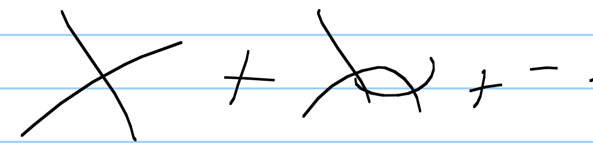
Renormalization 3

Note Title 3/30/2010

1) Philosophy

– use measured parameters

↙ bare



$$\mathcal{M}(s,t,u) = -6i\lambda_0 + \frac{i}{2}(-6i\lambda_0)^2\left[I(s)+I(t)+I(u)\right] + \mathcal{O}(\lambda_0^3)$$

Measure at $s=t=u=0$

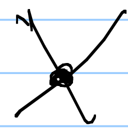
$$\left.\begin{array}{l}\mathcal{M}(0,0,0) \equiv -6i\lambda_r\end{array}\right\}\text{ Scheme}$$

Then

$$\mathcal{M} = -6i\lambda_r + \frac{1}{2}(-6i\lambda_r)^2\left[\underbrace{\Delta I(s)}_{I(s)-I(0)} + \Delta I(t) + \Delta I(u)\right] + \mathcal{O}(\lambda_r^3$$

2) <u>Technique</u> <u>Counterterms</u>

$$\mathcal{L} = \cdots \underbrace{\frac{\lambda_0}{4}}_{\text{bare}} \phi^4 = \text{------} - \underbrace{\frac{\lambda_r}{4}}_{\text{physical}} \phi^4 \mp \underbrace{\frac{\delta\lambda}{4} \phi^4}_{\text{counterterms}}$$

Feynman rule  $= -6i\delta\lambda$

Pert theory use λ_r

Choose $\delta\lambda$ to enforce renorm conditions $\qquad \delta\lambda = -\frac{1}{6}(-6i\lambda)^2 \, 3I(0)$

3) Do Loop integral

$$\Delta I(p^2) = \frac{-i}{32\pi^2}\left\{ 2 + \left(\frac{p^2-4m^2}{p^2}\right)^{1/2}\left\{\ln\left(\frac{p-(p^2-4m^2)^{1/2}}{p+(p^2-4m^2)^{1/2}}\right) + i\pi\right\}\right\} \qquad p^2>4m^2$$

$$= \quad '' \quad \left\{ \; '' \; + \left(\frac{4m^2-p^2}{|p|}\right)^{1/2} \ln\left(\frac{\sqrt{4m^2-p^2}-p}{\sqrt{}+p}\right)\right\} \qquad p^2<0$$

$$= \quad - - - - - \tan^{-1} \qquad 0<p^2<4m^2$$

Treatment

- two basic methods in books

1) <u>BPH renorm</u> – (with counterterms)

   - pert in physical parameter ✓
   - uses physical masses ✓
   - very good for higher orders

2) "<u>Conventional</u>"

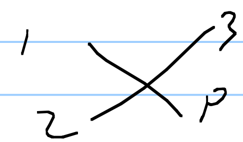
   - start with bare parameters, do pert th
   - identify physical parameters later
   - re-express everything in physical ones

Imaginary parts

$$\Delta I(p^2) = \frac{-i p^2}{32\pi^2} \int_0^1 dz \, \frac{(1-z)(1-2z)}{[\underbrace{m^2 - z(1-z)p^2}_{\text{pole?}} - i\varepsilon]} \leftarrow$$

A) **Where are imag parts?**

$p^2 = s, t, u$



$s = (p_1+p_2)^2 = E_{cm}^2 > 0$

$t = (p_1 - p_3)^2$

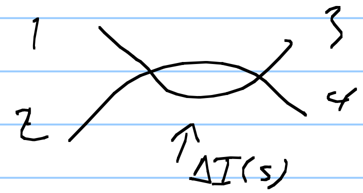
$= \underbrace{(E_1 - E_3)^2}_{0 \text{ in cm}} - (\vec{p}_1 - \vec{p}_3)^2 < 0$

$u = (p_1 - p_4)^2 < 0$

$i\Delta I(t)$, $i\Delta I(u)$ real $\quad (\varepsilon \to 0)$

$\max z(1-z) = \frac{1}{4} \implies$ no pole in $I(s)$ if $s < 4m^2$

goes through pole for $s > 4m^2$



B) Physics – Unitarity – physical intermediate state "on shell"

$\leftarrow T\exp\{\ \}$

Formal proof $\quad S_{fi} = \langle f | U_I(\infty, -\infty) | i \rangle$

S is unitary $\quad S^\dagger S = 1 \quad = \sum_f (S_{fi'})^\dagger (S_{fi}) = \delta_{ii'}$

$$= \langle i' | U^\dagger \underbrace{| f \rangle \langle f |}_{1} U | i \rangle = \langle i' | i \rangle = \delta_{ii'}$$

We define

$$S = 1 - iT \qquad \text{use } T = \frac{(2\pi)^4 \delta^4(\ )}{\sqrt{2\omega_1 2\omega_2 \cdots}} \mathcal{M}$$

$$S^\dagger S = 1 + iT^\dagger - iT + T^\dagger T = 1$$

$$\underbrace{(T - T^\dagger)_{fi}}_{\operatorname{Im} \mathcal{M}_{\phi\phi\to\phi\phi}} = -i (T^\dagger T)_{fi} = -i \sum_s \underbrace{T^\dagger_{sf} T_{si}}$$

$$= -i \left| T_{\phi\phi \to \phi\phi} \right|^2$$

Imag parts in $\mathcal{M}$ when on shell physical intermediate states

c) Techniques

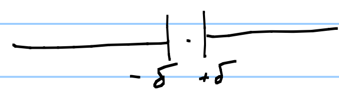
$$d = m^2 - p^2(1-z)z - i\varepsilon$$

Identity

$$\frac{1}{x - i\varepsilon} = \underbrace{P\,\frac{1}{x}}_{\text{Real}} + \underbrace{i\pi\,\delta(x)}_{\text{imag}} = \frac{x + i\varepsilon}{(x-i\varepsilon)(x+i\varepsilon)} = \frac{x}{x^2+\varepsilon^2} + \frac{i\varepsilon}{x^2+\varepsilon^2}$$

$\Longrightarrow$

Principal value



$$\frac{1}{d} = P\,\frac{1}{d} + i\pi\,\delta\big(m^2 - p^2 \underbrace{z(1-z)}_{\text{max of } 1/4}\big)$$

2) Analytic Continuation

$$\ln(-s) = \ln(|s|) - i\pi = \ln\left(e^{-i\pi} s\right) = \ln(e^{-i\pi}) + \ln s$$

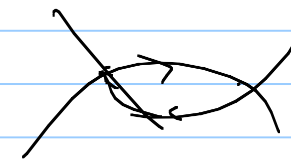
$\uparrow s > 0$

$$= \ln\left(-(s + i\varepsilon)\right) \Leftarrow i\varepsilon \text{ dictates}$$

Infinities

- hidden infinity

$$\delta\lambda = (\quad) \, I(0)$$



$$I(0) = \int \frac{d^4 \ell}{(2\pi)^4} \frac{i}{\ell^2 - m^2 + i\varepsilon} \frac{i}{\ell^2 - m^2 + i\varepsilon}$$

- log divergent

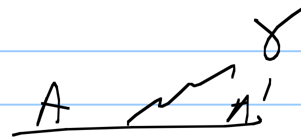
$$\Delta I = I(s) - I(0) \quad \text{finite}$$

Fact many loop integrals are divergent

Infinities in QM

- all over the place in QM

"Radiation reaction"

Radiation 

I

Radiation reaction $\Delta E_A = \sum \dfrac{\langle A | V | A'\gamma\rangle \langle A'\gamma | V | A\rangle}{E_A - E_{A'} - E_\gamma}$

- diverges badly (Bethe)

Why do QM calculations work?

- QM sum over <u>all</u> intermediate states
- physics experimental - don't know all intermediate states

Answer is renormalization

- uncertainty principle

high E stuff $\to$ very short distances

Look <u>local</u> to us $\Rightarrow$ looks like term in local $\mathcal{L}$, $\mathcal{H}$

- $\mathcal{L}$ has parameters $\lambda, m, \ldots g, e, \ldots$

- measure parameters

- all effect of unknown high E, $\Rightarrow$ goes into <u>measured</u> values

Appelquist Carrazone thm

- all effect of high energy (finite or not) goes into renormalized parameters

<u>or</u> is suppressed by powers of heavy scale $\frac{1}{E^2_{Heavy}}$.

Philosophy of Infinities

1) <u>CM folks don't worry</u>

ϕ is phonon — high E phonons don't exist

$\lambda < 1\,\text{Å}$ no phonons

— physical cutoff $\Rightarrow$ no infinites

2) HEP folks
— hope for a fund. theory - finite

3) Artifact of Pert Theory?

4) Perhaps gravity solves it?

5) "Just do it"