mu\,\delta\alpha_k}\,\partial_\mu\,\delta\alpha_k + \frac{\partial \mathcal{L}'}{\partial\alpha_k}\,\delta\alpha_k\right]$$

$$= \int d^4x \left[-\frac{\partial \mathcal{L}'}{\partial\,\partial_\mu\delta\alpha_k} + \frac{\partial \mathcal{L}}{\partial\alpha_k}\right]\delta\alpha_k$$

$$\partial_\mu \frac{\delta \mathcal{L}'}{\partial(\partial_\mu(\delta\alpha^k))} = \frac{\partial \mathcal{L}'}{\delta \delta_i \alpha^k}$$

$$\Rightarrow \text{ if invariance } \frac{\delta \mathcal{L}'}{\delta \delta \alpha_k} = 0$$

$$J^k_\mu = \frac{\partial \mathcal{L}'}{\partial(\partial_\mu \delta \alpha_k)} \equiv \text{Noether's current}$$

$$\partial^\mu J^k_\mu = 0$$

Phase invariance $\quad \phi \to \phi' = e^{-i\alpha}\phi$

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - m^2(\phi^*\phi) \qquad \mathcal{L}(\phi') = \mathcal{L}(\phi)$$

$$\alpha \to \delta\alpha(x)$$

$$\phi' = (1 - i\,\delta\alpha(x))\,\phi(x)$$

$$\mathcal{L}'(\phi', \partial_\mu \phi') = \left[\partial_\mu (1 + i\,\delta\alpha(x))\,\phi^*(x)\right]\left[\partial^\mu (1 - i\,\delta\alpha(x))\,\phi(x)\right] - m^2\phi^*\phi$$

$$= \mathcal{L}(\phi, \partial_\mu \phi) + i\underline{\left[\partial_\mu \delta\alpha(x)\right]}\left[\phi^*\partial_\mu \phi - (\partial_\mu \phi^*)\phi\right]$$

$$J_\mu = \frac{\delta \mathcal{L}'}{\delta\, \partial_\mu \delta\alpha} = i\,\phi^* \overleftrightarrow{\partial_\mu}\,\phi \qquad \text{as before} \checkmark$$

conserved current = "number" current = "probability" current

Dirac field

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi \to \bar{\psi} e^{i\alpha}(i\not{\partial} - m) e^{-i\alpha}\psi$$

$$\alpha \to \delta\alpha(x)$$

$$J_\mu = \frac{\delta \mathcal{L}'}{\delta \partial_\mu \delta\alpha(x)} = \bar{\psi}\gamma^\mu\psi \quad \checkmark$$

Energy & momentum

Symmetry $\phi(x) \to \phi(x+a)$ $\qquad x^\mu \to x^\mu + a^\mu$

$$S' = \int d^4x \, \mathcal{L}(\phi(x+a), \partial_\mu \phi(x+a)) = S$$

if no explicit x dependence

Path $a^\mu = \delta a^\mu(x)$

$$\delta S = 0 \text{ for any } \delta a^\mu(x)$$

$$= \int d^4x \, [\underbrace{\partial_\nu T^{\nu\mu}}_{0}] \, \delta a_\mu(x)$$

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - g_{\mu\nu} \mathcal{L}$$ ("Energy momentum tensor")

$$\underline{\underline{\partial^\mu T_{\mu\nu} = 0}}$$ $\Leftarrow$ 4 conserved currents, 4 symmetries a^ν $(\nu = 0,1,2,3)$

$$T_{00} = \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)}}_{\pi} \partial_0 \phi - \mathcal{L} = \mathcal{H}$$ energy density

$$T_{0i} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} \partial_i \phi = \partial_0 \phi \, \partial_i \phi$$ $\Leftarrow$ momentum density ("Basic 3")

$$\left.\begin{aligned} E &= \int d^3x \, T_{00} \\ \vec{p} &= \int d^3x \, T_{0i} \end{aligned}\right\}$$ conserved

Symmetries + Interactions

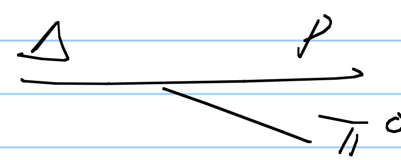
Example p, Δ^+

$$\mathcal{L}_0 = \psi_p^* [i\not\partial - m]\, \psi_p + \psi_\Delta^* [i\not\partial - m]\, \psi_\Delta$$

Has 2 invariances $\psi_p' = e^{-i\alpha_p}\psi_p$, $\psi_\Delta \to e^{-i\alpha_\Delta}\psi_\Delta$

$$J_p^\mu = \bar\psi_p \gamma^\mu \psi_p \qquad J_\Delta^\mu = \bar\psi_\Delta \gamma^\mu \psi_\Delta$$

Transition $\mathcal{L} = g\, \bar\psi_p\, \psi_\Delta\, \pi^0$



not invariant $\partial^\mu J_{p\mu} = g\, \psi_p \psi_\Delta \pi^0 \neq 0$

overall invariance $\left.\begin{array}{l} \psi_p \to e^{-i\alpha}\psi_p \\ \psi_\Delta \to e^{-i\alpha}\psi_\Delta \end{array}\right\}$ same α

$J^\mu_{\text{Baryon}} = J^\mu_p + J^\mu_\Delta$ <u>is</u> conserved "Baryon #"

To break Baryon #

$$\mathcal{L} = g' \, \bar{\psi}_{e^+} \psi_p \, \pi^0$$

not seen in nature (predicted in Grand Unified Theories)

Still

$$\left.\begin{aligned} \psi_p &\to e^{-i\alpha} \psi_p \\ \psi_\Delta &\to e^{-i\alpha} \psi_\Delta \\ \psi_{e^+} &\to e^{-i\alpha} \psi_{e^+} \end{aligned}\right\} \text{"}B-L\text{" conserved}$$