

## Anomalies:

- 1) Scale  $\rightarrow$  trace anomaly \*
- 2) Conformal  $\leftarrow *$
- 3) Axial

## 4 Ways

- 1) Path integral UV
- 2) Feynman diagrams UV
- 3) Non local actions IR -
- 4) Dispersion relations IR

Ch 3 from DSM

Currents in Path integral

$$Z[J, N_m] = \int d\phi e^{i \int d^4x \left[ \mathcal{L}(\phi) + J\phi - N_m \frac{\partial}{\partial J} \right]}$$

↑  
Source  
↓  
current

\*  $\overline{J}_m(x) = \frac{1}{Z} \frac{\delta Z[J_N]}{\delta N_m(x)} \Big|_{N=0}$

↑  
all matrix elements of  $J_m$

$$\langle 0 | T(J_m \phi \phi) | 0 \rangle = i \frac{\delta^2}{\delta J(x) \delta J(y)} \overline{J}_m(z)$$

Noether current

if  $\phi \rightarrow \phi' = \phi + \varepsilon f(\phi)$  is symmetry

$$\text{convert } \mathcal{E} \rightarrow \mathcal{E}(x) , \quad J^m = \frac{\partial \mathcal{L}'(\phi', \partial_m \phi')}{\partial \partial_m \mathcal{E}}$$

$$\text{or } \mathcal{L}(\phi', \partial_m \phi') = \mathcal{L}(\phi, \partial_m \phi) + J^m \partial_m \varepsilon$$

From \*

$$\ln Z[\bar{N}_m + \delta \bar{N}_m] - \ln Z[N_m] = -i \int d^4 x \bar{J}(x) \delta N_m(x) \quad **$$

$$\delta N_m = -\partial_m \mathcal{E}$$

$$\ln Z[N_p - \partial_\mu \varepsilon] - \ln Z[N_p] = -i \int d^4x \bar{J}(x) \partial_\mu \varepsilon \\ = +i \cdot \int d^4x \varepsilon(x) \partial^\mu \bar{J}_\mu(x)$$



Test of symmetry  $Z[N_p - \partial_\mu \varepsilon] = Z[N_p] \Rightarrow \partial^\mu \bar{J}_\mu(x) = 0$

for all matrix elements

Apply symmetry test

$$Z[\nu_r - \partial_\mu \varepsilon] = S[d\phi] e^{i \int d^4x [L(\phi, \partial_\mu \phi) - (\nu_r - \partial_\mu \varepsilon) J^\mu]}$$

$\underbrace{+ \partial_\mu \varepsilon J^\mu}_{-\nu_r J^\mu}$

$$\stackrel{?}{=} Z[\nu_r]$$

$$= S[d\phi] e^{i \int d^4x [L(\phi', \partial_\mu \phi') - \nu_r J^\mu]}$$

$\stackrel{\uparrow}{=} [d\phi']$

$$= Z[\nu_r]$$

Two conditions for invariance

$$1) \mathcal{L}(\phi', \partial_\mu \phi') = \mathcal{L}(\phi, \partial_\mu \phi) + J^\mu \partial_\mu \epsilon$$

Invariance  
 $\mathcal{E}(x) \rightarrow \mathcal{E}$

$$2) [\partial \phi] = [\partial \phi']$$

      

Violated with anomalies

Classical symmetries of  $\mathcal{L}$

not symmetries of P. I.  $\Rightarrow$  not quantum symmetries

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