Ling 510: Lab 2
Ordered Pairs, Relations, and Functions
Sept. 16, 2013

1. What we know about sets…

A set is a collection of members that are:

1. Not ordered
2. All different from one another

(1) a. A ={Elizabeth, Ilaria, 94} = {Ilaria, Elizabeth, 94} = …
b. B = {{Ilaria, Elizabeth}, {42}} = {{42}, {Ilaria, Elizabeth}}
= {{42}, {Elizabeth, Ilaria}} = …
c. C = {*x* | *x* is a student from the US}

We can perform operations on sets.

(2) a. A ∩ C =
b. A ∪ B =
c. A – C =

We can describe one set as a subset of another set.

(3) Write down the *set* of all of the *subsets* of A.

We differentiated between *subsets* and *members* of some set.

(4) Make the following true:
a. {Elizabeth, Ilaria} ∈
b. {Elizabeth, Ilaria} ⊆

2. Introducing ordered pairs and Cartesian Products

Ordered Pair: <x,y> is 'the pair where *x* comes first and *y* comes second'

Order matters!
{x,y} = {y,x} *but!* <x,y> ≠ <y,x>
So, while we won't see a set {James, James}, we can see ordered pairs <James, James>.

We aren't limited to ordered *pairs*. We also have *triples*, *quadruples*, and bigger ***n-tuples.***

<x, y, z> is an *ordered triple* <x, y, z, w> is an *ordered quadruple*
<ℵ, Ψ, δ, μ, σ, ξ> is an *ordered 6-tuple*

Now that we have the concept of ordered pairs (and n-tuples more generally), we can introduce another new operation that can be done to sets.

> **Cartesian Product:** The set of all the ordered pairs we can make where the first element is something from some set A and the second element is something from some set B.
>
> $$\mathbf{A \times B =_{def} \{\langle x,y \rangle \mid x \in A \text{ and } y \in B\}}$$

Let's see how this works…

(5) Let's say:
A = {John, Mary, James}   B = {Basil, Sybil}
A × B = { <John, Basil>, <John, Sybil>,
<Mary, Basil>, <Mary, Sybil>,
<James, Basil>, <James, Sybil> }

Notice that by taking the Cartesian product, we've made a *set*. Thus, we could also say:

(5)' A × B = { <James, Sybil>, <Mary, Sybil>,
<John, Sybil>, <James, Basil>,
<Mary, Basil>, <John, Basil> }

Watch out! Generally, A × B ≠ B × A, unless A = B (so, it's not like taking the product of two numbers).

Given the sets in (5), what is B × A?

(6) B × A =

3. Relations

3.1 Introducing relations

Ordered pairs can help us talk about *relations* (or, statements) that hold between two things. Examples of relations include *is the mother of*, *likes*, *kiss*, etc.

> **For some relation *R*, we write *Rab* (or, *aRb*) to say that the relation *R* holds from *a* to *b*.**

To see how this works, let's again take our sets A = {John, Mary, James} and B = {Basil, Sybil}.

Let's say that we know: John likes Sybil, Mary likes Sybil, and James likes Basil and Sybil.

(7) likes = { <John, Sybil>, <Mary, Sybil>, <James, Basil>, <James, Sybil> }

likes is a **relation** from A to B (or, from liker to likee)

Relations can be "related" in a particular way to Cartesian products.

We have seen that given the sets A = {John, Mary, James} and B = {Basil, Sybil} and our knowledge about who likes who, we can say the following things:

(8) a. A × B = { <John, Basil>, <John, Sybil>,
<Mary, Basil>, <Mary, Sybil>,
<James, Basil>, <James, Sybil> }

b. likes = { <John, Sybil>, <Mary, Sybil>, <James, Basil>, <James, Sybil> }

likes and A × B are both sets. Can you see a statement that can be made about them?

Generalization: Some relation R from a set A to a set B can be defined as a subset of the Cartesian product of A and B.
In other words: **For any relation R from A to B, R ⊆ A × B**

Another example to practice (from Partee et al.): is the mother of

Let's only worry about humans (which we'll call set H).

Here is what we know: Rose is the mother of Elizabeth. Mary is the mother of Rose. Blanche is the mother of Bill.

(9) a. is the mother of =

b. is the mother of ⊆ __ × __
H × H =

Linguistic Aside/Preview of Coming Attractions: What I have written in (9a) is perhaps a good first attempt at representing the things in natural language that we'll call *predicates*, which includes verbs like *likes* and bigger phrases like *is the mother of*.

Question: What about intransitive predicates (e.g., *is 10 years old*, *is human*)? What set B would we use?

3.2 Relations: some terminology

What if the relation R is between members of one set A? AKA, R is a **subset of A × A.**

- Then we say that R is **a relation in A.**

What if the relation R is between members of two sets, A and B? AKA, R is a **subset of A × B:**

- Then we say that R is **a relation from A to B**.

- **Domain of R:** The set of things x such that $x \in$ A and x is the first member of some ordered pair in R.
  - The domain of R is the things that R 'works on.'

- **Range (or, Co-domain) of R:** The set of things y such that $x \in$ B and y is the second member of some ordered pair in R.
  - The range of R is the things that R 'maps things onto.'

(10) *Practice with domains and ranges*

a. Given the relation is the mother of in (9), what is the domain of is the mother of? What is the range of is the mother of?

Domain:
Range:

b. Given the relation likes in (8), what is the domain of likes? What is the range of likes?

Domain:
Range:

Relations in pictures:

The following is a picture of the relation R, which is a relation from A to B.

Domain *Range/Co-domain*

Set A *Set B*

A = {a,b} B = {c, d, e}

R = { <a,d>, <a, e>, <b, c> }

4. Functions

A *function* is a special type of *relation*. We'll get back to how they're special momentarily.

4.1 Functions: a little preliminary terminology

Because functions are special kinds of relations, we can use the same terminology for functions as we did for relations.

Given a function F such that $F \subseteq A \times B$:

- A is the *domain* and B is the *range* (like we said for relations).
- We can say that F is a function *from A to B* (like we said for relations).
- We can write $F : A \rightarrow B$ to mean 'F is a function from A to B.'
- The things in the domain can be called **arguments**. The things in the range can be called **values**.

4.2 Introducing functions

Function: A relation R from A to B is a **function from A to B** if and only if:

- No member of A is paired with more than one member of B.
  - AKA: For $a \in A$ and $b \in B$, R cannot contain both <a,b> and <a,c> … unless b = c.
  - AKA: The *domain* of R contains each A just once.

General strategy: Some relation R is a function if the relation the number of ordered pairs in the relation is no bigger than the size of R's domain.

⇒ **<u>We have *two types* of functions:</u>**

- **Total function:** Every member of A is 'targeted' by R.
  - AKA: Every member of A appears on the lefthand side of an ordered pair of R.
  - AKA: The domain of R is equal to the domain of A.
- **Partial function:** *Not* every member of A is 'targeted' by R.
  - AKA: Some members of A are not paired with any member of B.
  - AKA: The domain of R is smaller than the domain of A.

(11) *Let’s assume the following sets:*

A = {John, Mary, James} B = {Basil, Sybil, Polly}

Are the following relations functions? If so, are they total or partial?

a. P ={ <John, Basil>, <Mary, Sybil>, <James, Polly> }

b. O ={ <John, Basil>, <Mary, Basil>, <James, Polly> }

c. Q ={ <John, Basil>, <Mary, Sybil> }

d. S = { <John, Basil>, <Mary, Basil> }

e. T = { <John, Sybil>, <John, Basil>, <Mary, Sybil>, <James, Polly> }

f. V = { <Mary, Basil>, <Mary, Sybil>, <James, Polly> }

Let’s look at some natural language functions, now.

(12) Is the relation was given birth to by a function? If so, is it a partial or a total function? Assume that was given birth to by is a relation from H (the set of humans) to H.

Terminology: And what are the arguments of was given birth to by? What are its values?

(13) Is the relation is the biological mother of a function? If so, is it a partial or a total function?

Again, assume that is the biological mother of is a relation from H (the set of humans) to H.

4.3 Some remaining terminology for functions

Recap from §4.1 and §4.2:

- We know what a *domain* is and that the things in the domain are called *arguments*.

- We know what a *range* is and that the things in the range are called *values*.

- We can say that F is a function *from A to B* (like we said for relations). We can write this F : A $\rightarrow$ B.

- We have already seen *total* vs. *partial* functions.

One-to-one function: A function F is one-to-one is no member of the range (AKA, no value) is assigned to more than one member of the domain (AKA, no argument).

Are the functions in (14a-c) one-to-one?

(14) A = {John, Mary, James} B = {Basil, Sybil, Polly}

a. A = { <James, Basil>, <Mary, Sybil>, <John, Polly> }

b. B = { <James, Basil>, <Mary, Sybil> }

c. C = { <James, Basil>, <Mary, Basil>, <John, Sybil> }

Functions *onto* some range: A function F is *onto* B if and only if every member of the range is the value of some element in the domain of F.

Are the functions in (15a-b) functions onto B?

(15) A = {John, Mary, James, Ted} B = {Basil, Sybil, Polly}

a. A = { <James, Basil>, <Mary, Sybil>, <John, Polly> }

b. B = { <James, Basil>, <Mary, Sybil>, <John, Basil>, <Ted, Sybil> }

A question that one might have: If a function is *one-to-one*, does it also have to be *onto*?

Another question that one might have: If a function is *onto*, does it also have to be *one-to-one*?