

# Electric Dipole Moment of the Electron

## I. INTRODUCTION AND EXPERIMENTAL STATUS

In the following, we extend the discussion of the neutron electric dipole moment given in Sect. IX-4 of DGH2 to that of the electron, and also comment on the remaining charged leptons  $\mu$  and  $\tau$ .<sup>1</sup>

The only vectorial quantum numbers associated with an electron are its momentum  $\mathbf{p}$  and spin  $\mathbf{s}$ . Thus, for an electron at rest its electric dipole moment  $\mathbf{d}_e$  must be proportional to the spin,

$$\mathbf{d}_e = d_e \frac{\mathbf{s}}{|\mathbf{s}|} . \quad (1)$$

The dipole interacts with an electric field as

$$H_{\text{edm}} = -\mathbf{d}_e \cdot \mathbf{E} = -d_e \frac{\mathbf{s} \cdot \mathbf{E}}{|\mathbf{s}|} . \quad (2)$$

Since  $\mathbf{E}$  is unchanged under time reversal  $T$ , but all angular momenta reverse sign, it follows that  $H_{\text{edm}}$  is odd under  $T$ .<sup>2</sup> For a relativistic spin-1/2 particle, the electric dipole moment contribution has the matrix element

$$\langle \mathbf{p}' | J_\mu^{\text{em}} | \mathbf{p} \rangle_{\text{edm}} = i d_e \bar{u}(\mathbf{p}') \sigma_{\mu\nu} q^\nu \gamma_5 u(\mathbf{p}) , \quad (3)$$

with  $q^\nu = (p' - p)^\nu$ , which is equivalent to an interaction density

$$\mathcal{H}_{\text{edm}} = -i d_e \partial_\mu (\bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi) A_\nu = i \frac{d_e}{2} \bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu} . \quad (4)$$

Since  $F^{0i} = -E^i$  and

$$\sigma^{0i} \gamma_5 = -i \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} , \quad (5)$$

this interaction attains the following hamiltonian form in the nonrelativistic limit,

$$H_{\text{edm}} = \int d^3x \mathcal{H}_{\text{edm}} \rightarrow -d_e \langle \boldsymbol{\sigma} \rangle \cdot \mathbf{E} . \quad (6)$$

Despite many attempts over the years to detect  $\mathbf{d}_e$ , only upper bounds have been obtained. The best, and most recent, of these is from the ACME Collaboration [1],

$$d_e^{(\text{expt})} < 8.7 \times 10^{-29} \text{ e cm} , \text{ CL} = 90 \% . \quad (7)$$

<sup>1</sup> Throughout, we refer to the 2nd Edition of *Dyanamics of the Standard Model* as DGH2.

<sup>2</sup> Note that an electric dipole moment is also odd under parity.

This represents a substantial improvement in experimental sensitivity, *e.g.* about a factor twelve smaller than the bound,  $d_e < 10.5 \times 10^{-28}$  e cm , CL = 90 % , in the most recent PDG listing [2]. The dramatic upgrade was achieved, in part, by probing an electron bound in the polar molecule thorium monoxide (*ThO*). A valence electron moving relativistically near the thorium nucleus experiences a huge internal electric field  $|\mathbf{E}_{\text{eff}}| \simeq 84 \times 10^9$  V/cm, and from Eq. (2), this allows the possible detection of a tiny electric dipole moment.<sup>3</sup>

There are also experimental limits on electric dipole moments for the muon [4],

$$d_\mu^{(\text{expt})} < 1.8 \times 10^{-19} \text{ e cm} , \text{ CL} = 95 \% , \quad (8)$$

and the tau lepton [2],

$$\text{Re } d_\tau^{(\text{expt})} = (-0.22 \rightarrow +0.45) \times 10^{-16} \text{ e cm} , \quad \text{Im } d_\tau^{(\text{expt})} = (-0.25 \rightarrow +0.008) \times 10^{-16} \text{ e cm} . \quad (9)$$

According to the values in Eqs. (7–9), the current limits on  $d_\mu$  and  $d_\tau$  are respectively about 9 and 12 orders of magnitude less precise than that on  $d_e$ .

## II. THEORETICAL STATUS

To have a nonzero electron dipole moment requires CP violation and thus involves the weak interaction sector of the Standard Model.<sup>4</sup> Just how this arises in detail depends on the issues of neutrino mass and of QCD.

### A. Zero mass neutrinos, no QCD

Consider the limit of zero neutrino mass with QCD effects neglected. For massless neutrinos, there is no mixing within the lepton sector (*cf.* comments on p.173 of DGH2), so Standard Model CP violation must arise from the complex phase occurring in the CKM matrix. As such, a nonzero value for  $d_e^{(\text{SM})}$  can arise only via the *quark* sector. The lowest order amplitude of this type is the two-loop process depicted in Fig. 1(a). The vertex to the left describes  $u_i + W^- \rightarrow d_j$  and the one to the right has  $d_j \rightarrow u_i + W^-$ , where  $i, j$  are respectively generation labels for up-type and down-type quarks. This (two-loop) amplitude is real-valued because the two CKM factors contribute as  $|V_{ij}|^2$  and so  $d_e^{(\text{SM})}$  vanishes at this level,

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<sup>3</sup> A more recent determination [3] of the effective electric field in ThO gives  $|\mathbf{E}_{\text{eff}}| \simeq 75.6 \times 10^9$  V/cm, a 10% decrease. This would have the effect of somewhat increasing the value of the bound cited in Eq. (7).

<sup>4</sup> We assume there is no effect from the so-called ‘theta term’  $\bar{\theta}$  described in Sect. IX–4 of DGH2.

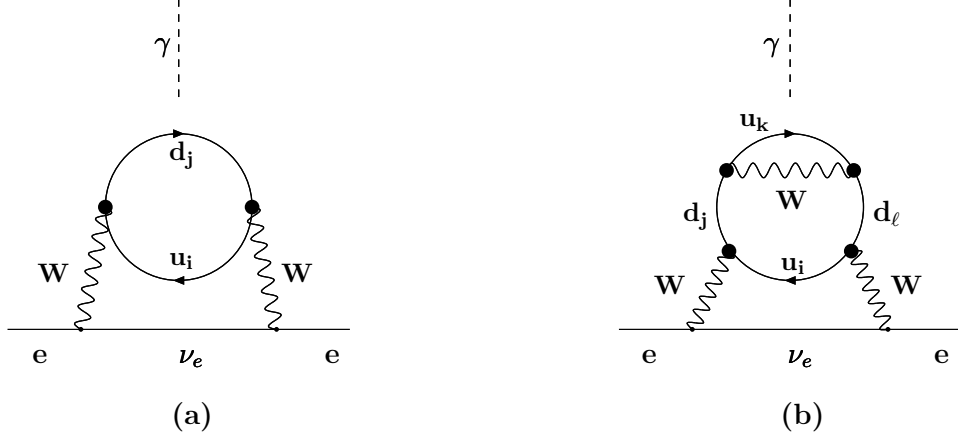


FIG. 1: (a) Two-loop diagram for  $\mathbf{d}_e$ . (b) Three-loop contribution. For both (a), (b), the detached photon line symbolizes attaching the photon to each charged particle and summing over all such contributions.

We must proceed to the three-loop contributions, an example of which appears in Fig. 1(b). Pospelov and Khriplovich [5] found that the electron electric dipole moment remains zero, but the effect is more subtle than for two loops. Although there are individual contributions which vanish (as in the two-loop case), others are found to be nonzero. However, the sum over all such nonzero components vanishes by cancellation, although no explanation is given in Ref. [5] for this behavior.

One can learn more by exploiting the association between the electric dipole moments of the electron and  $W$ -boson,  $d_e^{(\text{SM})}$  and  $d_W^{(\text{SM})}$ . For example, detaching the two  $W$ s in Fig. 1(a) from the electron yields the one-loop amplitude for the  $W$  electric dipole moment  $d_W^{(\text{SM})}$ . More generally, an  $(n+1)$ -loop expression for  $d_e^{(\text{SM})}$  is related to an  $n$ -loop expression for  $d_W^{(\text{SM})}$  (e.g. the two-loop vanishing of  $d_e^{(\text{SM})}$  implies the one-loop vanishing of  $d_W^{(\text{SM})}$ ). In a two-loop analysis of  $d_W^{(\text{SM})}$ , Booth [6] found that  $d_W^{(\text{SM})}$  vanishes and also provided the following explanation for this result. First recall the description of Standard Model CP-violation (*cf* DGH2 Eqs. (II-4.29),(II-4.30)) in terms of a Jarlskog invariant  $J$  [7],

$$\Phi_{ij}^{k\ell} \equiv \text{Im} \left[ V_{ij} V_{kj}^* V_{kl} V_{il}^* \right] \equiv J \sum_{m,n} \epsilon_{ikm} \epsilon_{jln} \quad , \quad (10)$$

where the current evaluation [2] gives  $J = (2.96_{-0.16}^{+0.20}) \times 10^{-5}$ . It follows directly from Eq. (10) that  $\Phi$  is antisymmetric in the the up-type quark labels  $i, k$  and also in those of the labels  $j, \ell$  for down-type quarks,

$$\Phi_{kj}^{i\ell} = -\Phi_{ij}^{k\ell} \quad , \quad \Phi_{i\ell}^{kj} = -\Phi_{ij}^{k\ell} \quad . \quad (11)$$

Now, the full amplitude for  $d_W^{(\text{SM})}$  at two loops will be the product of the CP-violating quantity  $\Phi_{ij}^{k\ell}$  and a dynamical function  $A_{ik}^{j\ell}(m_i^2, m_j^2, m_k^2, m_\ell^2)$  to be summed over all quark configurations in

the Feynman diagrams. It is shown in Ref. [6] that  $A_{ik}^{j\ell}$  is symmetric in the masses of up-type quarks and as a consequence of the first relation in Eq. (11),  $d_W^{(\text{SM})}$  vanishes at two-loops and thus  $d_e^{(\text{SM})} = 0$  at three-loops.<sup>5</sup>

Thus, fourth order is the next place to look for a nonzero evaluation of  $d_e$ . To our knowledge there is no published calculation carried out exactly to this order, although order-of-magnitude expressions exist. An oft-cited example is [8],

$$d_e^{(\text{SM})} \sim e \frac{G_F}{\pi^2} \left( \frac{\alpha}{2\pi} \right)^3 m_e J \simeq 6 \times 10^{-37} \text{ e cm} \quad (m_{\nu_i} = 0, \text{ no QCD assumed}) . \quad (12)$$

Compared to the experimental limit on  $d_e$  of Eq. (7), this SM prediction is 8 orders of magnitude smaller.<sup>6</sup>

If, in fact, the dependence of the electric dipole moment for lepton  $\ell$  is as occurs in Eq. (12) for the electron, then we can scale up the value given there to predict

$$d_\mu^{(\text{SM})} \sim e \frac{G_F}{\pi^2} \left( \frac{\alpha}{2\pi} \right)^3 m_\mu J \simeq 1 \times 10^{-34} \text{ e cm} , \quad d_\tau^{(\text{SM})} \sim e \frac{G_F}{\pi^2} \left( \frac{\alpha}{2\pi} \right)^3 m_\tau J \simeq 2 \times 10^{-33} \text{ e cm} . \quad (13)$$

Compared to the experimental bounds given in Eqs. (8), (9) for  $d_\mu^{(\text{expt})}$  and  $d_\tau^{(\text{expt})}$ , these theoretical estimates are even more remote than that for  $d_e^{(\text{SM})}$ .

## B. Zero mass neutrinos, QCD to first order

Thus far, we have not included effects of QCD. This situation might seem reminiscent to the problem of relating  $\alpha^{-1}(M_Z^2)$  to  $\alpha^{-1}(0)$  encountered in Chap. II and Chap. XVI of DGH2, where we first used one-loop quark corrections (*e.g.* Eq. (II-1.38) of DGH2), but later worked to all orders in QCD (Eqs. (XVI-6.13-6.15)) by using dispersion relations with input from cross section data.

Here, however, including even one gluon loop correction will affect the symmetric contributions of quark propagators mentioned in the preceding section, and thus affect the delicate cancellation mechanism that causes  $d_e^{(\text{SM})}$  to vanish at three loops. As a result, there will be non-zero four-loop contributions for  $d_e^{(\text{SM})}$  arising from a gluon correction to a three-loop  $\mathcal{O}(G_F \alpha^2)$  diagram (see Fig. 2

<sup>5</sup> This cancellation mechanism was noted earlier by Shabalin [9] to explain the vanishing of the quark electric dipole moment at two-loops.

<sup>6</sup> A value  $d_e^{(\text{SM})} \sim 10^{-38} \text{ e-cm}$  has long been cited in the literature and conference talks. However, it employs the outdated bound  $J < 10^{-4}$  and is based on a specious factor of  $10^{-2}$ , due to a numerical error in transforming units from  $\text{GeV}^{-1}$  to  $\text{e cm}$ .

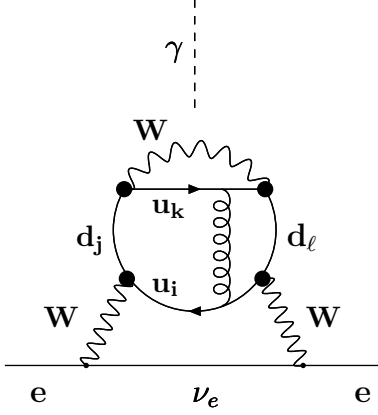


FIG. 2: Example of one-gluon four-loop diagram.

for an example). Several comparable order-of-magnitude estimates for this exist in the literature, *e.g.* Ref. [10] has

$$d_e^{(\text{SM})}[\text{one} - \text{gluon}] \sim \frac{\alpha_S}{4\pi} \cdot \frac{eG_F m_e J \alpha^2}{256\pi^4} \simeq 3 \times 10^{-37} \text{ e cm} \quad (m_{\nu_i} = 0 \text{ assumed}) \quad , \quad (14)$$

upon taking  $\alpha_s \simeq 0.4$ . This estimate for  $d_e^{(\text{SM})}$  is seen to remain tiny.

### C. Massive Dirac neutrinos, no QCD

Neutrino masses are known to be very small, *e.g.* the sum over neutrino mass eigenstates is bounded by astrophysical data to be no more than  $\sum_i m_i < 0.3 \text{ eV}$  (see DGH2 Eqs. (I-1.3a,b)) As such, if we were to continue taking into account, as above, only CP-violation arising from the quark sector, then only minor corrections would be expected to the estimates discussed above from effects of neutrino mass.

There is, however, a new class of contributions. CP-violation can now arise purely from the *leptonic* sector via the charged weak current (*cf.* Eq. (VI-2.1)),  $J_{\text{ch}}^\mu(\text{lept}) = 2 \sum_{i,j} \bar{\nu}_{L,i} \mathbf{V}_{ij}^{(\nu)} \ell_{L,j}$ , where  $\mathbf{V}^{(\nu)}$  is the *Dirac* leptonic mixing matrix of Eqs. (VI-2.2),(VI-2.11). This case was studied, among others, by Donoghue in Ref. [11]. His main finding was that  $d_e^{(\text{SM})}$  vanishes through *two-loop level* but is expected to be nonzero at three-loops.<sup>7</sup> An order-of-magnitude expression relevant to Standard Model expectations is

$$d_e^{(\text{SM})}[\text{Lepton}] \sim \frac{eG_F m_e J \alpha^2}{\pi^4 M_W^4} G(m_\nu^2/M_W^2, \dots) \quad , \quad (15)$$

<sup>7</sup> This parallels the behavior found for the electric dipole moments of quarks [12].

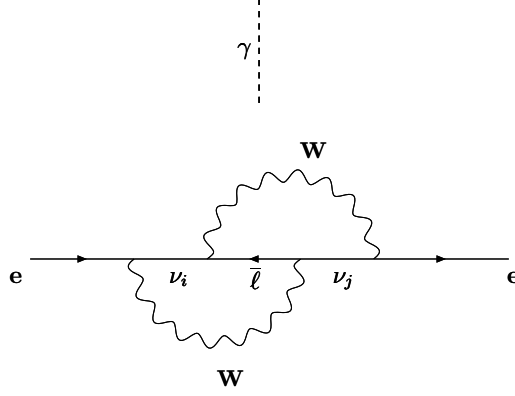


FIG. 3: Two-loop diagram with Majorana neutrinos, exhibiting lepton number nonconservation.

where the function  $G$  depends on neutrino mass  $m_\nu$  relative to the scale  $M_W$  as well as the mixing matrix  $\mathbf{V}^{(\nu)}$ . Already, the scale for  $d_e^{(\text{SM})}[\text{Lepton}]$  of a three-loop contribution with no mass suppression falls into the  $10^{-33}$ -to- $10^{-34}$  range. For a neutrino mass  $m_\nu \sim 1$  eV, the suppression from a quadratic factor  $m_\nu^2/m_W^2 \sim 1 \times 10^{-20}$  would reduce  $d_e^{(\text{SM})}[\text{Lepton}]$  to an unobservably small value.

#### D. Majorana neutrinos, no QCD

Our treatment thus far (for massless or massive Dirac neutrinos with or without effects of QCD) paints a convincing picture that  $d_e^{(\text{SM})}$  is too small to be observed in any foreseeable experiment. However, the topic of Majorana neutrinos remains. As mentioned at the beginning of Chap. VI in DGH2, we have chosen to include the topic of Majorana neutrinos in our treatment of the Standard Model, and we follow the usual three generation approach in the following.<sup>8</sup> Those Majorana contributions to  $d_e$  which conserve lepton number along the fermion line overlap with those already described. There is, however, the lepton-number violating process depicted in Fig. 2 which describes the fermion chain  $e_{\text{initial}} \rightarrow \nu_i \rightarrow \bar{\ell} \rightarrow \nu_j \rightarrow e_{\text{final}}$ , where  $\bar{\ell}$  is an intermediate state antilepton and the various transitions are induced by  $W$ -bosons. Denoting this contribution to the electron electric dipole moment as  $d_e^{(\text{ij})}$ , we then have [8]

$$d_e^{(\text{ij})} \sim \frac{eJ_{ij}^{(\ell)}}{256\pi^2} \cdot \frac{\alpha^2 m_e m_i m_j (m_i^2 - m_j^2)}{s_W^4 M_W^6} \cdot F(m_i^2, m_j^2, m_\ell^2) \quad , \quad (16)$$

where  $F$  is a dimensionless function of the fermion (leptons, neutrinos) masses. Inserting the current upper limit on the Majorana neutrino mass,  $(190 - -450) \times 10^{-3} \text{eV}$  [13], into the above

<sup>8</sup> For a New Physics model which employs just one fermion family, see Ref. [14].

formula results in a value for  $d_e^{(ij)}$  far tinier than anything encountered thus far.

### E. Final Comments

The current limit  $d_e^{(\text{expt})} < 8.7 \times 10^{-29}$  of Eq. (7) represents an impressive improvement on previous results, made possible by probing an electron contained within a polar molecule. Experimental work continues along this line and an order-of-magnitude improved bound may not be beyond reach.

Even so, the preceding sections, which consider estimates of  $d_e^{(\text{SM})}$  within various scenarios, yield predictions orders of magnitude beneath this. Thus, a signal detected for the electron electric dipole moment in any near-term experiment would presumably have an origin beyond the Standard Model.

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