EFT 4

Note Title 10/27/2009

Tree level – "integrating out"

One loop – "matching"

"Power counting"

Organize in energy expansion

$$\mathcal{M} = \frac{q^2}{v^2} + \frac{q^4}{M^2 v^2} + \cdots$$

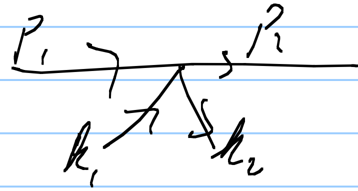
$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U \, \partial^\mu U^\dagger\right) + \alpha_1 \left[\mathrm{Tr}\left(\quad\right)\right]^2$$

– what is needed at a given order

Example

$$\mathcal{L} = \cdots \quad - \alpha_{100}\, \mathrm{Tr}\left(\partial_\mu \partial_\nu \partial_\alpha \cdots U\, \partial^\mu \partial^\nu \partial^\alpha \cdots U^\dagger\right)$$

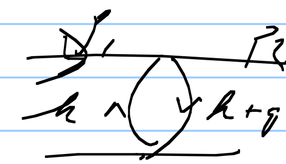
— Then



$$\frac{\alpha_{100}}{F^4} k_{1\mu} k_{1\nu} k_{1\alpha} \cdots k_2^\mu k_2^\nu k_2^\alpha \cdots$$

+ perm

In loop



$$\int d^4k \; \frac{k_\mu k_\nu \cdots (k+q)^\mu (k+q)^\nu \cdots}{k^2 (k+q)^2}$$

$$= \Lambda^n + \Lambda^{n-2} \underline{q^2} + \Lambda^{n-4} q^4 \cdots$$

$$\mathcal{L}_{eff} = \alpha_{100} \Lambda^n + \left(\frac{F^2}{4} + \alpha_{100} \Lambda^{n-2}\right) \mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right) + \left(\alpha_1 + \alpha_{100} \Lambda^{n-4}\right) [\ ]^2$$

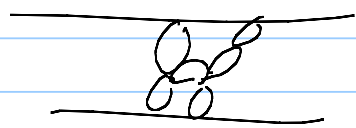
$$+ \cdots$$

Do we need to know α_{100} to predict anything?

No — "power counting"

Weinberg theorem

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots \mathcal{L}_n$$



N_E external legs

N_L = # loops

N_n = # of vertices from $\mathcal{L}_n$

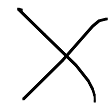
mass or

$$\mathcal{M} = (E)^{4-N_E}$$

$$= \# \, E^D$$

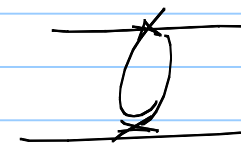
↑ external energies

$N_E = 4$



λ, $\frac{g^2}{\Lambda^2}$

$$\boxed{D = 2 + \sum_n N_n (n-2) + 2N_L}$$



Example $\pi\pi \to \pi\pi$

$$\mathcal{M} = \frac{g^2}{\Lambda^2} + \alpha^2 \frac{g^4}{\Lambda^4}$$

↑ $N_2 = 1$, $N_L = 0$

↑ $N_4 = 1$, $N_L = 0$ or $N_2 = 2$, $N_L = 1$

$E^2 \Rightarrow$. , $N_L = 0$

E^4, $N_L = 1$, $N_2 =$ anything, or $N_4 = 1$, $N_L = 0$

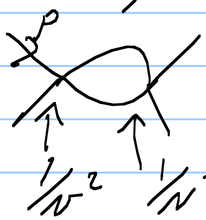
$E^6 = N_L = 2$, $N_L = 1$ & $N_4 = 1$, $N_L = 0$, $N_6 = 1$

$\mathcal{L}_{100} \Rightarrow E^{100}$ and higher

Proof – Example $\pi\pi \to \pi\pi$

$$\mathcal{L}_2 = \frac{g}{N^2}$$

dim reg.

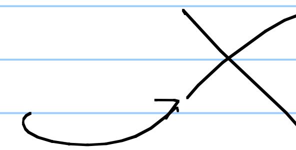
Loops 

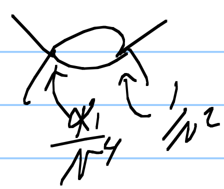
$$= \frac{1}{N^4} I(p_i) \sim \frac{1}{N^4} p_i^4 \left(\frac{1}{d-4} + \text{finite}\right)$$

$$= \frac{1}{N^6} I'(p_i) \sim \frac{1}{N^6} p_i^6$$

$$\mathcal{L}_4 = \alpha \left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 \sim \frac{\alpha_1}{N^4} p_1 \cdot p_2 \, p_3 \cdot p_4$$

dimensionless





$$\sim \frac{1}{N^6} I''(p_i) \sim \frac{1}{N^6} p^6$$

Measuring vs "Matching"

- can't match if
  - uncalculable - low E QCD
  - unknown - gravity

use eff $\mathcal{L}$ + loops, renormalize
  - measure parameters instead of matching

Rules of EFT

1) Use only low energy D.o.F. (π)

2) Write most general $\mathcal{L}_{eff}$

3) Order in energy expansion

4) Renormalize parameters
   - order by order   - power counting

5) Match or measure

6) Predictions
   - example $B(s,t,u) = [\quad s^2 \ln s/\mu^2 + \cdots \quad]$

σ Model + Real physics

1) Hands on example

2) Higgs sector of SM

$$\Sigma = (\underset{\substack{\uparrow \\ 246\,\mathrm{GeV}}}{v} + \underset{\substack{\uparrow \\ \text{Higgs}}}{S})\, e^{i\vec{\tau}\cdot\vec{\pi}/v}$$

$\vec{\pi} \leftarrow W_L$

Mass generation $\quad g\,\bar{\psi}_L \Sigma \psi \longrightarrow \underbrace{g\,\bar{\psi}_L (v+S)}_{m} \psi_R$

3) QCD at low energy

– symmetry is same

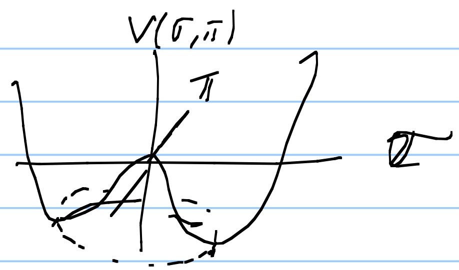
$\mathcal{L}_{eff}$ same form

- matching is different α_1, α_2 are different

Symmetry is not exact

Break symmetry $\mathcal{L} = a\sigma$

$$V(\sigma,\pi) = -\frac{\mu^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2) - a\sigma$$



1) expand about minimum

$$m_\pi^2 = \frac{a}{v}$$ to first order in a

2) eff $\mathcal{L}$

$$\sigma + i\vec{\tau}\cdot\vec{\pi} = \Sigma = (v+S)\,U$$

$$\mathcal{L} = a\sigma = \frac{a}{2}\,\mathrm{Tr}\,\underset{\substack{\uparrow \\ \frac{\Sigma+\Sigma^\dagger}{2}}}{\Sigma} = \frac{a}{2}(v+S)\,\mathrm{Tr}(U) = \frac{a}{4}(v+S)\,\mathrm{Tr}(U+U^\dagger)$$

Integrate out S

$$\mathcal{L}_{eff} = \underline{\frac{av}{4}}\,\mathrm{Tr}(U+U^\dagger) + \beta_1\left[\mathrm{Tr}(U+U^\dagger)\right]^2 + \beta_2\,\mathrm{Tr}(U+U^\dagger)\,\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) + \ldots$$

read off pion mass

$$\mathrm{Tr}(U+U^\dagger) = \mathrm{Tr}\left(\mathbb{1} - \frac{(\tau\cdot\pi)^2}{v^2} + \ldots\right) = 4 - \frac{\pi^i\pi^i}{v^2}\times 2$$

$$\mathcal{L}_{eff} = \frac{a}{2v}\,\pi^i\pi^i$$

$$m_\pi^2 = a/v \quad \checkmark$$

Ken Wilson + EFT

- EFT, Renorm group, Operator product expansion

- not only heavy particles, but also high energy parts of light particle
  - gluons or π remove these beyond some scale Λ

$$Z[J] = \int [d\pi]\, e^{iS_{\text{true}}} = \int [d\pi]_{<\Lambda} \underbrace{[d\pi]_{>\Lambda}\, e^{iS_{\text{true}}}}_{e^{iS_{eff}(\pi,\Lambda)}}$$

$$= \int [d\pi]_{<\Lambda}\, e^{iS_{eff}(\Lambda)}$$

Comment

1) Leads to R.G.

in $\mathcal{L}_{eff}(\Lambda)$ depend on Λ

ex. QED $\quad e_0^2(\Lambda_1)\left[1 + \frac{e^2}{15\pi^2} \ln \Lambda_1^2/\mu^2\right] = e_{ph} = 1/137$

$e_0^2(\Lambda_2)\left[1 \quad " \quad — \quad - \Lambda_2\right] = e_{ph}$

$\Rightarrow$ RGE $\quad e^2(\Lambda^2)$

2) Good in EFT when DOF change

- QCD $\quad \pi$ below 1 GeV $\quad$, q, G above 1 GeV

- $\int (d\pi)_{<1 GeV} \, e^{iS_{eff}}$

$(d\pi)_{>1 GeV}$ meaningless in QCD

3) Not used in practice – mostly

- cutoffs are difficult – "upset power counting"

  can make it work

- use dim reg
  - no cutoff
  - including false info in loops beyond 1 GeV
  - mistake is High E ⟹ local ⟹ change in α_i

4) <u>Operator Product Expansion</u>

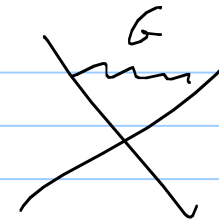
local operators

$$T(A(x)\,B(0)) = \sum_i C_i(x,\Lambda)\,[O_i(0)]_\Lambda$$

scale of Wilson EFT

Logic:

- high E part ($E > \Lambda$) effectively local
- low energy parts not local
- OPE $[O_i(0)]_\Lambda$ will have $E < \Lambda$ in matrix elements
- gluon $E > \Lambda$ get integrated out + local



- also dimensional reg in practice