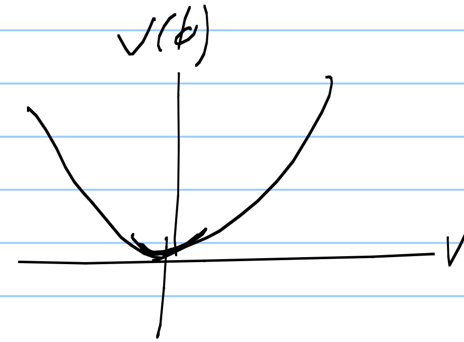
Calculating 4

Note Title 3/11/2010

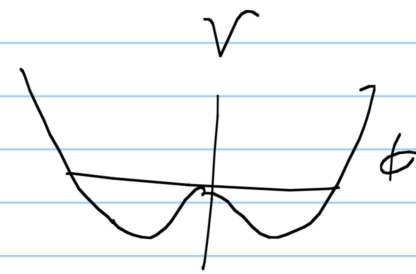
Masses

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\, \partial^\mu \phi\right) - V(\phi)$$



1) $V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

mass

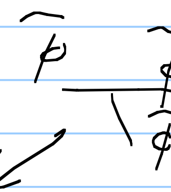


2) $V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

G.S. $\langle \phi \rangle = v = \sqrt{\frac{\mu^2}{\lambda}}$

$\phi = v + \tilde{\phi}$

$m = \sqrt{2}\,\mu$



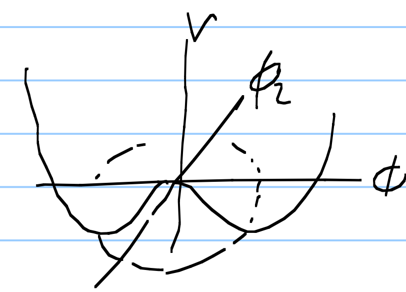
$$V(\tilde{\phi}) = -\frac{\mu^2}{2}(v+\tilde{\phi})^2 + \frac{\lambda}{4}(v+\tilde{\phi})^4 = \frac{-\mu^4}{4\lambda} + \mu^2\tilde{\phi}^2 + \lambda v\tilde{\phi}^3 + \frac{\lambda}{4}\tilde{\phi}^4$$

Symmetry Breaking + Goldstone Theorem

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$V = -\mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2$$

} Symmetry $\phi \to e^{-i\alpha}\phi$



Min when $|\phi| = \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\mu^2}{\lambda}}$

$$\phi = \frac{1}{\sqrt{2}}(v + \tilde{\phi}_1 + i\phi_2)$$

$$V(\phi) = -\frac{\mu^2}{2}\left[(v+\tilde{\phi}_1)^2 + \phi_2^2\right] + \frac{\lambda}{4}\left[(v+\tilde{\phi})^2 + \phi_2^2\right]^2$$

$$= -\frac{\mu^2}{4\lambda} + \mu^2\tilde{\phi}_1^2 + \underbrace{\left[-\frac{\mu^2}{2} + 2\frac{\lambda}{4}v^2\right]}_{0}\phi_2^2 + \lambda v\tilde{\phi}_1^3 + \lambda v\tilde{\phi}_1\phi_2^2 + \frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2$$

$m_1^2 = 2\mu^2$

$m_2^2 = 0$ ← massless

Goldstone's theorem

1) Symmetry is "hidden" or "broken" $\phi \to e^{-i\alpha}\phi$

2) G.S. has many continuous possibilities

— any given G.S. breaks symmetry

3) G.T. ⟹ massless particle

Names

$$\phi = \frac{1}{\sqrt{2}}(v + \tilde{\phi})\, e^{i\chi(x)/v}$$

$$= \frac{1}{\sqrt{2}}(v + \tilde{\phi} + i\chi + \cdots)$$

$$V(\phi^*\phi) = -\frac{\mu^2}{2}(v+\tilde{\phi})^2 + \frac{\lambda}{4}(v+\tilde{\phi})^4 \quad \leftarrow \text{no } \chi$$

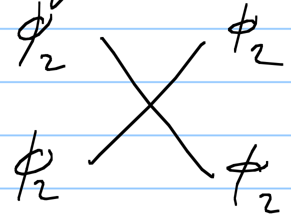
$$(\partial_\mu \phi^*)(\partial^\mu \phi) = \frac{1}{2}(\partial_\mu \tilde{\phi})^2 + \frac{1}{2}\left(\frac{v+\tilde{\phi}}{v}\right)^2 (\partial_\mu \chi)^2$$

Two different results

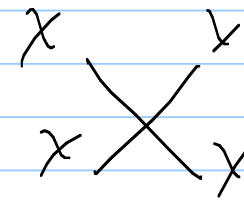
$$\mathcal{L}_1 = \frac{1}{2}(\partial_\mu \tilde{\phi}_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - m_1^2 \tilde{\phi}_1^2 - \lambda v \tilde{\phi}_1(\tilde{\phi}_1^2 + \phi_2^2) - \frac{\lambda}{4}(\tilde{\phi}_1^2 + \phi_2^2)^2$$

$$\mathcal{L}_2 = \frac{1}{2}(\partial_\mu \tilde{\phi})^2 + \frac{1}{2}\left(1 + \frac{\tilde{\phi}}{v}\right)^2 (\partial_\mu \chi)^2 - m_1^2 \tilde{\phi}^2 - \lambda v \tilde{\phi}^3 - \frac{\lambda}{4}\tilde{\phi}^4$$

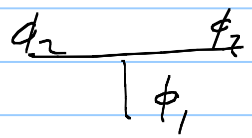
Feynman rules



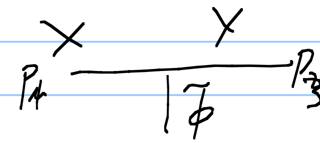
$= -6i\lambda$



$= 0$



$= -2i\lambda v$

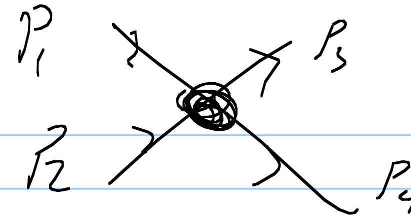


$= -\frac{2i}{v} p_1 \cdot p_3$

Names don't matter "Haag's Thm"

— all physical results are same!

Prelude - kinematics



Define $s = (p_1 + p_2)^2 = (E_{cm})^2$

$$t = (p_1 - p_3)^2 = q^2$$

$$u = (p_1 - p_4)^2$$

For massless particles

$$s = p_1^2 + p_2^2 + 2p_1\cdot p_2 = 0 + 0 + 2p_1\cdot p_2$$

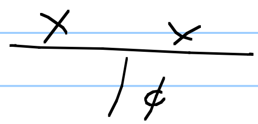
$$t = -2p_1\cdot p_3$$

$$u = -2p_1\cdot p_4$$

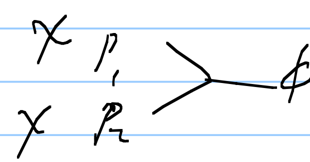
In general

$s+t+u = 4m^2$

$$s + t + u = 2p_1\cdot(p_2 - p_3 - p_4) = -2p_1\cdot p_1 = 0$$



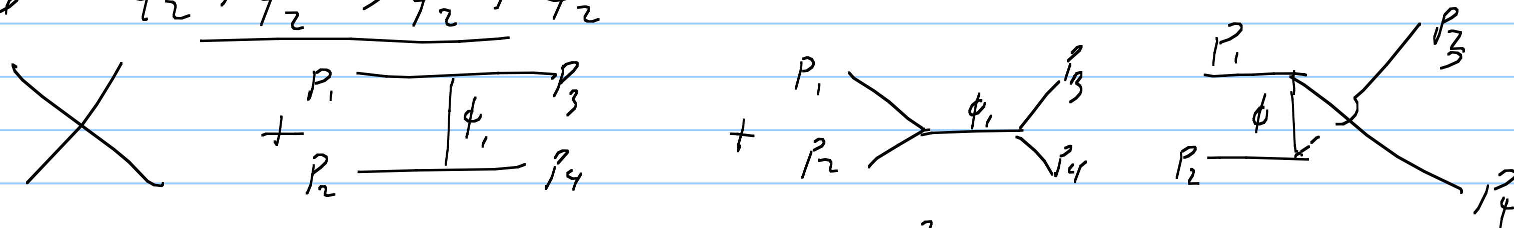
$= \frac{it}{N}$



$= -\frac{is}{N}$

Calculate $\underline{\phi_2 + \phi_2 \to \phi_2 + \phi_2}$

$-i\mathcal{M} =$



$$= -6i\lambda + (-2i\lambda v)\frac{i}{t-m^2}(-2i\lambda v) + (-2i\lambda v)^2\frac{i}{s-m^2} + (-2i\lambda v)^2\frac{i}{u-m^2}$$

$$= -2i\lambda\left[\frac{s-m^2}{s-m^2} + \frac{t-m^2}{t-m^2} + \frac{u-m^2}{u-m^2} + \underbrace{2\lambda v^2}_{\substack{2\mu^2\\ = m^2}}\left(\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right)\right]$$

m^2 cancel

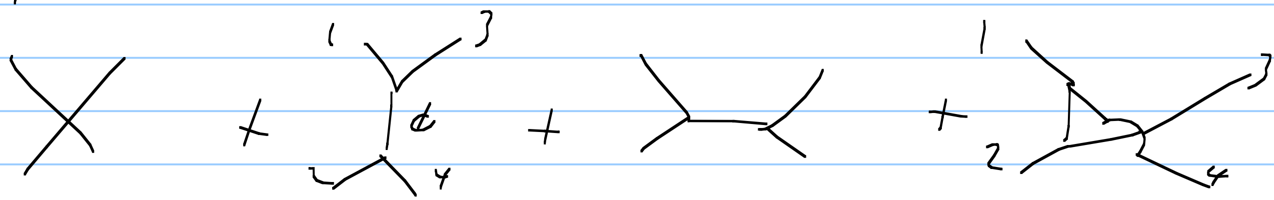
$$= -2i\lambda\left[\frac{s}{s-m^2} + \frac{t}{t-m^2} + \frac{u}{u-m^2} + \frac{\overbrace{s+t+u}^{0}}{m^2}\right]$$

$$= \underbrace{\frac{-2i\lambda}{m^2}}_{\frac{-i}{v^2}}\left[\frac{s^2}{s-m^2} + \frac{t^2}{t-m^2} + \frac{u^2}{u-m^2}\right]$$

(using $m^2 = 2\mu^2 = 2\lambda v^2$)

Consider $\chi + \chi \to \chi + \chi$

$-i\mathcal{M} =$



$$= 0 + \frac{+it}{N}\left(\frac{i}{t-m^2}\right)\left(\frac{it}{N}\right) + \left(\frac{is}{N}\right)^2 \frac{i}{s-m^2} + \left(\frac{iu}{N}\right)^2 \frac{i}{s-m^2}$$

$$= -\frac{i}{N^2}\left[\frac{s^2}{s-m^2} + \frac{t^2}{t-m^2} + \frac{u^2}{u-m^2}\right] \checkmark$$

Same result!

<u>Haag's thm</u>

- $\chi_i = f_{ij}(\phi)\,\phi_j$ if $\underline{f_{ij}(0) = \delta_{ij}}$ (kinetic energies unchanged)

Higgs mechanism

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi)^* - V(\phi^* \phi)$$

$\swarrow$ same

Symmetry $\quad \phi \to \phi' = e^{-iq\Lambda(x)} \phi$

$A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda(x) \quad \leftarrow$ gauge

Do S.S.B. $\quad \langle \phi \rangle = \frac{v}{\sqrt{2}}$

$\swarrow m_\gamma = qv$

as in HW $\quad (D_\mu \phi)^* (D^\mu \phi) = \cdots \quad \frac{q^2 v^2}{2} A_\mu A^\mu$

Puzzle:

massless photon $\Rightarrow$ 2 D.O.F.

$2\ \phi_1\ \phi_2$ $\Rightarrow$ 2 D.O.F

$\overline{\quad 4 \quad}$

But

massive photon 3 D.O.F

$\phi_1\ \phi$ 2 D.O.F

$\overline{\quad 5 \quad}$??

Names $\phi = \frac{1}{\sqrt{2}}(v + \tilde{\phi})\, e^{i\chi/v}$

$$D_\mu \phi = \frac{e^{i\chi/v}}{\sqrt{2}}\left[\partial_\mu \tilde{\phi} + i(v+\tilde{\phi})\frac{\partial_\mu \chi}{v} + i q A_\mu (v+\tilde{\phi})\right]$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu \tilde{\phi}\right)^2 + \frac{q^2}{2}(v+\tilde{\phi})^2\left[A_\mu + \frac{\partial_\mu \chi}{qv}\right]^2$$

$$+ \frac{\mu^2}{2}(v+\tilde{\phi})^2 - \frac{\lambda}{v}(v+\tilde{\phi})^2$$

$$A'_\mu = A_\mu + \frac{\partial_\mu \chi(x)}{q v} \qquad \Rightarrow F^2 \text{ unchanged}$$

χ Has Disappeared!

$\Rightarrow$ massive photon + $\underline{1}$ scalar = 4 D.O.F.

Longitudinal Gauge Boson = Goldstone boson

Φ residual Higgs Boson

SM SU(2) symmetry broken $\Rightarrow$ 3 G.B.s

$\Rightarrow W^+, W^-, Z^0$