Introducing the fields 1

Note Title 1/28/2010

1) Real scalar field $\phi(x)$ - no spin

2) Complex scalar ϕ, ϕ^* - antiparticles if charge

3) Non rel fields a) boson ψ^*, ψ

b) fermion

4) Dirac field } spin $1/2$

5) Photons or vector fields Spin 1 (non rel $\vec{M}$)

6) Gravitational field

Real scalar - magnitude $\phi(\vec{x},t) \neq \phi(x)$

- phonons $\Rightarrow$ wave eq

$$S = \int d^4x \ \underbrace{\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi}_{\Rightarrow \text{wave eq}}$$

$$H = \int \frac{d^3p}{(2\pi)^3} \ \omega_p \ a^+(p) \, a(p) \quad + \underset{\sim}{E_0}$$

More generally

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi}\;-\frac{\mu^2}{2}\phi^2 - \frac{\lambda_3}{3}\phi^3 - \frac{\lambda_4}{4}\phi^4 + \cdots$$

$\mathcal{L} = K - V(\phi)$

$H = \cdots + V(\phi)$

could forbid by $\phi \to -\phi$ symmetry

$\mathcal{L} \sim E^4$

$\partial_\mu \sim E^1$

$\phi \sim E^1$

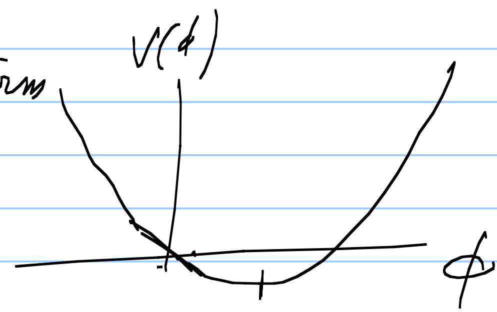
$\mu \sim E^1 \implies \mu = \text{mass}$

$\phi^4 \sim E^4$

λ_4 dimensionless

$\lambda_3 \sim E^1$

Why no linear term



$V(\phi)$

$\phi = \tilde{\phi} + \phi_{min}$

can always redefine linear term away

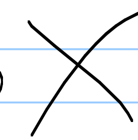
$\implies$ around ϕ_{min} $\quad V(\tilde{\phi}) = \frac{\mu^2}{2}\tilde{\phi}^2 + \frac{\lambda_3}{3}\phi^3 + \cdots$

Eq of motion

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

$\Box \phi$ $\qquad$ $\frac{\partial \mathcal{L}}{\partial \phi} = -\mu^2 \phi - \lambda_3 \phi^2 - \lambda_4 \phi^3$

$$\underbrace{(\Box + \mu^2)\phi}_{\text{Klein Gordon eq}} = \underbrace{-\lambda_3 \phi^2 - \lambda_4 \phi^3}_{\text{"interactions"}}$$

$\frac{\lambda_4}{4}\phi^4 \Rightarrow$ 

Free field (no interactions

$$(\Box + \mu^2)\phi = 0$$

Solutions $\phi \sim e^{-i(Et - \vec{p}\cdot\vec{x})}$

$$-E^2 + \vec{p}^{\,2} + \mu^2 = 0 \quad \Longrightarrow \quad E^2 = \vec{p}^{\,2} + \mu^2$$

rest mass m

Why strings & phonons "massless"

$$\Box \phi = 0 \quad \text{vs} \quad (\Box + \mu^2)\phi = 0$$

"Shift symmetry"

$$V(y_i) = V((y_{i+1} - y_i))$$

no energy cost $y_i \to y_i + c$ for <u>all i</u>

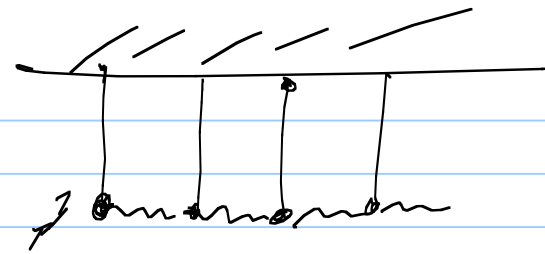
Then go to field $\phi \to \phi + c$

$$\mathcal{L} = - \ldots \cancel{\frac{\mu^2}{2}\phi^2} \leftarrow \text{forbidden}$$

<u>Phonons</u>: $\lambda \to \infty$, no energy cost $E \sim k$ ← wave eq. $\left.\begin{matrix} \omega \to 0 \\ k \to 0 \end{matrix}\right\}$

$\sim k^2/2m$

Simple field theory with mass terms



θ_i or $(R\theta_i) \Longrightarrow \phi$

$V_{spring} = \frac{1}{2} k (R\theta_{i+1} - R\theta_i)^2 \leftarrow$

$V_{gravity} = mgR\cos\theta \sim \cdots \frac{1}{2}\frac{mg}{R}(R\theta)^2 \leftarrow \frac{\mu^2}{2}\phi^2 + \lambda\phi^4$

N

$(\Box + \mu^2)\phi = 0$

$H = KE + \frac{\mu^2\phi^2}{2} + \frac{\lambda\phi^4}{4}$

$E^2 = p^2 + \mu^2 \Rightarrow E = \mu + \frac{p^2}{2\mu} + \cdots$

Quantization

$$e^{-i(Et - \vec{p}\cdot\vec{x})}$$

$$\phi(x,t) = \int \frac{d^3p}{(2\pi)^3} \sqrt{\frac{1}{2\omega_p}} \left\{ e^{-ip\cdot x} a(p) + e^{+ip\cdot x} a^\dagger(p) \right\}$$

Feynman Propagator

$$i D_F(\overset{\vec{x},t}{x} - x') = \langle 0 | T(\phi(x,t)\ \phi(x',t')) | 0 \rangle$$

$T(\ )$ time ordering, early on right

$$T(\phi(x)\,\phi(x')) = \theta(t-t')\,\phi(x)\,\phi(x') + \theta(t'-t)\,\phi(x')\,\phi(x)$$

Green function

$$(\Box_x + m^2)\, i D_F(x-x') = -i\,\delta^4(x-x') \qquad \star$$

if $(\Box + m^2)\,\phi(x) = 0$

To show this

$$\frac{\partial^2}{\partial t^2} T\left(\phi(x,t)\,\phi(x',t')\right) = \frac{\partial}{\partial t}\left\{ T\left(\frac{\partial}{\partial t}\phi(x)\,\phi(x')\right) + \underbrace{\delta(t-t')\,\phi(x)\phi(x') - \delta(t'-t)\,\phi(x')\phi(x)}_{\delta(t-t')\,\underbrace{[\phi(x),\,\phi(x')]}_{0}} \right\}$$

$$= \left\{ T\left(\frac{\partial^2}{\partial t^2}\phi(x)\,\phi(x')\right) + \delta(t-t')\underbrace{\left[\frac{\partial}{\partial t}\phi(x,t),\,\phi(x',t')\right]}_{t'=t} \right\}$$

$$[\pi(x,t),\,\phi(x',t)] = -i\,\delta^3(\vec{x}-\vec{x}')$$

$$= -i\,\delta^4(x-x') + \underbrace{\text{normal terms}}_{(\Box+m^2)\phi=0}$$

Rep in terms of solution

$$iD_F(x-x') = \langle 0|T\Big(\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}\left(a(p)e^{-ip\cdot x}+a^\dagger(p)e^{+ip\cdot x}\right)$$

$$\int\frac{d^3p'}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{p'}}}\left(a(p')e^{-ip'\cdot x'}+a^\dagger(p')e^{+ip'\cdot x'}\right)\Big)|0\rangle$$

$a(p)|0\rangle = 0$ $\qquad\langle 0|\, a(p)\, a^\dagger(p')\,|0\rangle$

$$\underbrace{[a(p),a^\dagger(p')]}_{(2\pi)^3\delta^3(p-p')} + \cancel{a^\dagger(p')a(p)} \to 0$$

$$iD_F(x-x') = \int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\left\{\Theta(t-t')e^{-i(p-p')\cdot x}+\Theta(t'-t)e^{+i(p-p')\cdot x}\right\}$$

Solution by Fourier Transform

$$i D_F(x-x') = \int \frac{d^4q}{(2\pi)^4} \, e^{-iq\cdot(x-x')} \, D_F(q)$$

$$(\Box_x + m^2)\, i D_F(x-x') = \int \frac{d^4q}{(2\pi)^4} \underbrace{(-q_0^2 + \vec{q}^{\,2} + m^2)}_{-q^2+m^2 \;=\; -q_\mu q^\mu + m^2} e^{-iq\cdot(x-x')} D_F(q)$$

$$= -i\,\delta^4(x-x') = -i \int \frac{d^4q}{(2\pi)^4} \, e^{-iq\cdot(x-x')}$$

$\uparrow$

$\cancel{\alpha}$ $\Rightarrow$ rel invariant

$$\Rightarrow i D_F = \frac{i}{q^2 - m^2}$$

$\uparrow$ $q^2 = q_0^2 - \vec{q}^{\,2}$