Renormalization 1

Note Title 3/10/2010

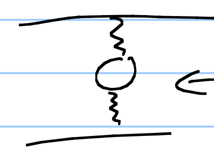
Predictions + Measurements

$\mathcal{L}_{QED}$ $\quad e \longrightarrow \alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

Coulomb scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \, 4m^2}{\vec{q}^{\,4}}$$

Calculate to next order



← vac. pol

$$\frac{d\sigma}{d\Omega} = \left[\alpha\left(1 - \alpha\,\Pi(q)\right)\right]^2 \frac{4m^2}{\vec{q}^{\,2}}$$

Do we plug in $\alpha = \frac{1}{137}$? NO!

Measuring e Coulomb field at long distance (low $\vec{q}$)

- long distance $\leftrightarrow \vec{q} \to 0$

Call charge e_0 , $\alpha_0 = e_0^2/4\pi$ "bare charge"

$e_0 \leftarrow \mathcal{L}$

$$\frac{d\sigma}{d\Omega} = \left[\alpha_0 \left(1 - \alpha_0 \Pi(q^2)\right)\right]^2 \frac{4m^2}{\vec{q}^{\,2}} \Longrightarrow \text{Coulomb potential}$$

$$\alpha_0 \underbrace{\left(1 - \alpha_0 \Pi(q \to 0)\right)}_{\Omega})$$

Measurement

$$\alpha = \frac{1}{137} = \alpha_0 \left(1 - \alpha_0 \Pi(0)\right)$$

Express in terms of α

$$\frac{d\sigma}{d\Omega} = \left[\alpha \left(1 - \alpha \left(\Pi(q^2) - \Pi(0)\right)\right)\right]^2 \frac{4m^2}{\vec{q}^{\,2}}$$

$\leftarrow + \mathcal{O}(\alpha^2)$

Charge renormalization – express predictions in charge of measured α

<u>Renorm. "program"</u>

1) Renorm. conditions – how do you plan measurement?

2) Measure

3) Express all results in term of measured value

$$\alpha\left(\Pi(q^2) - \Pi(0)\right) = \frac{\alpha}{15\pi}\frac{q^2}{m^2} \quad \checkmark$$

Comments

1) Different renorm conditions are possible

- could measure at $q^2 = q_0^2$

$$\left.\frac{d\sigma}{d\Omega}\right|_{q^2=q_0^2} = \left[\alpha(q_0^2)\right]^2 \frac{4m^2}{q_0^4}$$

$$\alpha(q_0^2) = \alpha\left(1 - \alpha(\Pi(q_0^2) - \Pi(0))\right) \quad \leftarrow \text{running coupling}$$

2) Sometimes conditions chosen for theoretical convenience

- calculate shift to physical measurement

3) Complications in books - all orders

Two techniques

1) $\mathcal{L}$ contains e_0

calculate

identify $e_{ren} = e_{phys}$

go back + express everything in e_{phys}

2) $\mathcal{L}$ contains e_0

$e_0 = e_{phys} - \delta e$

rewrite $\mathcal{L}$ using e_{phys} + $\underbrace{\delta e}_{\text{small}}$ "counterterm"

calculate directly with e_{phys} + δe

choose δe to enforce renorm condition (disappears)

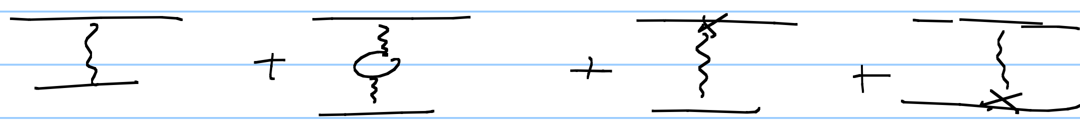
$\Longrightarrow$ everything in terms of e_{phys}

Counterterm method

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar\psi(i\not\partial - m)\psi - e_0 A_\mu \bar\psi \gamma^\mu \psi$$

$$= \qquad '' \qquad - e_{phy} A_\mu \bar\psi \gamma^\mu \psi + \delta e\, A_\mu \bar\psi \gamma^\mu \psi$$

$(e_{phy} - e_0)$



$i\delta e\, \gamma_\mu$

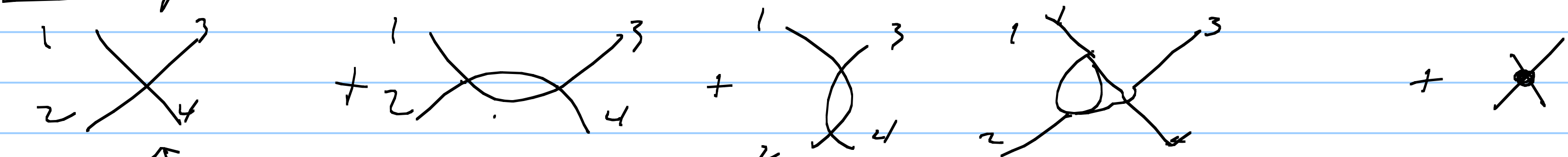
$$\frac{d\sigma}{d\Omega} = \left[\alpha_{phy} + \frac{2\,\delta e\, e}{4\pi} + \alpha_{ph}\,\Pi(q^2)\right]^2 \frac{4m^2}{q^2} \xrightarrow[q^2\to 0]{} \alpha_{phy}^2 \frac{4m^2}{q^2}$$

Choose $\frac{2\,\delta e\, e}{4\pi} = -\alpha_{phy}\,\Pi(0)$

Then

$$\frac{d\sigma}{d\Omega} = \left[\alpha_{phy} + \alpha_{phy}\left(\Pi(q^2) - \Pi(0)\right)\right]^2 \frac{4m^2}{q^2}$$

Example $\quad \phi\phi \to \phi\phi \quad$ in $\lambda\phi^4$



$$-i\mathcal{M} = -6i\lambda + \frac{1}{2}(-6i\lambda)^2\Big[\underbrace{I(p_1+p_2)}_{I(s)} + \underbrace{I(p_1-p_3)}_{I(t)} + \underbrace{I(p_1-p_4)}_{I(u)}\Big] + 6i\delta\lambda$$

$$I(p) = \int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}\,\frac{i}{(k-p)^2-m^2}$$

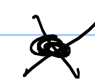
Really should do $\qquad \leftarrow$ bare

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda_0}{4}\phi^4$$

$\leftarrow \lambda - \lambda_0$

$$= \quad '' \quad -\frac{\lambda}{4}\phi^4 + \frac{\delta\lambda}{4}\phi^4$$

$\leftarrow$ small

$\Rightarrow$  $= +6i\delta\lambda$

Renorm. condition # 1

- measure λ at threshold

$s = (p_1+p_2)^2 \to 4m^2$
$t = (p_1-p_3) \to 0$
$u = (p_1-p_4) \to 0$

Define

$$\mathcal{M}(s=4m^2, t=0, u=0) \equiv -6i\lambda_{R1}$$

$$\Rightarrow 6i\delta\lambda = -\frac{1}{2}(-6i\lambda)^2\left[I(4m^2) + 2I(0)\right]$$

$$-i\mathcal{M} = -6i\lambda_{R1} + \frac{1}{2}(-6i\lambda_{R1})^2\left[I(s) + I(t) + I(u) - I(4m^2) - 2I(0)\right] + \mathcal{O}(\lambda_{R1}^3)$$

Renorm condition # 2 $\qquad s=0,\ t=0,\ u=0$

$$-i\mathcal{M}(s=t=u=0) \equiv -6i\lambda_{R2}$$

$$\delta\lambda = -\frac{1}{2}(-6i\lambda)^2\left[3I(0)\right] \leftarrow$$

$$-i\mathcal{M} = -6i\lambda_{R2} + \frac{1}{2}(-6i\lambda_{R2})^2\left[I(s)+I(t)+I(u)-3I(0)\right] + O(\lambda_{R2}^3)$$

For measurement $\to$ go to $s=4m^2,\ t=u=0$

$$-i\mathcal{M}(4m^2,0,0) = \underset{\text{measure}}{-6i\lambda_{R1}} = -6i\lambda_{R2} + \frac{1}{2}(-6i\lambda_R)^2\left[I(4m^2)-I(0)\right] + O(\lambda^3)$$

find