

GR QFT

March 12

Note Title

2/10/2014

First organizing principle - locality

- high energy local
- low energy - not $m_0 = \ln \delta^2$

$$m_0 \sim \bar{T}_{\mu\nu} = (g_\mu g_\nu - g_{\mu\nu} \delta^2) \frac{\alpha}{16\pi} \frac{q^2}{M_H^2}$$

$$L_{eff} = \frac{\alpha}{16\pi M_H^2} F_{\mu\nu} \square F^{\mu\nu}$$

\Rightarrow $\square F^{\mu\nu} = -i \cdot M = i$

Also

$$\bar{T}_{\mu\nu}$$

Euler Heisenberg

$$L_{eff} = \frac{\alpha^2}{96 M_H^4} \left[(F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu} \hat{F}^{\mu\nu})^2 \right]$$

$$\hat{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Integrating out heavy field

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_h^2 \phi^2) + \phi F(\psi) + \underbrace{\mathcal{L}(\psi)}_{\text{light}}$$

$$\begin{aligned} Z &= \int d\phi d\psi e^{i \int d^4x [\mathcal{L}(\phi) + \mathcal{L}(\phi, \psi) + \mathcal{L}(\psi)]} \\ &= \int d\psi e^{i \mathcal{L}(\psi)} \underbrace{\left[\int d\phi e^{i \int d^4x [\mathcal{L}(\phi) + \phi F(\psi)]} \right]}_{Z_1(\psi)} \end{aligned}$$

Complete the square

$$\mathcal{L}(\phi) + \phi F(\psi) = -\frac{1}{2} \phi (\square + m^2) \phi + \phi F(\psi)$$

$$\text{Let } (\square + m^2) D_F(x-y) = -\delta^4(x-y)$$

$$\tilde{\phi} = \phi(x) + \int d^4y D_F(x-y) F(y)$$

Then

$$-\frac{1}{2} \tilde{\phi} (\square + m^2) \tilde{\phi} = -\frac{1}{2} \phi (\square + m^2) \phi + \frac{1}{2} \star^2 \phi F(y) + \frac{1}{2} \int d^4y F(\psi_x) D_F(x-y) F(\psi_y)$$

$$Z[\psi] = \int d\tilde{\phi} e^{-i \int d^4x \frac{1}{2} \tilde{\phi} (\square + m^2) \tilde{\phi}} e^{-\frac{i}{2} \int d^4x d^4y F(\psi_x) D_F(x-y) F(\psi_y)}$$

$$d\phi = d\tilde{\phi}$$

$$Z = \int d\psi e^{i \int d^4x L(\psi)} \int \frac{1}{2} \langle FDF \rangle$$

Now locality

$$D_F(x-y) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-iq \cdot (x-y)}}{q^2 - M_H^2} = \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot (x-y)} \left[\frac{-1}{M_H^2} - \frac{q^2}{M_H^4} + \dots \right]$$

$$= \left(-\frac{1}{M_H^2} + \frac{\square}{M_H^4} + \dots \right) \underbrace{\int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot (x-y)}}_{\delta^4(x-y)}$$

$$\mathcal{L} = \mathcal{L}(\psi) + \frac{1}{2} F(\psi) \perp \frac{1}{M_H^2} F - \frac{1}{2M_H^4} F(\psi) \square F(\psi) + \dots$$

Light particles not local

Local

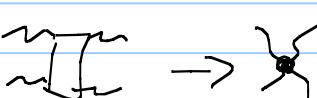
$$D_F(x-y)|_{m=0} = \frac{1}{16\pi^2} \frac{1}{(x-y)^2 - i\varepsilon}$$

Second organizing principle Energy expansions

- powers of $\frac{E}{M_H^2}$ }

But tree level is not all:

- Effective Z (operator)

EFT is loops also 

$$\text{F_finite} = \text{I}_{\text{cut}} + \text{II}_{\text{cut}}$$

Third org. principle - Loops, renormalization matching/measuring

A) Divergences from High E

- not reliable

- but local

\Rightarrow something in L to renormalize

B) When you renormalize you measure

B1) Known HE theory - express parameters in terms of fundamental one
(Matching) *

B2) Unknown theory - measure directly

C) Separate HE parts (local) from nonlocal (Low E)

Predictions live here

Linear sigma model

- construct EFT by hand

- EFT closest in style to GR

$$(\sigma, \vec{\pi})$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_0 + i\phi_2 \end{pmatrix}$$

$$\begin{pmatrix} \vec{\pi} \\ N \end{pmatrix}$$

$$\tilde{\zeta}^i = \sigma^i$$

$$\mathcal{L}_\sigma = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + \frac{m^2}{2} (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + \bar{\psi}_1 \not{D} \psi_1 + \bar{\psi}_2 (\sigma + i \vec{\zeta} \cdot \vec{\pi}) \not{D} \psi_2$$

$$\boxed{\mathcal{L}_{eff} = \frac{F^2}{4} Tr (\partial_\mu U \partial^\mu U^\dagger) + \dots}$$

$$U = \exp \left(i \frac{\vec{\zeta} \cdot \vec{\pi}}{F} \right)$$

$$F = \sqrt{\frac{m^2}{\lambda}}$$

$$\Sigma = \sigma + i\vec{\zeta} \cdot \vec{\pi} \quad , \quad \frac{1}{2}\text{Tr}(\Sigma^+ \Sigma) = \frac{1}{2}\sigma(\sigma^2 + (\vec{\zeta} \cdot \vec{\pi})^2) = \sigma^2 + \vec{\pi} \cdot \vec{\zeta}$$

Then

$$\mathcal{L} = \frac{1}{4}\text{Tr}(\partial_\mu \Sigma^+ \partial^\mu \Sigma) + \frac{m^2}{4}\text{Tr}(\Sigma^+ \Sigma) - \frac{\lambda}{16}[\text{Tr}(\Sigma^+ \Sigma)]^2 + \bar{\psi}_i \gamma^\mu \psi_i + \bar{\psi}_R \gamma^\mu \psi_R$$

Symmetry

$$\begin{aligned} \psi_L &\rightarrow L \xleftarrow{\text{SU}(2)} \psi_L \\ \psi_R &\rightarrow R \xleftarrow{\text{SU}(2)} \psi_R \\ \Sigma &\rightarrow L \Sigma R^+ \end{aligned}$$

$\psi_{\frac{L+R}{2}} = \frac{1}{2}(1 \pm \gamma_5)\psi$

invariant
 $\text{SU}(2)_L \times \text{SU}(2)_R$

$$\text{Tr}(\Sigma^+ \Sigma) = \text{Tr}(R \underbrace{\Sigma_L^+ \Sigma_L}_{\Sigma} R^+)$$

SSB

$$\langle \sigma \rangle = \sqrt{\frac{m^2}{\lambda}}, \quad \langle \vec{\pi} \rangle = 0$$

$\sigma \approx F$

$$\sigma = \nu + \tilde{F}$$

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left[(\partial_\mu \tilde{\sigma})^2 - 2m^2 \tilde{\sigma}^2 \right] + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \lambda \nu \tilde{\sigma} (\tilde{\sigma}^2 + \vec{\pi}^2) \\ & - \frac{\lambda}{4} (\tilde{\sigma}^2 + \vec{\pi}^2)^2 + \bar{\psi} (i \gamma^\mu - g \nu) \gamma^\mu + \bar{\psi} (g + i \vec{\epsilon} \cdot \vec{\pi}) \gamma^\mu \end{aligned}$$

To get \mathcal{L}_{eff}

$$\begin{aligned}\Sigma &= (\sigma + i \vec{\tau} \cdot \vec{\pi}) = (N + S) U ; U = \exp(i \frac{\vec{\tau} \cdot \vec{\pi}}{N}) \\ &= (N + \vec{\tau}' + i \vec{\tau} \cdot \vec{\pi}') = (N + S + i \vec{\tau} \cdot \vec{\pi}' + \dots)\end{aligned}$$

$$\tilde{\sigma} = S + \dots$$

$$\vec{\pi}' = \vec{\pi}' + \dots$$

Rewrite \mathcal{L}

$$\mathcal{L} = \frac{1}{2} [\bar{\rho}_n S]^2 - 2\mu^2 S^2] + \frac{(N+S)^2}{4} \text{Tr}(\bar{\rho}_n u^\dagger u^\dagger) - \lambda S^3 - \frac{\lambda}{4} S^4$$

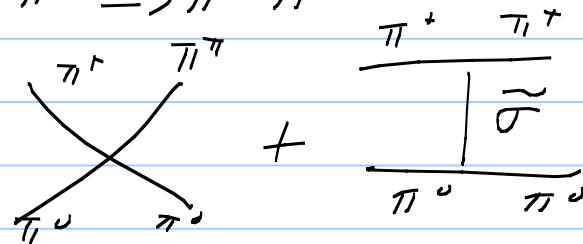
no approx

$\nwarrow \nearrow$
no π' 's

First approx drop S (Appelquist Carazzone)

$$\mathcal{L}_{\text{eff}} = \frac{n^2}{4} \text{Tr}(\partial_\mu u \partial^\mu u^+)$$

Calculate $\pi^+ \pi^- \rightarrow \pi^+ \pi^-$
Full theory

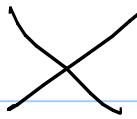


$$\mathcal{L}_I = -\frac{\lambda}{4} (\vec{\pi})^2 - \lambda n \vec{\sigma} \vec{\pi}^2$$

$$-iM = -2i\lambda + \frac{(-2i\lambda n)^2}{\vec{q}^2 - m_0^2} = -2i\lambda \left[1 + \frac{2\lambda n^2}{\vec{q}^2 - 2m_0^2} \right]$$

$$= i \frac{\vec{q}^2}{n^2}$$

Effective theory



$$+ \frac{TS}{\sim Tn(\partial_\mu \partial^\mu u^*)} \sim \frac{g^4}{M_S^4}$$

$$\mathcal{L} = \frac{1}{6N^2} \left[(\vec{\pi}' \partial_\mu \vec{\pi}')^2 - \vec{\pi}'^2 (\partial_\mu \vec{\pi}' \cdot \partial^\mu \vec{\pi}') \right]$$

← corrected
from class

$$\cancel{= i M = i \frac{g^2}{N^2}}$$

Haag's Theorem — names don't matter

$$\sigma = S + \dots$$

$$\pi = \pi' + \dots$$

\cancel{R}