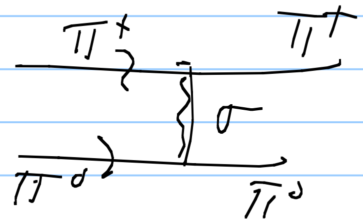
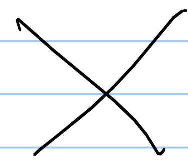
EFT 2

Note Title — 10/20/2009

Linear σ model





$$\mathcal{M} = i\frac{g^2}{v^2}\left[1 + \frac{g^2}{2\lambda v^2} + \cdots\right]$$

$$\mathcal{L}_{eff} = \frac{-1}{v^2}\,\partial_\mu \pi^0 \partial^\mu \pi^0 \;\pi^+\pi^-$$

$$\mathcal{L}_4 = \partial_\mu \pi^0 \partial^\mu \pi^0 \;\partial_\sigma \pi^+ \partial^\sigma \pi^- \,\frac{2}{v^2 m_\sigma^2}$$

$$\to \frac{1}{\lambda v^4} \cdots$$

Exponential param.

$$\Sigma = \sigma + i\vec{\tau}\cdot\vec{\pi} = (v + \tilde{\sigma} + i\vec{\tau}\cdot\vec{\pi}) \equiv (v+S)\,U$$

$$U = e^{i\vec{\tau}\cdot\vec{\pi}'/v} \qquad \left(1 + i\frac{\vec{\tau}\cdot\vec{\pi}'}{v} + \dots\right)$$

Haag's thm – same physics

$$S = \tilde{\sigma} + \cdots$$

$$\pi'^{i} = \pi^{i} + \cdots$$

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) - V(\Sigma^\dagger\Sigma) + \bar{\psi} i \partial\!\!\!/ \psi + g(\bar{\psi}_L \Sigma \psi_R + h.c.)$$

$$\Sigma^\dagger\Sigma = (v+S)^2 U^\dagger U = (v+S)^2$$

$$\partial_\mu(U^\dagger U) = 0$$

$$\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) = \partial_\mu S \partial^\mu S\, \underbrace{\mathrm{Tr}(U^\dagger U)}_{2} + \partial_\mu S\, \underbrace{\mathrm{Tr}(U^\dagger \partial^\mu U + (\partial^\mu U^\dagger) U)}_{0} (v+S)$$

$$+(v+s)^2\, \mathrm{Tr}(\partial_\mu U\, \partial^\mu U^\dagger)$$

$$V(\sigma,\pi) = \frac{\mu^2}{2}(v+s)^2 - \frac{\lambda}{4}(v+s)^4$$

Then

$$\mathcal{L} = \frac{(v+s)^2}{4}\, \mathrm{Tr}(\partial_\mu U^\dagger \partial U^\dagger) + \frac{1}{2}\left[\partial_\mu s\, \partial^\mu s - m_\sigma^2 s^2\right]$$

$$- \lambda v s^3 - \frac{\lambda}{4} s^4 + \bar{\psi}\, i \not\partial \psi - g(v+s)(\bar{\psi}_L U \psi_R + \bar{\psi}_R U^\dagger \psi_L),$$

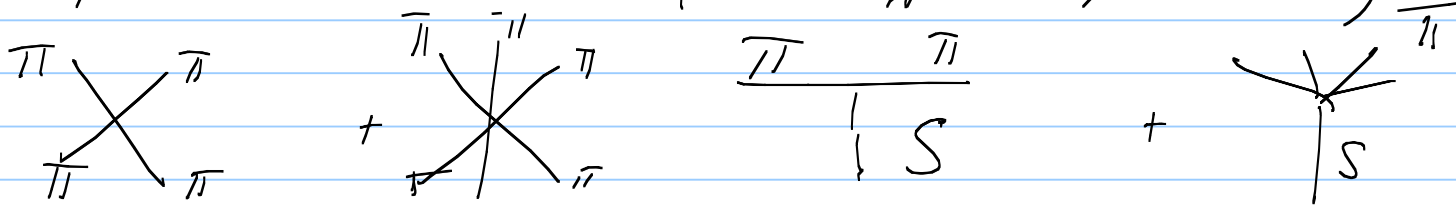
Renormalizability hidden

Symmetry visible $\quad U \to L\, U R^\dagger \quad$ (SU(2)), $\psi_L \to L \psi_L$, $\psi_R \to R \psi_R$

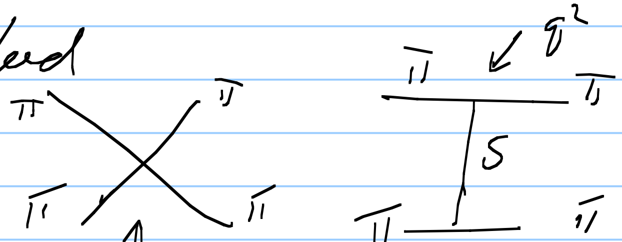
Scattering - drop ψ

$$\frac{(v+S)^2}{4}\,\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) = \frac{(v+S)^2}{4}\,\mathrm{Tr}\Big(\frac{\partial_\mu \pi}{v}\,\partial^\mu \frac{\vec{\tau}\cdot\vec{\pi}}{v} + \partial^2 \pi^4 + \cdots\Big)$$

$\vec{\tau}\cdot\vec{\pi}$



Need



q^2 $\quad$ q^2 $\quad$ q^2

only one at $O(q^2)$ $\qquad$ $O(q^4)$

To order q^2

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^+) \quad \leftarrow \text{eff } \mathcal{L}$$

$$= \frac{v^2}{4} \mathrm{Tr}\left(\left[(\partial_\mu U) U^+\right]\left[\partial_\mu U U^+\right]^+\right) = \frac{v^2}{4} \mathrm{Tr}(L_\mu L^\mu) = \frac{v^2}{2} l_\mu^i l_\mu^i$$

$$L_\mu = (\partial_\mu U) U^+ \to L(\partial_\mu U U^+) L^+$$

$$= \left(i \partial_\mu \frac{\vec{\tau} \cdot \vec{\pi}}{v} - \frac{1}{2} \partial_\mu \phi^2 - \frac{i}{6} \partial_\mu \phi^3\right)\left(1 - i\phi - \frac{1}{2}\phi^2 - + - \right) \quad \frac{\vec{\tau} \cdot \vec{\pi}}{v} = \phi$$

$$= \tau^i l^i$$

$$= \left(i \partial_\mu \phi - \underbrace{\frac{1}{2}(\phi \partial_\mu \phi + \partial_\mu \phi \phi) + 1(\partial_\mu \phi)\phi}_{+\frac{1}{2}\left((\partial_\mu \phi)\phi - \phi \partial_\mu \phi\right)} - \frac{i}{6}\left[\phi, [\phi, \partial_\mu \phi]\right]\right)$$

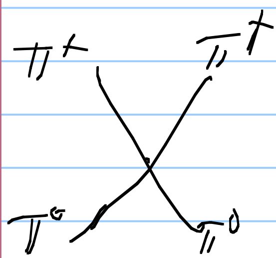
$$\frac{1}{2}\underbrace{[\tau^i, \tau^j]}_{2i\,\varepsilon^{ijk}\tau^k} \partial_\mu \phi^i \phi^j$$

$$l_\mu^i = i\,\partial_\mu \phi^i + \varepsilon^{ijk} \phi^j \partial_\mu \phi^k + \varepsilon^{ijk}\varepsilon^{klm} \phi^j \phi^l \partial_\mu \phi^m + \dots$$

$$\mathcal{L} = \frac{1}{6N^2}\left[(\vec{\pi}\cdot\partial_\mu \vec{\pi})(\vec{\pi}\cdot\partial^\mu \vec{\pi}) - \vec{\pi}\cdot\vec{\pi}\,(\partial_\mu \vec{\pi}\,\partial^\mu \vec{\pi})\right]$$

$$= \frac{1}{6N^2}\left[-\partial_\mu \pi^0 \partial^\mu \pi^0 \left(\pi^+\pi^- + \pi^-\pi^+\right) + \dots\right]$$

2 deriv

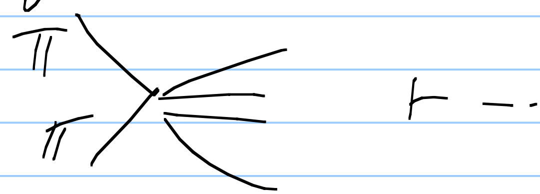


$$= i\,\frac{p^2}{N^2}$$

$$\Longrightarrow \mathcal{L}_{eff} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^\dagger)$$

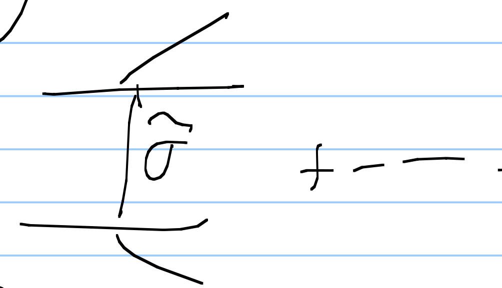
no S, σ needed

This gives all matrix element to $\mathcal{O}(q^2)$



$+ \ldots$

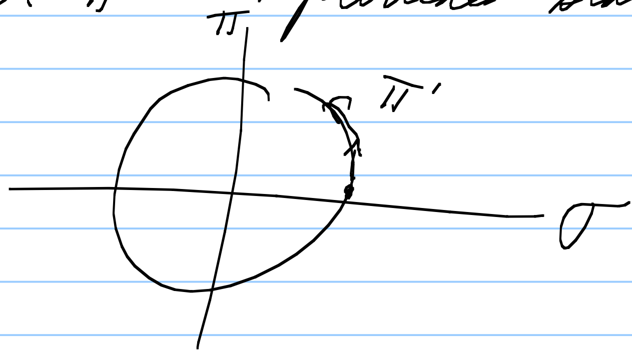
old way



$+ \ldots$

lots of cancelations $\mathcal{O}(q^2)$

All π amplitudes start at $\mathcal{O}(q^2)$



$\leftarrow$ no π^M terms at all

$U = e^{i \vec{\tau} \cdot \vec{\pi} / v}$

angular factor

$U \to L U R^\dagger$

Path integral

$$Z[\vec{J}_\pi] = \int [ds][d\vec{\pi}]\, e^{i\int d^4x\,[\mathcal{L}(s,\pi) + \vec{J}_\pi \cdot \vec{\pi}]}$$

$$= \int d\pi \left[\int ds\, e^{i\langle \qquad \rangle} \right] \leftarrow e^{iW[\pi]}$$

$$= \int d\pi\, \underbrace{e^{i\int d^4x\,[\mathcal{L}_{eff}(\pi) + \vec{J}_\pi \cdot \vec{\pi}]}}_{e^{iW[\pi]}}$$

Subtle point - locality

$$e^{iW[\pi]} = e^{i\int d^4x\, \mathcal{L}_{eff}(\pi)}$$

- "obvious" by uncertainty principle

- objects are singular J-D J
  - Operator Product Expansion

$$A(x)\ B(0) = \sum_i C_i(x)\ \mathcal{O}_i(0)$$

functions local operators

- proven in pert theory (assumed non-pert.)

For σ model

$$e^{iW[\pi]} = \int dS \, e^{i\int d^4x \left[\left(\frac{N^2}{4} + \frac{2NS}{4} + \frac{S^2}{4}\right) Tr(\partial_\mu U \partial^\mu U^\dagger) + KE - \lambda N S^3 - \frac{\lambda}{4} S^4\right]}$$

$$= e^{i\int d^4x \frac{N^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)} \; e^{i\hat{W}[\pi]}$$

Next order, integrate out

$$e^{i\int d^4x [S \mathcal{O} S + SJ]} \Rightarrow e^{i\int d^4x\, d^4y\, J(x) D_F J(y)}$$

$$\mathcal{L}_{eff} = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{v^4}{8 m_S^2} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) \mathrm{Tr}\left(\partial_\nu U \partial^\nu U^\dagger\right)$$

$$\left[m_S^2 = 2\mu^2 = 2\lambda v^2 \Rightarrow \frac{v^2}{8 m_S^2} = \frac{1}{16}\frac{1}{\lambda} \right.$$

$$\mathcal{M} = i\frac{s}{v^2}\left[1 + \frac{q^2}{2\lambda v^2} + \cdots\right]$$ $\Leftarrow$ get full matrix element

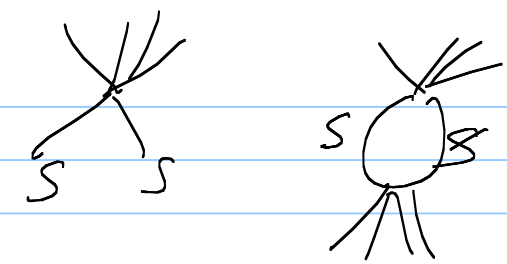
- Locality
- Energy expansion
- Non renorm. $\mathcal{L}_{eff}$
- symmetry not hidden

} EFT

SSB $\Leftarrow$ "breaking" $\to$ hidden

Loops

$S^2 \, \mathrm{Tr}\, \partial_\mu U \, \partial^\mu U^+$



$\Longrightarrow$ do loops

"Matching"

- σ renormalizeable $\Longrightarrow$ all predictions λ, m_σ^2 $\swarrow N^2$
- EFT nonrenorm. $\Longrightarrow$ many parameters

matching parameters of EFT $\Longleftrightarrow$ $\lambda \, m_\sigma^2$ (order by order)