Standard Model 3

Note Title 11/17/2009

$$\mathcal{L} = -\frac{1}{\sqrt{2}}(v+H)\left[\Gamma^{(u)}_{ij}\,\bar{u}_{iL}u_{jR} + h.c.\right]$$

$$u_i = \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}$$

Mass term

$$\mathcal{L} = -\frac{v}{\sqrt{2}}\begin{pmatrix}\bar{u}_0 & \bar{c}_0 & \bar{t}_0\end{pmatrix}_L \underbrace{\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{21} & \Gamma_{22} & \vdots \\ \cdots & \cdots & \cdots \end{pmatrix}}_{\text{not real not hermitian}} \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}_R + h.c.$$

Define ↙ Unitary 3x3

$$u_{0L} = S_L u_L$$

$$u_{0R} = S_R u_R$$

$$M_0 = \frac{v}{\sqrt{2}} \cdot \Gamma$$

$$M_D = S_L^\dagger M_0 S_R = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

<u>Can we do it?</u>

Consider $$M_D^2 = S_L^\dagger M_0 S_R S_R^\dagger M_0^\dagger S_L = S_L^\dagger \underbrace{M_0 M_0^\dagger}_{\text{Hermetian}} S_L$$

$$\left.\begin{array}{l} S_L \text{ diagonalizes } M_0 M_0^\dagger \\ S_R \text{ diagonalizes } M_0^\dagger M_0 \end{array}\right\} \det M_0 M_0^\dagger = \det M_0^\dagger M_0$$

$\Longrightarrow$ can diagonalize it

$\Longrightarrow$ state of definite mass

Neutrino masses

$$\mathcal{L} = -\frac{y}{\sqrt{2}} \bar{\nu}_L \Gamma \nu_R \implies \text{same story}$$

Recall ν_R is singlet under all gauge groups

Majorana mass $\mathcal{L}_M = -M_M \underbrace{\psi_R^T i\gamma_0\gamma_2}_{\bar{\psi}^c} \psi_R$

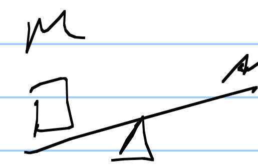
allowed $\Rightarrow$ add it

$$\mathcal{L}_\nu = -\frac{y}{\sqrt{2}} \bar{\nu}_L \Gamma \nu_R - \bar{\nu}_R^c M_M \nu_R$$

$\uparrow$ 3x3 matrix

1 Flavor

$$(\bar{\nu}_L, \bar{\nu}_R^c)\begin{pmatrix} 0 & m_D \\ m_R & M_M \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

seesaw 

$M_M \gg m_D$

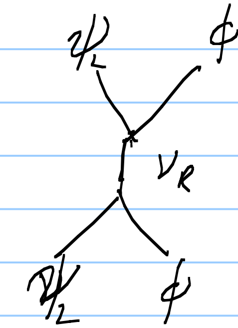
$m_1 = \dfrac{m_D^2}{M_M}$ ← Majorana mass, $m_2 = M_M - \cancel{\dfrac{m_D^2}{M_M}} \to 0$

$\theta = \dfrac{m_D}{M_M}$

$\Longrightarrow$ explain small ν masses $m_\nu \approx 0.05\,eV = \dfrac{m_t^2}{(\underbrace{6\times10^{10}\,GeV}_{M_M})}$

Dim 5 operator

– if $\nu_R \sim M_M \sim 10^{10}$ GeV $\Longrightarrow$ integrate out



$$\mathcal{L} = -\Gamma \, \underline{\bar{\psi}_{eL} \, \bar{\phi} \, \nu_R} - \bar{\nu}_R^c \, M_M \, \nu_R$$

$$\mathcal{L}_{eff} = -\frac{\Gamma^2}{M_M} \underbrace{\left(\bar{\psi}_{eL} \, \bar{\phi}\right) \left(\bar{\phi} \, \bar{\psi}_{eL}^c\right)}_{\text{dim 5}} + h.c. \quad \Longleftarrow \underline{\text{more general}}$$

dim 5, gauge inv

After sym breaking $\bar{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$

$$\mathcal{L} = -\underbrace{\frac{\Gamma^2 v^2}{2 M_M}}_{\sim m_D^2/M_M} \left(\bar{\nu}_L \, \bar{\nu}_L^c + h.c.\right) = \text{Majorana mass}$$

Neutrino mixing $\quad 3\times 3$ case

$$\begin{pmatrix} 0 & M_0^D \\ M_0^D & M_0^M \end{pmatrix}$$

$\leftarrow$ 3×3 symmetric

Integrate out Majorana

$$M_0^{\text{seesaw}} = M_0^D \frac{1}{M_0^M} (M_0^D)^T$$

Can diagonalize $M_0^D = S_L M^D S_R^\dagger$ $\qquad M^D = \begin{pmatrix} M_1^D & & \\ & M_2^D & \\ & & M_1^D \end{pmatrix}$

$$M_\nu^{\text{Seesaw}} = S_L \left(M^D S_R^{\dagger} \frac{1}{M_\nu^M} S_R^{T} M^D \right) S_L^T = S\, C\, S_L^T$$

central matrix

Diagonalize Central matrix

$$C = F\, m_\nu\, F^T \qquad m_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$$

physical state

Overall LH Trans

$$\nu_R = (S_L\, F)$$

Dirac   Majorana_ central

Effect of diagonalization

Neutral current - no effect

singlets

$$J_\mu^{em} = \bar{\Psi} Q \gamma_\mu \Psi = \frac{2}{3}\Big(\bar{u}_{0L}\gamma_\mu u_{0L} + \bar{c}_{0L}\gamma_\mu c_{0L} + \bar{t}_{0L}\gamma_\mu t_{0L} + L \to R + \cdots\Big)$$

$$= \frac{2}{3}\left[\bar{u}\gamma_\mu u + \bar{c}\gamma_\mu c + \cdots\right]$$

Charged currents

$$(\bar{u}_0\ \bar{c}_0\ \bar{t}_0)_L\, \gamma_\mu \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_L \to (\bar{u}\ \bar{c}\ \bar{t})_L\, \gamma_\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{CKM} = \left(S^\dagger_{(u)L}\ S_{dL}\right) = 3\times 3 \text{ Unitary}$$

Leptons $V_{PMNS} = \left(F^{T} S^{\dagger}_{\nu_L} S_e \right) = 3\times 3$ Unitary

<u>Parameters in V_{CKM}</u>

Unitary $3\times 3 \Rightarrow 9$ parameters

Unphysical parameters

1) diag elements real by $u \to e^{i\phi_1} u_L$, $c \to e^{i\phi_2} c$, $t \to e^{i\phi_3} t$

$9 \to 6$

2) Make V_{ud} real $d \to e^{i\phi_4} d$, readjust $\phi_1\ \phi_2\ \phi_3 \to 5$

3) One more $t + b$ Make V_{cb} real also $\Rightarrow 4$

$$V = \begin{pmatrix} \text{real} & \text{real} & \text{complex} \\ & \text{real} & \text{real} \\ & & \text{real} \end{pmatrix} \quad \cancel{=} \quad 3 \times 3 \text{ real} \rightarrow 3 \text{ param} + \text{one phase}$$

$$= \begin{pmatrix} 1 & \lambda & \lambda^3 e^{i\delta} \\ & 1 & \lambda^2 \\ & & 1 \end{pmatrix} \quad \leftarrow \text{expt pattern}$$

For neutrinos

– almost same, but if Majorana mass can't rephase ν_L

$\nu_L^T \nu_L$ is not invariant

$$V_{PMNS} = \begin{pmatrix} 3+1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix} \quad \leftarrow \text{2 extra phases}$$

– unobservable except with Majorana mass

Wolfenstein Param

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda = 0.22$

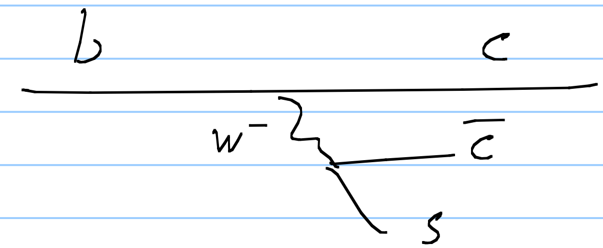
$A \sim O(1)$

$\rho, \eta \sim O(1)$

Weak decay

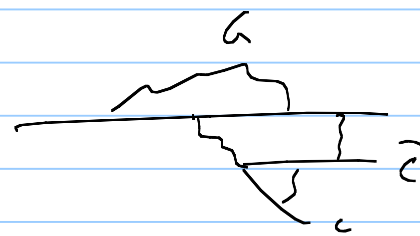
- Tree level Trivial

$b \to c\,\bar{c}\,s$



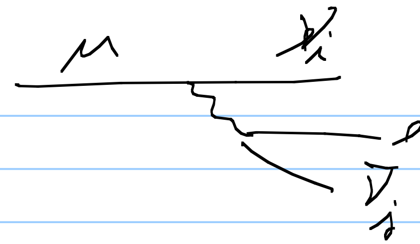
$$\mathcal{H}_W = \left(\frac{g_2}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2}\, V_{cs}^*\, V_{cb}\quad \bar{s}\,\gamma^\mu(1+\gamma_5)\,c\quad \bar{c}\,\gamma^\mu(1+\gamma_5)\,b$$

Deeper, complex



$\Longrightarrow$ OPE

Leptons - easier



$$H = G_F \, V_{je} V^*_{i\mu} \; \bar{e}\gamma_\mu(1+\gamma_5)\nu_j \cdot \bar{\nu}_i \gamma^\mu (1+\gamma_5)\mu$$

$\to 10^{-4}$ $\quad 10^{-6}$ $\quad 10^{-15}$

$$\Gamma(\mu \to e \nu_i \bar{\nu}_j) = \frac{G_F^2 m_\mu^5}{192\pi^2} \underbrace{|V_{je}|^2 |V^*_{i\mu}|^2} \left[1 - \frac{8m_e^2}{m_\mu^2} + \frac{3}{3}\frac{m_\mu^2}{M_W^2} - \# \frac{m_\nu^2}{m_\mu^2} - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{24}{4}\right) + \dots \right]$$

$\uparrow 4\times10^{-3}$

If don't measure ν types

$$\sum_{i,j} |V_{je}|^2 |V^*_{i\mu}|^2 = 1$$

$$\Gamma = \frac{G_\mu^2 m_\mu^5}{192\pi^2} \implies G_\mu = 1.16637(2)\times 10^{-5}\,\text{GeV}^{-2} \approx \frac{10^{-5}}{M_p^2} \checkmark$$