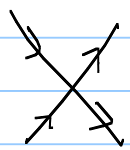
# Interactions 3

Note Title 2/23/2010

## Pert. Theory

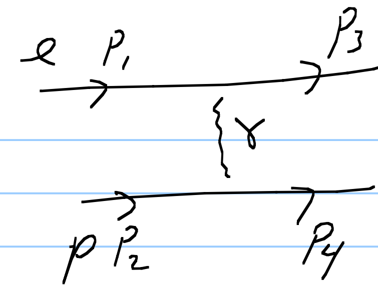
$$T_{fi} = \langle f | U_I(\infty, -\infty) | i \rangle = \frac{(2\pi)^4 \delta^4(p_i - p_f)}{\sqrt{2\omega_1 \cdots}} \; (-i\mathcal{M})$$



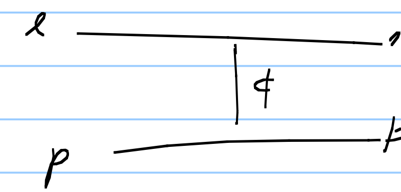
$$\mathcal{L} = -\frac{\lambda}{4}\phi^4 \quad \Rightarrow \quad -i\mathcal{M} = -6i\lambda$$

$$\mathcal{M}_{fi} = \langle f | V | i \rangle + \sum_I \frac{\langle f | V | I \rangle \langle I | V | i \rangle}{E - E_I} + \cdots$$

# Mimic $ep$ scattering




QED $\mathcal{L}_I = -q_e \bar{\psi}_e \gamma_\mu \psi_e A^\mu - q_p \psi_p \gamma_\mu \psi_p A^\mu$




Toy model $\mathcal{L} = -q_e \chi_e^* \chi_e \phi - q_p \chi_p^* \chi_p \phi$

$$-i\mathcal{M} = (-iq_e)\frac{i}{q^2 - m^2 + i\varepsilon}(-iq_p)$$

$(\mathcal{L}_e + \mathcal{L}_p)$

$\downarrow$

$$S = U_I(\infty, -\infty) = T\exp\left\{ i\int d^4x\, \mathcal{L}_I(x) \right\}$$

$$= 1 + i\int d^4x\, \mathcal{L}_I(x) - \frac{1}{2} T\left\{ \int d^4x\, \mathcal{L}_I(x) \int d^4y\, \mathcal{L}_I(y) \right\} + \cdots$$

$$= -\frac{2}{2}\int d^4x\, d^4y\, T\left\{ \mathcal{L}_e(x)\, \mathcal{L}_p(y) \right\}$$

. .

$$T_{fi} = \langle e(p_3)\, p(p_4) \,|\, -\int d^4x\, d^4y\; T\big(g_e\, \chi_e^\dagger \chi_e\, \phi(x)\;\; g_p\, \chi_p^\dagger(y)\, \chi_p(y)\, \phi(y)\big) \;|\, e(p_1)\, p(p_2)\rangle$$

$a^\dagger_e \quad a_e \qquad a^\dagger(p') \quad a_p(p)$

$\chi_e^\dagger\, \chi_e$ commute with $\chi_p^\dagger, \chi_p$

$$T_{fi} = -\int d^4x\, d^4y \,\langle 0|\, \frac{g_e\, e^{-i(p_1-p_3)\cdot x}}{\sqrt{2\omega_1 2\omega_3}}\; T(\phi(x)\,\phi(y))\; \frac{g_p\, e^{-i(p_2-p_4)\cdot y}}{\sqrt{2\omega_2 2\omega_4}} \,|0\rangle$$

Use $i D_F(x-y) = \langle 0|\, T(\phi(x)\,\phi(y))\,|0\rangle = \int \frac{d^4q}{(2\pi)^4}\, i\, \frac{e^{-iq\cdot(x-y)}}{q^2-m^2+i\epsilon}$

$$T_{fi} = -\frac{g_e\, g_p}{\sqrt{2\omega_1 \cdots 2\omega_4}} \int \frac{d^4q}{(2\pi)^4} \int d^4x\, d^4y\;\; e^{-i(p_1-p_3)\cdot x}\; \frac{i\, e^{-iq(x-y)}}{q^2-m^2+i\epsilon}\; e^{-i(p_2-p_4)\cdot y}$$

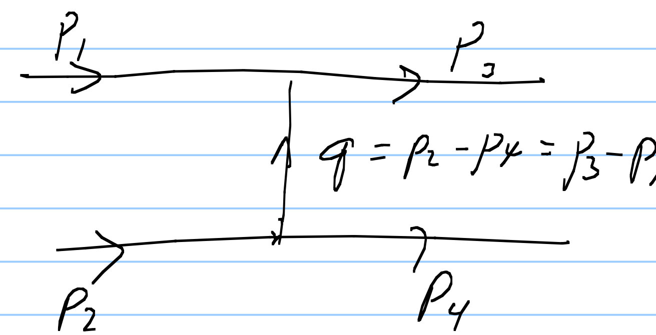
"

$$(2\pi)^4\delta^4(p_1-p_3+q) \qquad (2\pi)^4\delta^4(p_2-p_4-q)$$

$$T_{fi} = -i\,\frac{(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)}{\sqrt{2\omega_1 \cdots 2\omega_4}} * \frac{g_\alpha g_\beta}{q^2-m^2+i\epsilon}$$

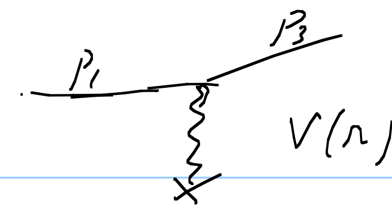
$$q = (p_2-p_4) = (p_3-p_1).$$

$$-i\mathcal{M} = (-i g_\alpha)\,\frac{i}{q^2-m^2+i\epsilon}\,(-i g_\beta)$$




Features

1) 4 Momentum conserved at each vertex
2) $(2\pi)^4\delta^4(p_i-p_f)\ \frac{1}{\sqrt{2\omega}}$
3) internal line $\Rightarrow$ Feynman propagator




# First Born Approx

$M$ is the same

$$f(\theta) = (\quad) \int d^3x \; \underbrace{V(r)}_{\frac{q_e q_p}{4\pi r}} \; e^{i\underbrace{(\vec{p}_1 - \vec{p}_3)}_{\vec{q}}\cdot\vec{x}} = (\#) \frac{q_e q_p}{\vec{q}^{\,2}}$$

$$V(r) = e\,\phi_p(x) = e \int d^3x \; \underbrace{G(x-y)}_{\text{Green function}} \; \rho(y) \qquad \swarrow q_1 \delta^3(y)$$

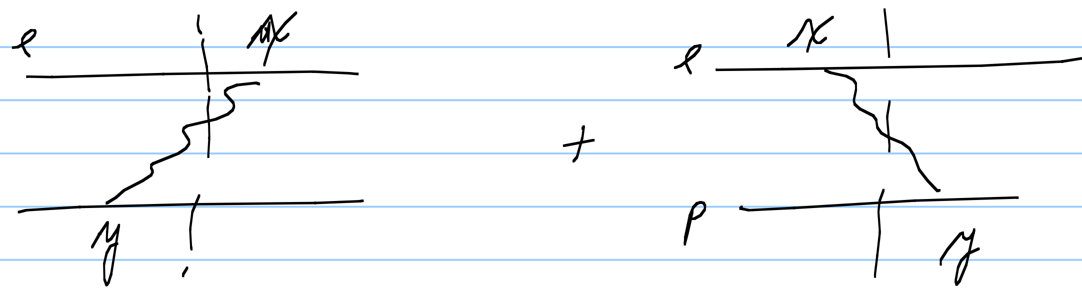
$$\Rightarrow f(\theta) = (\quad) \int d^3x \, d^3y \; \underbrace{e^{i(\vec{p}_1 - \vec{p}_3)\cdot x}}_{\psi_e^* \psi_e} \; q_e \; G(\vec{x}-y) \; \underbrace{\rho_p(y)}_{\psi_p^* \psi_p}$$

## QM Pert theory – 2nd order

$$\mathcal{M}_{fi} = \sum_I \frac{\langle f | V | I \rangle \langle I | V | i \rangle}{E - E_I} \Longleftrightarrow \Longrightarrow \frac{1}{q^2 \pm i\varepsilon}$$

← E not conserved

Time ordering

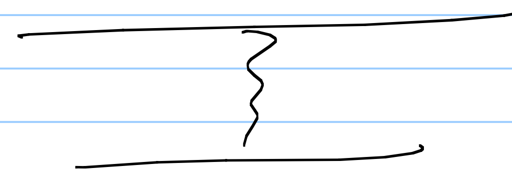



$I = e + p + \gamma$

Feynman rules ⟹ $E, \vec{p}$ conserved at each vertex!

# Variations

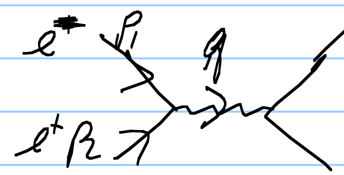
## 1) Real QED



$$-i\mathcal{M} = -iq_e \underbrace{\bar{u}_e(p_3)\gamma^\mu u_e(p_1)} \frac{-i g_{\mu\nu}}{q^2+i\epsilon} (-iq_p)\,\bar{u}_p(p_4)\gamma^\nu u_p(p_2)$$

$\uparrow$ $iD_{F\mu\nu}(x-y) = \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle$

## 2) Scalar case "$e^+e^- \to p^+p^-$"




$$-i\mathcal{M} = (-iq_e)\frac{i}{q^2-m^2+i\epsilon}(-iq_p)$$

$\uparrow$ $q=(p_1+p_2)=(p_3+p_4)$

$$u(p) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \chi \\ \frac{\sigma \cdot p}{E+m} \chi \end{pmatrix}$$

3) <u>Spin 0 QED</u>

$q \phi^* i \overleftrightarrow{\partial}_\mu \phi$

$$\langle p_3 | J_\mu | p_1 \rangle = \frac{e^{i(\ )}}{\sqrt{2\omega_1 2\omega_3}} q_1 (p_1 + p_3)^\mu$$

$$-i\mathcal{M} = -i q_1 (p_1 + p_3)^\mu \quad \frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} \quad -i q_2 (p_2 + p_4)^\nu$$

## Wick Theorem

$$\phi \qquad \phi \qquad |p_1 \ldots\rangle$$

In matrix elements

- act on external state

or

- act on another field in operator

Wick $\Rightarrow$ book keeping of these choices

Defining "normal ordering" $\quad :\phi(x)\,\phi(y)\cdots\phi(z):$ $\quad$ ✓ normal ordering signs

- operationally fields act only on external states
- technically move all $a(p)$ to <u>the right</u> of $a^\dagger(p')$

$$\langle 0|:\phi(x_1)\cdots\phi(x):|0\rangle = 0 \quad \text{always}$$

$$\langle f|:\phi(x_1)\,\phi(x_2):|i\rangle \neq 0$$

Wick says

$$T(\phi(x)\,\phi(y)) = :\phi(x)\,\phi(y): + \langle 0|T(\phi(x)\,\phi(y))|0\rangle$$

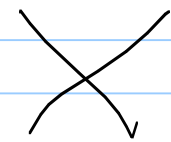
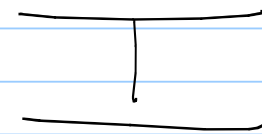
$$T(\phi(x_1)\,\phi(x_2)\,\phi(x_3)) = :\phi(x_1)\,\phi(x_2)\,\phi(x_3): + \phi(x_1)\langle 0|T(\phi(x_2)\,\phi(x_3))|0\rangle$$
$$+ \phi(x_2)\langle 0|T(\phi(x_1)\,\phi(x_3))|0\rangle$$
$$+ \phi(x_3)\langle 0|T(\phi(x_1)\,\phi(x_2))|0\rangle$$

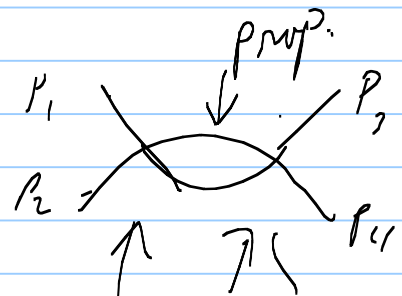
$\phi(x_1)$

$$T(\phi_1\,\phi_2\,\phi_3\,\phi_4) = :\phi_1\,\phi_2\,\phi_3\,\phi_4: + :\phi_1\,\phi_2:\langle 0|T(\phi_3\,\phi_4)|0\rangle + \text{perm}$$
$$+ \langle 0|T(\phi_1\,\phi_2)|0\rangle\langle 0|T(\phi_3\,\phi_4)|0\rangle + \text{perms}$$

$\vdots$

# Loop diagrams

Tree diagrams  

Loop 


$$\underbrace{-\frac{\lambda}{4}\phi^4(x) \quad -\frac{\lambda}{4}\phi^4(y)}_{-i\mathcal{M} \,=\, ?}$$