

General Relativity as a Quantum Field Theory / QFT 2

Note Title

2/10/2014

GR is the most interesting QFT

Goal : Treat GR like other fundamental interactions — QFT

Goal #2 : Advanced QFT methods

Course website : blogs.umass.edu/grft/

Note : "Trust, but verify"

Preliminary list of topics

General Relativity QFT

Constructing GR through gauge invariance
The weak field limit and tree level Feynman rules

Path integral quantization of General Relativity

General relativity as an effective field theory
Fermions, torsion, Holst and all that

Anomalies general relativity

Solutions to the wave equations in curved space
Quantum fluctuations during inflation
Black holes and Hawking radiation

Non-local effective actions general relativity

(If time) Gravity as the square of a gauge theory

QFT II

Yang Mills

Some field theory techniques:

Heat kernel methods
Background field method
Gauge fixing and ghosts

Effective field theory

Anomalies in gauge theory

(If time) Schwinger-Keldysh, in-in or
closed time path formalism

Non-local effective actions

(If time) Spinor helicity techniques

Gravity

$$\nabla(r) = -G \frac{m_1 m_2}{r}$$

$$\leftarrow h = c = 1$$

$$G = \frac{1}{M_p^2} = (1.22 \times 10^{19} \text{ GeV})^{-2}$$

$$\text{Reduced } M_p = \frac{1}{\sqrt{8\pi G}} = (2 \times 10^{18} \text{ GeV})^{\frac{1}{2}}$$

Higgs?

$$\mathcal{L} = -\frac{m_i}{\sqrt{2}N} \bar{\psi} \psi H$$

$$\begin{array}{c} m_1 \\ \hline m_2 \end{array} \xrightarrow{H} -im = -\frac{im_1}{\sqrt{2}N} \frac{i}{g^2 - m_H^2} -\frac{im_1}{\sqrt{2}N}$$

$$\text{FT.} \Rightarrow \nabla(r) = -\underbrace{\frac{1}{8\pi N^2} \frac{m_1 m_2}{r}}_G e^{-m_H r}$$

$m_H \rightarrow 0$

Work?

- Quark masses $\ll M_{N,p}$ Trace anomaly

$$M_p = \langle p | \bar{T}_\mu^\mu | p \rangle = \underbrace{\langle p | \beta F^2}_{900 \text{ MeV}} + \underbrace{m_u \bar{u} u + m_d \bar{d} d}_{40 \text{ MeV}} | p \rangle$$

- $B.E. = 10 \frac{\text{MeV}}{\text{Nucleon}}$

- Photons don't couple

- Momentum $\Delta E \sim \frac{h}{\Delta t} \sim \frac{1}{\Delta R}$

Equivalence Princ

$$F = M_G g = M_{\perp} a \implies g = a$$

$$\text{but } E = mc^2 = \sqrt{p^2 + m_{\perp}^2}$$

↳

1) \implies Focus on total energy, \vec{p} $\leftarrow *$

2) No combination of couplings of scalar works

$$\phi F^2$$

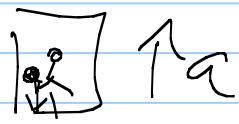
$$\sim \alpha E^2 B^2 \rightarrow 0$$

]

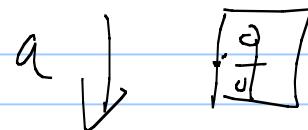
Conceptually



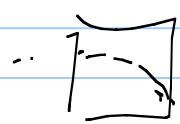
\Rightarrow



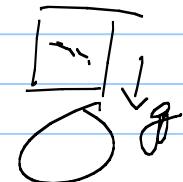
Also
freefall



also light



\Rightarrow



B) Gravity like non-inertial frame

4) + Coordinate system with no ^{local} grav effects (free fall) -

Energy + Momentum

E, p associated with time, space translations *

Rel. Theory E + Mom tensor $T_{\mu\nu}$

$$H = \int d^3x T_{00}, \quad \vec{p} = \int d^3x T_{01}$$

Conserved $\partial^\mu T_{\mu\nu} = 0 \Leftarrow 4 \text{ cons.}$

* Noether's Thm

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial_\nu \phi - g_{\mu\nu} \mathcal{L}$$

Example $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2)$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial_\lambda \phi \partial^\lambda \phi - m^2 \phi^2)$$

Cons.

$$\partial^\mu T_{\mu\nu} = \cancel{[(\partial^\mu \partial_\nu \phi - \partial_\lambda \phi \partial_\nu \partial^\lambda \phi + m^2 \phi \partial_\nu \phi)} = 0$$

$$+ \cancel{\partial_\mu \phi \partial^\mu \partial_\nu \phi}$$

Reasonable to make $T_{\mu\nu}$ source

$$\frac{T_{\mu\nu}}{T_{\mu\nu}} \sim K T_{\mu\nu} \frac{1}{g^2} K T_{\mu\nu} \sim K \frac{M_1 M_2}{R}$$

Note on norm

$$\langle p | p' \rangle = \int_{(2\pi)^3}^{(2\pi)^3} d^3p \frac{1}{2E} \delta^3(p - p')$$

$$\phi(x) = \int_{(2\pi)^3} d^3p \frac{1}{2E} \left(a(p) e^{-ip \cdot x} + a^\dagger(p) e^{+ip \cdot x} \right)$$

$$H = \int_{(2\pi)^3} d^3x T_{00} = \int_{(2\pi)^3} d^3p E_p a^\dagger(p) a(p) + \dots$$

$$\langle p' | T_{\mu\nu} | p \rangle = \frac{1}{16\pi^2 2E} \left[(p_\mu p'_\nu + p'_\mu p_\nu) - g_{\mu\nu}(p \cdot p' - m^2) \right]$$

↑ don't write (goes with $\int_{(2\pi)^3} d^3p \frac{1}{2E} (1/16\pi^2)$)

$T_{\mu\nu}$ as source:

Gauge symmetries

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu - m)\psi$$

Global invariance $\psi \rightarrow \psi' = e^{i\theta}\psi$

Noether $J_\mu = \bar{\psi} \gamma^\mu \psi$ $\partial^\mu J_\mu = 0$

$$Q = \int d^3x J_0 \rightarrow \int \frac{d^3p}{(2\pi)^3} (b^\dagger b - d^\dagger d)$$

$Q|f\rangle = +|f\rangle$
 $Q|\bar{f}\rangle = -|\bar{f}\rangle$

To make J_μ a source - make gauge theory (QED)

$$\psi' = e^{-i\theta(x)}\psi, \quad A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta$$

Covariant deur $D_\mu = \partial_\mu + i\epsilon A_\mu$

$$D_\mu \psi \rightarrow D'_\mu \psi' = e^{-i\theta(x)} D_\mu \psi$$

$$= \left[\partial_\mu + i\epsilon \left(A_\mu + \frac{1}{e} \partial_\mu \theta \right) \right] e^{-i\theta(x)} \psi$$

$$\mathcal{L} = \bar{\psi} (i D - m) \psi \quad \text{invariant}$$

$$D_\mu \Rightarrow D'_\mu = e^{-i\theta} D_\mu e^{+i\theta}$$

$$[D_\mu, D_\nu] = i\epsilon (\partial_\mu A_\nu - \partial_\nu A_\mu) = i\epsilon F_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{4} \underbrace{F_{\mu\nu} F^{\mu\nu}} + \overline{\psi} (iD - m) \psi$$

$\overbrace{\overbrace{\overline{\psi} (iD - m) \psi}^{\text{mass term}} - e A_\mu J^\mu}$

↑ source of A_μ

$T_{\mu\nu}$ as source \Rightarrow gauge space time translation

Yang Mills $SU(N)$ gauge theory

Take $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$

Look for invariance $\psi \rightarrow U\psi$

$$U = e^{-i\frac{\lambda^A}{2}}$$

Hermitian
✓

$\curvearrowleft N \times N$ matrix-unitary

$$U = \exp \left\{ -i \left(\chi_0 + \alpha^A \frac{\lambda^A}{2} \right) \right\} \quad \lambda^A \quad N \times N \text{ Hermitian, traceless}$$

\uparrow

$U(1)$ phase \Rightarrow drop

basis

$$\left[\frac{\lambda^A}{2}, \frac{\lambda^B}{2} \right] = i f^{ABC} \frac{\lambda^C}{2}$$

$$\text{Tr} \frac{\lambda^A}{2} \frac{\lambda^B}{2} = \frac{1}{2} \delta^{AB}$$

Gauge symmetry

$$\psi \rightarrow \psi' = u(x) \psi$$

$$D_m \psi \rightarrow D'_m \psi' = u(x) D_m \psi$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \mathcal{L} = \overline{\psi} (iD - m) \psi$$

$$D_m = \partial_m + ig \frac{\lambda^A}{2} A_m^A = \partial_m + ig \underline{A}_m \quad \underline{A}_m = \frac{\lambda^A A_m^A}{2}$$

Then if $\underline{A}_m \rightarrow \underline{A}'_m = u \underline{A}_m u^{-1} + \frac{i}{g} (\partial_m u) u^{-1}$

we have

$$D'_m \psi' = \left[\partial_m + ig(u \underline{A}_m u^{-1} + \frac{i}{g} (\partial_m u) u^{-1}) \right] u \psi$$

$$= U(x) D_\mu \psi$$

$$D'_\mu = U D_\mu U^{-1}$$

To complete

$$[D_\mu, D_\nu] \psi = ig \underline{F}_{\mu\nu} \psi = ig \frac{1}{2} F^A_{\mu\nu} \psi$$

$$\underline{F}_{\mu\nu} = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu + ig [\underline{A}_\mu, \underline{A}_\nu]$$

$$\underline{F}^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu$$

$$\left. \begin{aligned} \mathcal{L} &= -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} \\ &+ \overline{\psi} (i \not{D} - m) \psi \end{aligned} \right\}$$

Preview

- gauge Poincaré trans.
- new field $g_{\mu\nu}(x)$
- covariant deriv D_μ
- invariant matter action $\sum \frac{\delta}{\delta g^{\mu\nu}} L_m = \# T_{\mu\nu}^{\text{matter}}$
- $[D_\mu, D_\nu] \sim R_{\mu\nu} - \dots$
- $L_{\text{grav}} \quad \frac{\delta}{\delta g_{\mu\nu}} L_{\text{grav}} = G_{\mu\nu} \implies G_{\mu\nu} = \# T_{\mu\nu}$
 $\Rightarrow \boxed{GR}$

Lorentz invariance review

Coord change $x^m \rightarrow x'^m = \Lambda_{\nu}^m x^{\nu}$

Invariant $x_\mu x^\mu = \eta_{\mu\nu} x^\mu x^\nu$ $\eta_{\mu\nu} = \begin{pmatrix} 1 & -1 & & \\ -1 & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$$\begin{aligned} &\rightarrow \Lambda_{\alpha}^m \Lambda_{\beta}^{\nu} \eta_{\mu\nu} x^\alpha x^\beta \\ &= \eta_{\alpha\beta} x^\alpha x^\beta \end{aligned}$$

$$\Rightarrow \underbrace{\Lambda_{\alpha}^m \Lambda_{\beta}^{\nu} \eta_{\mu\nu}}_{\eta_{\alpha\beta}} = \eta_{\alpha\beta}$$

$$x_\mu = \eta_{\mu\nu} x^\nu \quad -x'_\mu = \Lambda_\mu^\nu x_\nu$$

$$\underbrace{\Lambda_\mu^{\;\nu}}_{\equiv} \underbrace{\Lambda^\mu_{\;\nu}}_{\equiv} = \delta^\nu_\mu$$

Field transform $\phi(x) \rightarrow \phi'(x') = \phi(x)$

$$A^\mu \rightarrow A^{\mu'}(x') = \Lambda^\mu_{\;\nu} A^\nu(x)$$

Also

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\partial_t, \vec{\nabla}) \quad \text{such that } \partial_\mu x^\mu = 1 + 1 + 1 + 1 = 4$$

$$- \partial_\mu x^\mu = \partial_t(x^\alpha \eta_{\alpha\beta} x^\beta) = 2 x_\mu$$

$$\partial^\mu = \eta^{\mu\nu} \partial_\nu = (\partial_t, -\vec{\nabla})$$

Can include rotations $\Lambda^\mu_{\;\nu} = \begin{pmatrix} 1 & 0 \\ 0 & R_{ij} \end{pmatrix}$ + translations $x'^\mu = \Lambda^\mu_{\;\nu} x^\nu + a^\mu$