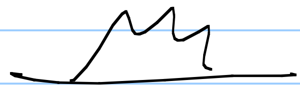
QED 2

Note Title 4/13/2010



$$\Sigma(p) \quad \longrightarrow \quad Z_2$$

$$i\,\Pi_{\mu\nu}(q^2) = i\left(q^2 g^{\mu\nu} - q_\mu q_\nu\right)\Pi(q^2)$$

$$\longrightarrow \quad \frac{-i\,g^{\mu\nu}}{q^2\left(1-\Pi(q^2)\right)} \quad \Longrightarrow \quad Z_3 = \frac{1}{1-\Pi(0)}$$

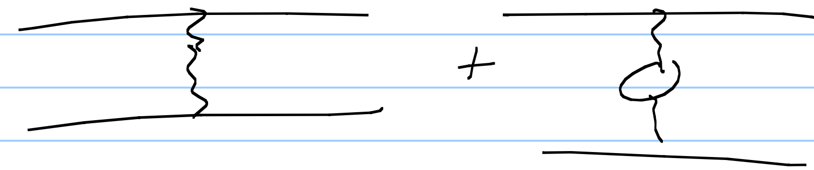
$$\Pi(q^2) = \Pi(0) + \hat{\Pi}(q^2)$$

$$\Pi(0) = \frac{e^2}{6\pi^2}\left[\frac{1}{\varepsilon} + \ln\sqrt{4\pi} \; \cdots \right]$$

$$\hat{\Pi}(q^2) = \frac{e^2}{6\pi^2} \times \frac{q^2}{10\,m^2} + \cdots \qquad q^2 \ll m^2$$

$$= \frac{e^2}{6\pi^2}\left[-\frac{1}{2}\ln q^2/m^2\right] \qquad q^2 \gg m^2$$

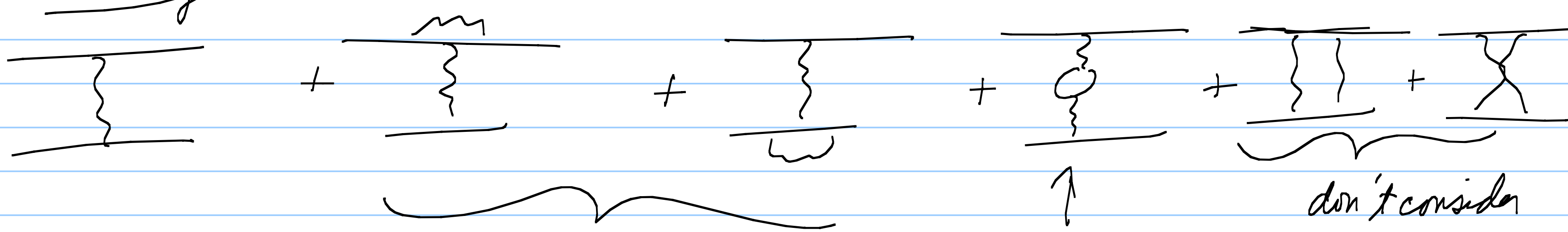
Charge renorm.



Recall:

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{q^4}\left[\underbrace{\alpha_0\left(1 - \alpha_0 \Pi(q^2)\right)}_{\alpha = \alpha_0(1-\alpha\Pi(0)) = \frac{1}{137}}\right]$$

Really



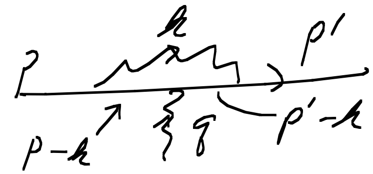
Vertex correction

Z_1

don't consider

not $\propto \gamma_\mu \cdot \gamma^\mu$

Vertex correction



$$-ie\gamma_\mu \quad + \quad -ie\Gamma_\mu(p,p') \equiv -ie\Lambda_\mu(p,p')$$

$$-ie\Gamma_\mu = \int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2+i\epsilon}\left[(-ie\gamma^\lambda)\frac{i}{\not p' - \not k - m}(-ie\gamma_\mu)\frac{i}{\not p - \not k - m}(-ie\gamma_\lambda)\right]$$

$$\langle e(p')| J_\mu | e(p)\rangle = \bar u(p')\left[-ie\gamma_\mu - ie\Gamma_\mu(p,p')\right]u(p)$$

$$q^\mu \langle J_\mu\rangle = 0 \implies \quad = -ie\,\bar u(p')\left[\gamma_\mu F_1(q^2) - i\frac{\sigma^{\mu\nu}}{2m}q_\nu F_2(q^2)\right]u(p)$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu] = -\sigma^{\nu\mu}$$

Define

$$-ie\Gamma_\mu(p,p') = -ie(Z_1^{-1}-1)\gamma_\mu + (\quad)$$

vanishes when $p \to p'$

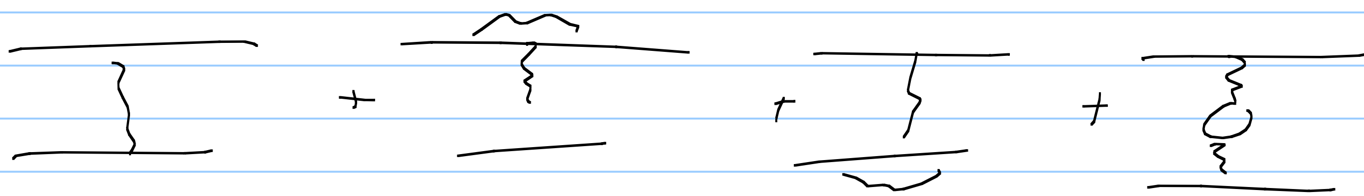
$q \to 0$ limit

Overall

$$\langle J_\mu \rangle = -ie\,\bar u \left[\gamma_\mu + (Z_1^{-1} - 1)\gamma_\mu + \overset{O(q^2)}{\cdots} \right] u = -ie \frac{1}{Z_1} \bar u \gamma^\mu u$$

Charge renorm. $\quad q \to 0 \equiv$ "on shell" renorm.

e

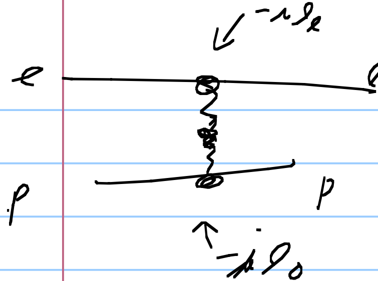


p

Remember Wavefunction renorm for fermions

$$\mathcal{L}_0 = \bar\psi_0 (i\not{D} - m)\psi_0 \equiv \bar\psi_r Z_2 (i\not{D} - m)\psi_r \qquad = \qquad \psi_0 = Z_2^{1/2}\psi_r$$

$\Longrightarrow$ add $Z_2^{1/2}$ for each external state.



$$= (-ie_0)^2 Z_{2e} Z_{2p} \frac{1}{Z_{1e}} \bar{u} \gamma^\mu u \frac{-i}{q^2(1-\Pi(q^2))} \frac{1}{Z_{1p}} \bar{u} \gamma_\mu u + O(q)$$

$\uparrow \Pi(0)$ with $Z_3 = \frac{1}{1-\Pi(0)}$

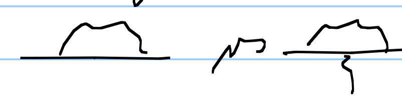
$\Rightarrow q^2 \to 0$

$$= (-i)^2 e_0^2 \frac{Z_{2e} Z_{2p}}{Z_{1e} Z_{1p}} Z_3 \times \underbrace{\bar{u} \gamma^\mu u \frac{1}{q^2} \bar{u} \gamma_\mu u}$$

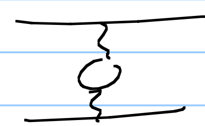
Define $e_r^2 = e_0^2 \frac{Z_{2p}}{Z_{1p}} \frac{Z_{2e}}{Z_{1e}} Z_3$

Ward identity $Z_1 = Z_2$ for all particles

Magic!



$$e^2 = e_0^2 Z_3 = \frac{e_0^2}{1-\Pi(0)}$$

$\leftarrow$ all from 

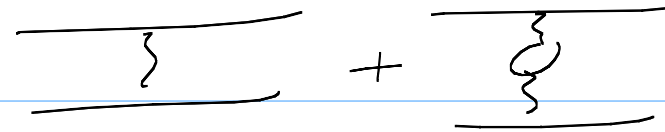
<u>Crucial</u> renorm is the same for <u>all</u> particles

- if not true $e_p \neq e_e$ — atoms not neutral

— earth blows apart ----

<u>Residual predictions</u>

1) Running coupling
2) Lamb shift
3) IR divergences + soft photons
4) $(g-2)$



Running coupling constant

$$\frac{e^2}{4\pi} = \frac{e_0^2}{4\pi}\,\frac{1}{1-\Pi(0)} = \frac{1}{137}$$

Different renorm conditions

- measure at $q^2 = q_0^2$

$$e^2(q_0^2) = \frac{e_0^2}{1-\Pi(q_0^2)} = \frac{e^2}{1-(\Pi(q^2)-\Pi(0))}$$

If $q^2 \gg m^2$

all particles $m^2 < q^2$

$$\left[\Pi(q^2) - \Pi(0)\right] = \frac{\alpha}{3\pi}\ln -q^2/m^2 \longrightarrow \sum_i \frac{\alpha}{3\pi}(Q_i)^2 \ln\frac{q^2}{m_i^2}$$

$$\frac{e^2(q_0^2)}{4\pi} = \frac{e^2}{4\pi} \frac{1}{1 - \frac{\alpha}{3\pi} \ln(-q_0^2/m^2)}$$

$\frac{e^2}{4\pi} \uparrow \frac{1}{137}$

$\uparrow \frac{1}{(\ )}$ gets larger as $q^2 \uparrow$

$$\boxed{q_0^2 < 0}$$

"Renorm group eq." $\Leftarrow$ defines how $e^2(\mu^2)$ changes with μ^2

$q_0^2 = -\mu_0^2$, $q_1^2 = -\mu_1^2$

$\alpha(\mu_0^2)$ in pert th.

$$\frac{e^2(\mu_1^2)}{4\pi} = \frac{1}{4\pi} \frac{e^2(\mu_0^2)}{1 - [\Pi(\mu_1^2) - \Pi(\mu_0^2)]} = \frac{1}{4\pi} \frac{e^2(\mu_0^2)}{1 - \frac{\alpha}{3\pi} \ln \mu_1^2/\mu_0^2} = \frac{e^2(\mu_0^2)}{4\pi}\left[1 + \frac{\alpha}{3\pi} \ln \mu_1^2/\mu_0^2\right]$$

$\uparrow$ mass has dropped out

"Beta function"

$$\mu_1 \frac{\partial}{\partial \mu_1} \alpha(\mu_1) = \alpha \times \frac{2}{3\pi} \alpha \equiv \beta(\alpha) = b_0 \alpha^2 \qquad b_0 = \frac{2}{3\pi}$$

↑ diff eq

"<u>Solve</u>:"

$$\int_{\alpha(\mu_0)}^{\alpha(\mu)} \frac{d\alpha}{\alpha^2} = b_0 \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} = b \ln \mu/\mu_0 = -\frac{1}{\alpha(\mu)} + \frac{1}{\alpha(\mu_0)}$$

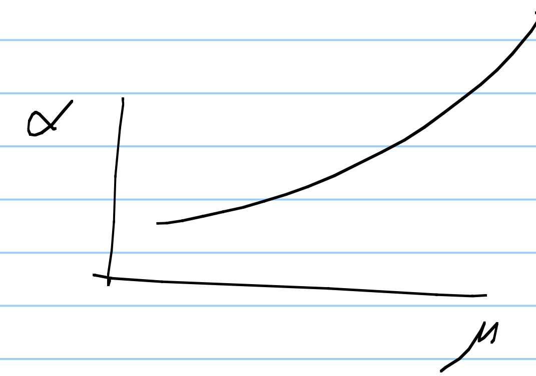
$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} - b_0 \ln \mu/\mu_0 \qquad \boxed{\alpha^{-1} \propto \ln \mu}$$

dimensionful

Define $\frac{1}{\alpha(\mu_0)} \equiv b_0 \ln \Lambda/\mu_0$ for some $\Lambda \gg \mu_0$ since $\alpha > 0$

$$\frac{1}{\alpha(\mu)} = b_0 \ln \Lambda/\mu_0 - b_0 \ln \mu/\mu_0 = b_0 \ln \Lambda/\mu$$

$$\boxed{\alpha(\mu) = \frac{1}{b_0 \ln \Lambda/\mu}}$$



When done carefully

$$\alpha(\mu = M_Z) = \frac{1}{128} > \frac{1}{137}$$

Landau pole

$\alpha \to \infty$ as $\mu \to \Lambda$ in pert theory

but pert th. breaks down as $\alpha \gg 1$

don't really know

Where is Landau pole?

$b_0 = \frac{10}{3\pi}$ with all SM particles

$$\alpha(M_Z) = \frac{1}{b_0 \ln \Lambda / M_Z} = \frac{1}{128}$$

$$\Rightarrow \Lambda = M_Z \, e^{\frac{3\pi}{10}\frac{1}{\alpha}} = M_Z \, e^{116} = \underline{\underline{2 \times 10^{32} \text{ GeV}}}$$

M_P

!!

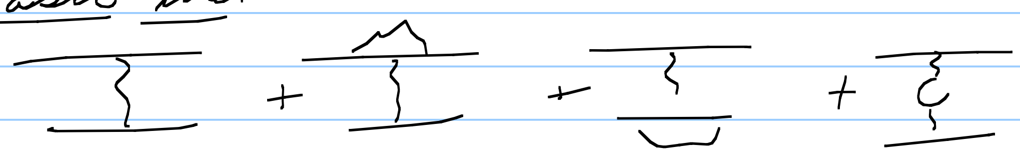
Lamb Shift – first for QFT

Context – in H atom $S_{1/2}$, $P_{1/2}$ degenerate (even in Dirac)

Lamb measured splitting

Bethe resolved QFT loop

Basic idea



$$= \bar{u}\gamma_\mu u \, \underset{\substack{\uparrow \\ \text{ren}}}{(-ie)^2} \underbrace{\left[\frac{1}{q^2} + \underset{\substack{\uparrow \\ \dots\, q^2 \,\dots}}{f(q^2)} \frac{1}{q^2} \right]}_{\left[\frac{1}{q^2} + \text{const} + \mathcal{O}(q^2) \right]} \bar{u}\gamma^\mu u$$

$$V(r) = \int \frac{d^3q}{(2\pi)^3} \frac{e^{-iq\cdot r}}{q^2} \longrightarrow \frac{1}{r}$$

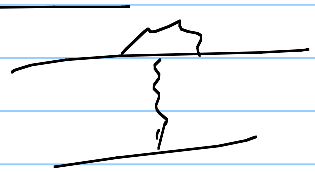
$$\int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} \text{ const} \longrightarrow \delta^3(x)$$

S waves shifted $\psi_s(0) \neq 0$ $\quad \delta^3(x)$ shifts E

P waves $\psi_p(0) = 0 \Rightarrow$ no ΔE

$\Big\}\ 2S_{1/2} - 2P_{1/2}$

2 places



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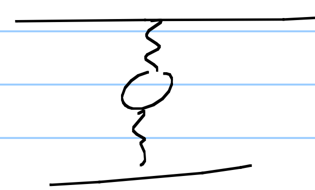

A) Vertex

$$\langle e | J_\mu | e \rangle = \bar{u}\left[\gamma_\mu F_1(q^2) + i \frac{\sigma^{\mu\nu}}{2m} q_\nu F_2(q^2) \right] u$$

does not contribute

$$\rightarrow F_1(q^2) = 1 + a \frac{q^2}{m^2} + \dots$$

$\rightarrow$ Lamb shift (hiding something)

2) Vacuum pol

$$\frac{1}{1 - \Pi(q^2) - \Pi(0)} = 1 + \frac{\alpha}{15\pi} \frac{q^2}{m^2}$$

$\leftarrow$ only need electron vertex

$$\frac{1}{m_e^2} \gg \frac{1}{M_p^2}$$

Relative amounts

Expt $\Delta \nu = 1057.873 \pm 0.020$ MHz

Vertex corr 1085 MHz

Vac pol -27 MHz

Both needed