Introduction to Set Theory

LING 510: Introduction to Semantics
Lab 1: September 9, 2013

Elizabeth Bogal-Allbritten
eba@linguist.umass.edu
Office: South College 311
Office Hours: Wednesdays, 1:15 – 3:15
(or by appointment)

- I'm a PhD candidate in linguistics.
- My research is on semantics. I'm most interested in the semantics of adjectives and comparison across languages, and the semantics of modals (*maybe*, *might*, *must*, etc.).
- I also do semantic fieldwork on Navajo.

What is this "lab" business?

•The lectures are for introducing bigger ideas, the lab is for implementing and practicing those ideas.

•We'll practice the formal tools and notation that semanticists (i.e., you and me) use.

•We also may discuss the homework assignments in greater depth once they've been returned to you.

To do by Thursday:

•Read chapter 1 of Partee *et al*. (*Mathematical Methods in Linguistics*)

•Read scanned section from Chierchia & McConnell-Ginet

•Read scanned section from Portner

***Complete Assignment 1, due Sept. 12 ***

What we're doing today: Set theory!

•This will help you do the first part of your homework assignment.

•If you've read Partee *et al.* chapter 1 already, this will be review.

•You may have come across set theory in math classes in high school or college. I've included linguisticky asides to show how this formalism is relevant to semantics and linguistics.

What is a set?

•**Set:** An abstract collection of objects…

- •…that are not ordered, and…
- •…which are all different from one another.

•The objects in a set are called the **members** of that set.

- •Members of a set can be concrete (a cat, DuBois library, my left ear,…) or abstract (the number 3, the phoneme /p/, the Navajo language, …)

What is a set?

94

What do we know about this set?

•All of the members are different from one another (each member only occurs once).

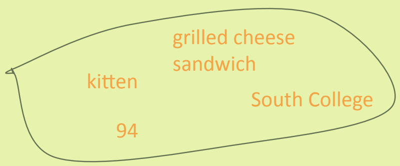
•There are four members.

•These members aren't ordered in any way.

A faster way to write down a set…

•Although the members of a set are the actual objects, we don't want to spend all of our time finding kittens, grilled cheese sandwiches, and South College and pinning them to paper.

…so as a shortcut, we'll just write down the words that refer to the set's members.



List notation

•When we ask you write down a set, we'll want you to write it using list notation. This is just a nice, convenient way to represent a set.

•Put the members between braces/curly brackets { }. Put a comma between each member.

•Order does not matter! Just pick an order and write your set down.

{kitten, grilled cheese sandwich, 94, South College}
= {grilled cheese sandwich, South College, 94, kitten}
= {South College, grilled cheese sandwich, kitten, 94}
= etc.

Referring to sets

Semanticists use capital letters to refer to sets.

S = {kitten, grilled cheese sandwich, 94, South College}
'The set S whose members are kitten, grilled cheese sandwich, 94, and South College.'

A = {Ilaria, Elizabeth}
'The set A whose members are Ilaria and Elizabeth}

The size of sets

•Sets can be finite. Finite sets can be large or small.

•What is an example of a large but finite set?
•What is an example of a small but finite set?
•What is an example of a singleton set (AKA, a set with just one member)?

•Sets can also be infinite.

•What is an example of an infinite set?

Predicate Notation

•The list notation I just showed you will be sufficient if the set is finite and small.

- •For very large sets (e.g., the set of all students at UMass), this notation will be unwieldy.
- •…and if the set is infinitely large, then we simply can't list all the members.

•…So we use a different notation – **predicate notation** – instead.

Predicate Notation

•Predicate notation specifies the conditions that need to be met in order for something to be a member of a set.

B = {*x* | *x* is a current instructor of 510}
'The set B of all *x* such that *x* is a current instructor of 510'

C = {*y* | *y* is a prime number}
'The set C of all *y* such that *y* is a prime number'

Predicate Notation

S = { *x* | *x* is a student at UMass}

Variable: The thing to the left of the |. *x* is the variable above.
(choice of letter for the variable is not important)

Predicate: The thing to the right of the |. It is what must be the case of *x* in order for that *x* to be a member of the set.

The variable is what the predicate **applies to**.

Note: Sometimes you'll see : instead of |. They're used interchangeably.

Predicate Notation

The predicate can be complex. For instance:

{*x* | *x* is a book and Mary owns *x*}
'The set of all *x* such that *x* is a book and Mary owns *x*'
AKA, 'The set of all *x* such that *x* is a book that Mary owns'

{*z* | *z* is a UMass student and *z*'s advisor is Ilaria}
'The set of all *z* such that *z* is a UMass student and Ilaria is *z*'s advisor'
AKA, 'The set of all *z* such that *z* is a UMass student whose advisor is Ilaria'

Linguisticky Aside

Other than being more concise than set notation, why else might predicate notation be preferable to set notation?

Answer: Using predicate notation allows us to be more general than set notation does.

Linguisticky Aside

Other than being more concise than set notation, why else might predicate notation be preferable to set notation?

Answer: Using predicate notation allows us to be more general than set notation does.

For certain predicates, the individuals which instantiate these predicates change from situation to situation.

Writing {*z* | *z* is a current instructor of 510} will always deliver us the right set of people.

If we write {Ilaria, Elizabeth}, this will not be the right set of people next semester.

Notation: The empty set

There exists a set without any members at all. We call this the empty set and write it with the symbol Ø.

{*z* | *z* is a mermaid} = Ø

{*y* | *y* is Elizabeth BA's brother} = Ø

The empty set is written as Ø, without curly brackets around it. This is a notational convention.

What about {Ø}? What is this?

Notation: The empty set

There exists a set without any members at all. We call this the empty set and write it with the symbol Ø.

{*z* | *z* is a mermaid} = Ø

{*y* | *y* is Elizabeth BA's brother} = Ø

The empty set is written as Ø, without curly brackets around it. This is a notational convention.

What about {Ø}? What is this?

That is the set whose only member is the empty set.

Notation: Set Membership

We have said that sets have members. The symbol for ‘is a member of’ is $\in$

kitten $\in$ {kitten, grilled cheese sandwich, South College, 94}
‘kitten is a member of the set whose members are kitten, grilled cheese sandwich, South College, and 94.’

Ilaria $\in$ {z | z is a current instructor of 510}
How would I write this out in prose? How else could I write it?

Notation: Set Membership

The symbol for ‘is not a member of’ is $\notin$

tennis shoes $\notin$ {kitten, South College, 94, grilled cheese sandwich}

Barack Obama $\notin${z | z is an instructor of 510}

Be careful!

Is the following true?

{kitten} $\in$ {kitten, grilled cheese sandwich, South College, 94}

What about the following?

{Ilaria} $\in$ {z | z is an instructor of 510}

Be careful!

Neither is true!

{Ilaria} is a set. The set {Ilaria} is not a member of the set of instructors of 510.

However, {Ilaria} would be a subset of the set of instructors of 510.

Subsets

The symbol for 'is a subset of or equal to' is $\subseteq$.

Subset: A set A is a subset of a set B if and only if **every member** of A is also a member of B.

$\not\subseteq$ means 'not a subset of'

Subsets

The symbol for 'is a subset of or equal to' is $\subseteq$.

Subset: A set A is a subset of a set B if and only if **every member** of A is also a member of B.

$\not\subseteq$ means 'not a subset of'

{kitten} $\subseteq$ {kitten, grilled cheese sandwich, South College, 94}

'The set {kitten} (AKA, 'the set whose member is kitten') is a subset of the set whose members are kitten, grilled cheese sandwich, South College, and 94.'

Subsets

Are the following true?

{kitten, South College} $\subseteq$ {kitten, grilled cheese sandwich, South College, 94}

{Ilaria} $\subseteq$ {z | z is an instructor of 510}

Subsets

Are the following true?

{kitten, South College} $\subseteq$ {kitten, grilled cheese sandwich, South College, 94} **True!**

{Ilaria} $\subseteq$ {z | z is an instructor of 510}

Subsets

Are the following true?

{kitten, South College} ⊆ {kitten, grilled cheese sandwich, South College, 94} **True!**

{Ilaria} ⊆ {*z* | *z* is an instructor of 510} **True!**

Subsets

Given our definition of subset, ⊆ can also be true if the two sets are equal.

Subset: A set A is a subset of a set B if and only if **every member** of A is also a member of B.

{Ilaria, Elizabeth} ⊆ {*x* | *x* is a current instructor of 510}

{Ilaria, Elizabeth} = {*x* | *x* is a current instructor of 510}

Be careful (again)!

Is the following true?

{Ilaria, Elizabeth} ∈ {South College, kitten, {Elizabeth, Ilaria}, 94}

Be careful (again)!

Is the following true?

{Ilaria, Elizabeth} ∈ {South College, kitten, {Elizabeth, Ilaria}, 94}

Yes, yes it is true!

A set is an abstract collection of any objects – either concrete or abstract – whose order does not matter. A set can be a member of an another set.

Still be careful!

Let S = {South College, kitten, {Elizabeth, Ilaria}, 94}

Is the following true:

{Ilaria, Elizabeth} $\subseteq$ S

Still be careful!

Let S = {South College, kitten, {Elizabeth, Ilaria}, 94}

Is the following true:

{Ilaria, Elizabeth} $\subseteq$ S

No, no it is not true!

The set {Ilaria, Elizabeth} is a member of S, not a subset of it. Remember that Ilaria and Elizabeth are not, on their own, members of S.

Still be careful!

Let S = {South College, kitten, {Elizabeth, Ilaria}, 94}

What about the following, is it true?

{ {Ilaria, Elizabeth} } $\subseteq$ S

Still be careful!

Let S = {South College, kitten, {Elizabeth, Ilaria}, 94}

What about the following, is it true?

{ {Ilaria, Elizabeth} } $\subseteq$ S

Yes, yes it is true!

The set { {Ilaria, Elizabeth} } is a subset of S.

{ {Ilaria, Elizabeth} } is a set with just one member. This member is the set {Ilaria, Elizabeth}. And if we look in S, we find that set.

A note on the empty set…

Remember the empty set?

{z | z is a mermaid} = Ø
{y | y is Elizabeth BA's brother} = Ø

The empty set Ø is a subset of any set.

The empty set is written as Ø, without curly brackets around it. This is a notational convention.

What about {Ø}? What is this?

A note on the empty set…

Something to remember: The empty set is a subset of any set.

Ø ⊆ {x | x is a human}

Ø ⊆ {1, 2, 3}

Ø ⊆ { {Ilaria, Elizabeth}, {kitten, house} }

Doing things with sets

We can add numbers together, or subtract them, or count them…

Doing things with sets

We can add numbers together, or subtract them, or count them…

Similarly, we can perform various operations on sets.

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

{1, 2, 3} ∪ {kitten, house, {94} } =

{*z* | *z* is a current instructor of 510} ∪ {*y* | *y* is a member of the US government executive branch} =

{1, 2, 3} ∪ {2, 3, 4} =

{1, 2, 3} ∪ Ø =

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

{1, 2, 3} ∪ {kitten, house, {94} } = {kitten, 1, 2, 3, {94}, house}

{*z* | *z* is a current instructor of 510} ∪ {*y* | *y* is a member of the US government executive branch} =

{1, 2, 3} ∪ {2, 3, 4} =

{1, 2, 3} ∪ Ø =

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

{1, 2, 3} ∪ {kitten, house, {94} } = {kitten, 1, 2, 3, {94}, house}

{*z* | *z* is a current instructor of 510} ∪ {*y* | *y* is a member of the US government executive branch} = {Ilaria, Elizabeth, Barack Obama, Joe Biden}

{1, 2, 3} ∪ {2, 3, 4} =

{1, 2, 3} ∪ Ø =

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

{1, 2, 3} ∪ {kitten, house, {94} } = {kitten, 1, 2, 3, {94}, house}

{*z* | *z* is a current instructor of 510} ∪ {*y* | *y* is a member of the US government executive branch} = {Ilaria, Elizabeth, Barack Obama, Joe Biden}

{1, 2, 3} ∪ {2, 3, 4} = {1, 2, 3, 4}

Delete any duplicates!

{1, 2, 3} ∪ Ø =

Set Union

Set union: Take two sets and construct the set that contains all the members of both sets.

Set union is written with the symbol ∪

{1, 2, 3} ∪ {kitten, house, {94} } = {kitten, 1, 2, 3, {94}, house}

{*z* | *z* is a current instructor of 510} ∪ {*y* | *y* is a member of the US government executive branch} = {Ilaria, Elizabeth, Barack Obama, Joe Biden}

{1, 2, 3} ∪ {2, 3, 4} = {1, 2, 3, 4}

Delete any duplicates!

{1, 2, 3} ∪ Ø = {1, 2, 3}

Set Intersection

Set intersection: Take two sets and make the new set consisting of only the members that they share.

Set intersection is written with the symbol ∩.

Set Intersection

Set intersection: Take two sets and make the new set consisting of only the members that they share.

Set intersection is written with the symbol ∩.

{1, 2, 3, 4} ∩ {3, 4, 5} =

{*z* | *z* is a current instructor for 510} ∩ {*y* | *y* is Italian} =

Set Intersection

Set intersection: Take two sets and make the new set consisting of only the members that they **share.**

Set intersection is written with the symbol $\cap$.

$\{1, 2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$

$\{z \mid z$ is a current instructor for 510$\} \cap \{y \mid y$ is Italian$\}$ =

Set Intersection

Set intersection: Take two sets and make the new set consisting of only the members that they **share.**

Set intersection is written with the symbol $\cap$.

$\{1, 2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$

$\{z \mid z$ is a current instructor for 510$\} \cap \{y \mid y$ is Italian$\}$ = {Ilaria}

Set Intersection

What happens if the sets share no members?

$\{1, 2, 3\} \cap \{5, 6, 7\}$ =

$\{ z \mid z$ is a current instructor for 510$\} \cap \{x \mid x$ is French$\}$ =

Set Intersection

What happens if the sets share no members?

Then we get the empty set!

$\{1, 2, 3\} \cap \{5, 6, 7\} = \emptyset$

$\{ z \mid z$ is a current instructor for 510$\} \cap \{x \mid x$ is French$\} = \emptyset$

Linguisticky Aside

Connectives are words like 'or' and 'and.' We'll talk (much) more about them later in the course.

Out of set union and set intersection, which seems intuitively related to 'or'?

...and which seems intuitively related to 'and'?

Set Difference

Set difference: Take two sets (A and B) and make a new set in which all the members shared by A and B are subtracted out.

Set difference is written with the symbol –.

Set Difference

Set difference: Take two sets (A and B) and make a new set in which all the members shared by A and B are subtracted out.

Set difference is written with the symbol –.

A = {1, 2, 3, 4}
B = {3, 4, 5}

A – B =

Set Difference

Set difference: Take two sets (A and B) and make a new set in which all the members shared by A and B are subtracted out.

Set difference is written with the symbol –.

A = {1, 2, 3, 4}
B = {3, 4, 5}

A – B = {1, 2}

Set Complement (...we're almost done)

The **complement** of a set A is everything not in A.

The complement of set A is written as A'

Set Complement (...we're almost done)

The **complement** of a set A is everything not in A.

The complement of set A is written as A'

Let A = {Ilaria, Elizabeth}.

What is A'?

Set Complement (...we're almost done)

The **complement** of a set A is everything not in A.

The complement of set A is written as A'

Let A = {Ilaria, Elizabeth}.

What is A'?

...to answer this, we need to know what is meant by "everything."

Set Complement

Formally, we talk about the '**universe of discourse**' *U*. *U* contains everything that we're concerned with in a particular context.

A' =

Set Complement

Formally, we talk about the 'universe of discourse' U. U contains everything that we're concerned with.

A' = U – A

Set Complement

Formally, we talk about the 'universe of discourse' U. U contains everything that we're concerned with.

A' = U – A

Let U = {x | x is human} and A = {y | y is over 50}

Then A' =

Set Complement

Formally, we talk about the universe of discourse, U. U contains everything that we're concerned with.

A' = U – A

Let U = {x | x is human} and A = {y | y is over 50}

Then A' = {z | z is 50 or under}

Cardinality

Cardinality: The number of members that a set has.

| A | is read: the cardinality of the set A

Cardinality

Cardinality: The number of members that a set has.

| A | is read: the cardinality of the set A

| {1, 2, 3} | = 3

| {*z* | *z* is a current instructor for 510}| = 2

| {Elizabeth, {Ilaria, Elizabeth}, iceberg} | =

Cardinality

Cardinality: The number of members that a set has.

| A | is read: the cardinality of the set A

| {1, 2, 3} | = 3

| {*z* | *z* is a current instructor for 510}| = 2

| {Elizabeth, {Ilaria, Elizabeth}, iceberg} | = 3

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \qquad \}$

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\},$ $\}$

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\}, \{2\},$ $\}$

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\}, \{2\}, \{1,2\}$ $\}$

The Power Set

The power set of a set A is the set of all the distinct subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset\}$ The empty set $\emptyset$ is a subset of any set!

The Power Set

The **power set** of a set A is the set of **all** the **distinct** subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset\}$ The empty set $\emptyset$ is a subset of any set!

Question: How can we represent the power set of set A in predicate notation?

The Power Set

The **power set** of a set A is the set of **all** the **distinct** subsets you can make out of A.

Power set of A is notated $\mathcal{P}(A)$

$\mathcal{P}(\{1,2\}) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset\}$ The empty set $\emptyset$ is a subset of any set!

Question: How can we represent the power set of set A in predicate notation?

$\mathcal{P}(A) = \{X \mid X \subseteq A \}$