

# GRQFT

March 26

Note Title

27/03/2014

## EFT Exercises

### 1) U(1) effective lagrangian

Consider a theory with a complex scalar field  $\varphi$  with a  $U(1)$  global symmetry  $\varphi \rightarrow \varphi' = \exp(i\theta)\varphi$ . The lagrangian will be

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

- a) Minimize the potential to find the ground state and write out the lagrangian in the basis

$$\varphi = \frac{1}{\sqrt{2}}(v + \varphi_1(x) + i\varphi_2(x))$$

Show that  $\varphi_2$  is the Goldstone boson.

- b) Use this lagrangian to calculate the low-energy scattering of  $\varphi_2 + \varphi_2 \rightarrow \varphi_2 + \varphi_2$ . Show that despite the non-derivative interactions of the lagrangian, cancellations occur such that leading scattering amplitude starts at order  $p^4$ .
- c) Instead of the basis above express the lagrangian using an exponential basis,

$$\varphi = \frac{1}{\sqrt{2}}(v + \Phi(x))e^{i\chi(x)/v}$$

Show that in this basis a ‘shift symmetry’  $\chi \rightarrow \chi + c$  is manifest.

- d) Calculate the same scattering amplitude using this basis and show that the results agree. Note that the fact that the amplitude is of order  $p^4$  is more readily apparent in this basis.

### 2) Why is the sky blue?

Write some examples of gauge invariant effective Lagrangians for the scattering of light off of a neutral object. Use these to calculate the frequency dependence of light scattered from molecules in the sky – showing that the sky is blue.

For more detail about this calculation, see Barry R. Holstein, *Blue skies and effective interactions*, American Journal of Physics, 67, 422 (1999)

## Rules for EFT

- 1) Most general  $L$  ( $\sim$  symmetries), order by energy expansion
- 2) Quantize using lowest order
- 3) Renormalize  $\xrightarrow{R}$
- 4) Match / measure
- 5) Predictions  $\leftarrow$  low energy propagations

## GR as EFT

$$R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2 = \underset{\text{deriv}}{\text{Total}}$$

$$S = \int d^4x F_F \left[ -\Lambda - \frac{2}{k^2} R + C_1 R^2 + C_2 R_{\mu\nu} R^{\mu\nu} + C_3 R_{\mu\nu\rho\sigma} \cancel{R^{\mu\nu\rho\sigma}} + R^3 \right]$$

↑  
 C.c. (drop)      ↗ G.R.

Bound coeff. (Exercise)

$$R \sim \partial^2 h + \dots, \quad R^2 \sim \partial^2 h \partial^2 h + \dots$$

$$\left( \square + C_1 \partial^4 \right) h = T_{\mu\nu} \implies \boxed{T} = T_{\mu\nu} = \frac{1}{g^2 + Cg^4} T^{\mu\nu}$$

$$\frac{1}{g^2 + Cg^4} = \left[ \frac{1}{g^2} - \frac{1}{g^2 - m^2} \right] \xrightarrow[m^2 \approx 1/C]{} \frac{1}{r} - \frac{e^{-mr}}{r}$$

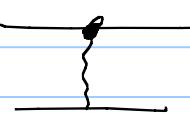
Yukawa  
limit to  $\delta(r)$

K. Stelle GRG 9 358 (1978)

Gravity works at 1 mm  $\Rightarrow$  Bound worse than  $C < 10^{50}$   
 $\ell \sim 10^{-3}$

Matter also

$$L = \frac{1}{2} \left[ (\partial_\mu \phi)^2 - m^2 \phi^2 \right] + d_1 R \phi^2 + d_2 R^{\mu\nu} \partial_\mu \phi \partial^\nu \phi + d_3 R (\partial_\mu \phi \partial^\mu \phi) + \dots$$



$$\partial^\mu \times \frac{1}{\partial} \rightarrow \delta(x)$$

# Quantize 't Hooft Veltman

One loop gravity

$$\frac{1}{16\pi^2} \frac{1}{\epsilon} \left[ \frac{1}{120} R^2 + \frac{7}{120} R_{\mu\nu} R^{\mu\nu} \right]$$

→ Energy expansion

Renormalize

$$C_1^n = C_1 + \frac{1}{16\pi^2} \left[ \frac{1}{\epsilon} + \gamma - \ln 4\pi \right] \frac{1}{120}$$

$$C_2^n = C_2 + \frac{1}{16\pi^2} \left[ \frac{1}{\epsilon} - \frac{7}{120} \right] \dots$$

Two loop (pure gravity)

$$R_{\mu\nu}, R \rightarrow 0$$

$$\frac{209}{880} \frac{1}{(16\pi^2)^2} \frac{1}{\epsilon} R^{\mu\nu\rho\sigma} R_{\alpha\beta\gamma\delta} R^{\gamma\delta}_{\mu\nu}$$

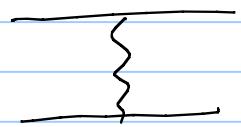
Energy expansion

Power counting

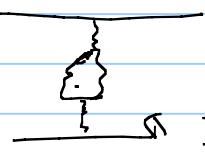
$$\left. \begin{array}{l} E^2 \sim R \text{ at tree level} \\ E^4 \sim R^2 \text{ at tree or } R \text{ at 1 loop} \\ E^6 \sim \text{two loops} \dots \end{array} \right\}$$

Predictions

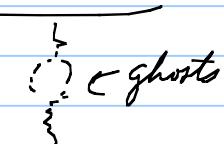
Newtonian Potential



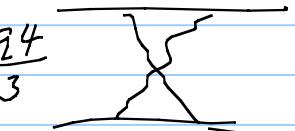
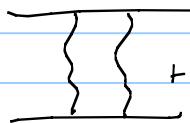
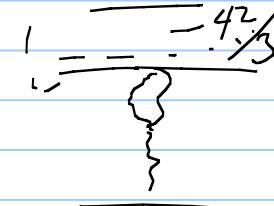
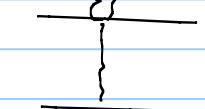
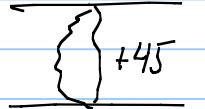
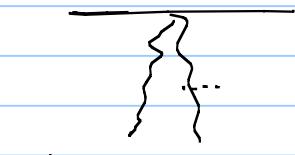
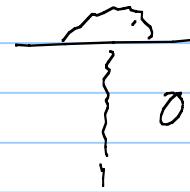
+



$$+\frac{43}{15}$$



ghosts



Nonlocal / nonanalytic  $\swarrow$  Nonrel.

$$V(r) = \int \frac{dq^3}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} M(q)$$

$$M = \frac{GMm}{g^2} \left[ 1 + Gm^2 \sqrt{\frac{g^2}{m}} \right] + Gg^2 \ln g^2 + Gg^2$$

↙ non analytic      ↘ analytic  
 ↓                          ↓                          ↑  
 ↗                          ↗                          ↓

$$\sqrt{a} = \frac{GMm}{r^2} \left[ 1 + \frac{Gm}{rc^2} + \frac{G\hbar}{r^2 c^3} \right] + \delta(r)$$

Pull out  $\sqrt{g^2}$ ,  $\ln g^2$

$$\approx \left( \frac{GM}{rc^2}, \frac{\hbar c}{mc^2}, \frac{1}{r} \right)$$

$$\hbar c = 200 \text{ MeV}_F$$

$$L = mg^2 \quad , \quad S = \sqrt{-\frac{g^2}{m^2}} \pi^2$$

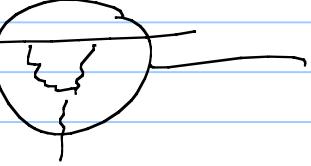
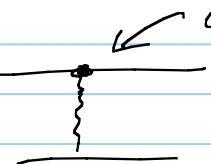
$$\begin{aligned}
I &= \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k-q)^2} \frac{1}{[(k-p')^2 - m^2]} = \frac{i}{32\pi^2 m^2} [-L - S] , \\
I_\mu &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{k^2 (k-q)^2 [(k-p')^2 - m^2]} = \frac{i}{32\pi^2 m^2} \left[ p'_\mu \left\{ \left( 1 + \frac{q^2}{2m^2} \right) L + \frac{1}{4} \frac{q^2}{m^2} S \right\} + q_\mu \left\{ -L - \frac{1}{2} S \right\} \right] , \\
I_{\mu\nu} &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^2 (k-q)^2 [(k-p')^2 + m^2]} = \frac{i}{32\pi^2 m^2} \left[ p'_\mu p'_\nu \left\{ -\frac{q^2}{2m^2} L \right. \right. \\
&\quad \left. \left. - \frac{q^2}{8m^2} S \right\} + (p'_\mu q_\nu + p'_\nu q_\mu) \left\{ \frac{1}{2} \left( 1 + \frac{q^2}{m^2} \right) L + \frac{3}{16} \frac{q^2}{m^2} S \right\} + q_\mu q_\nu \left\{ -L - \frac{3}{8} S \right\} + q^2 g_{\mu\nu} \left\{ \frac{1}{4} L + \frac{1}{8} S \right\} \right] + \dots \\
I_{\mu\nu\alpha} &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu k_\alpha}{k^2 (k-q)^2 [(k-p')^2 + m^2]} \\
&= \frac{i}{32\pi^2 m^2} \left\{ \left[ p'_\mu p'_\nu p'_\alpha \left\{ \frac{1}{6} \frac{q^2}{m^2} L \right\} \right] + (p'_\mu p'_\nu q_\alpha + p'_\mu q_\nu p'_\alpha + q_\mu p'_\nu p'_\alpha) \left\{ -\frac{1}{3} \frac{q^2}{m^2} L - \frac{q^2}{16m^2} S \right\} \right. \\
&\quad + (q_\mu q_\nu p'_\alpha + p'_\mu q_\nu q'_\alpha + q_\mu p'_\nu q_\alpha) \left[ \frac{1}{3} L \right] + q_\mu q_\nu q_\alpha \left[ -L - \frac{5}{16} S \right] \\
&\quad \left. + \left( g_{\mu\nu} p'_\alpha + g_{\mu\alpha} p'_\nu + g_{\nu\alpha} p'_\mu \right) \left[ -\frac{q^2}{12} L \right] + \left( g_{\mu\nu} q_\alpha + g_{\mu\alpha} q_\nu + g_{\nu\alpha} q_\mu \right) \left[ \frac{q^2 L}{6} + \frac{q^2 S}{16} \right] \right\} + \dots .
\end{aligned}$$

Result

$$V(r) = \frac{Gm_1 m_2}{r} \left[ 1 + \frac{3G(m_1 + M_\odot)}{r} + \frac{41}{10\pi^2} \frac{G\hbar}{r^2} \right] \quad \text{classical} \quad \text{quantum}$$

1) What about  $c_1, c_2$ ? — analytic  $\Rightarrow \delta^3(r)$

$$\underbrace{\text{cloud}}_{c_1, c_2} \sim \frac{1}{\delta^2} \times \delta^4 \quad \frac{1}{\delta^2} \sim \text{constant} \Rightarrow \delta^3(r)$$

Also   $\frac{1}{\rho} \Rightarrow$    $\Rightarrow \delta^3(r)$

2) Classical terms  $\rightarrow$  precession of Mercury ---  
 $\rightarrow$  Post Newtonian expansion

But (classical from loop)? Book loops  $\sim$  h expansions  
Not correct  $L = \frac{1}{2}[(\partial_\mu \phi)^2 - \frac{m^2}{\lambda^2} \phi^2]$   
D+H PRL 2004  $\sim$   $\square$

3) Low energy theories of quantum gravity

# Infrared Photons and Gravitons

Weinberg PR 140, B576 (1963)

## Field theories IR divergences

1) Soft divergences

$$\underbrace{\quad}_{\{ \}} + \underbrace{\quad}_{\{ \}} \text{ hard}$$

2) Collinear div

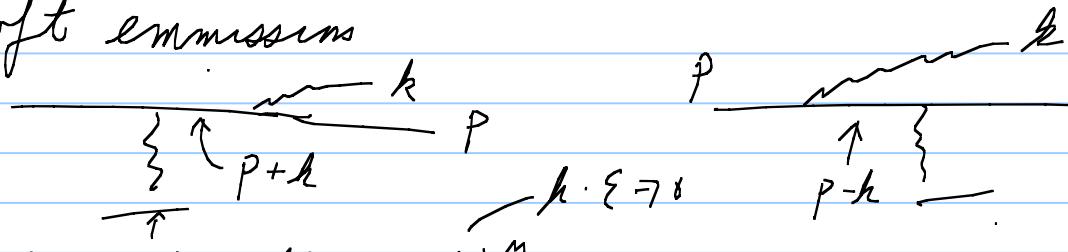
$$\underbrace{\quad}_{\{ \}} \text{ collinear both hard}$$

Gravity has only soft divergences (Weinberg)

- YM ~ collinear also

- Quadrupole?

Soft emmissions



$$\text{Photons } \mathcal{M}_0 \propto \frac{(2p + \eta k)^m}{(p + \eta k)^2 - m^2} = \mathcal{M}_0 \eta \frac{p_m}{p-k}$$

$\eta = \pm 1$  outgoing  
incoming

$$\text{Gravity } \frac{k}{2} \mathcal{M}_0 \frac{(2p + \eta k)^m (2p + \eta k)^n}{(p + \eta k)^2 - m^2} = -k \mathcal{M}_0 \eta \frac{p_m p_n}{p-k}$$

$$\mathcal{M} = \mathcal{M}_0 \sum_i \eta_i \frac{p_m^n p_n^m}{p_i - k}$$

Feynman diagram showing a soft gluon exchange between two external lines labeled  $p$  and  $p'$ . A wavy line labeled  $k$  represents the gluon, which is exchanged between the two lines. The incoming momenta are  $p$  and  $p'$ , and the outgoing momenta are  $p + k$  and  $p' - k$ .

$$\rightarrow e^{2p^m} \frac{1}{-2p \cdot k} M_0 e^{2p'^n} \frac{1}{-2p' \cdot k}$$

$\underbrace{\qquad\qquad\qquad}_{\text{same factors}}$

Soft div cancel in both QED + GR in same way

Soft factor exponential

Coulomb phase + Newton phase

$$\underbrace{\{ \text{hard} \quad \{ \quad \{ \quad \} \quad \} \quad \} }_{\leftarrow \text{soft}} \quad \swarrow \text{IR cutoff}$$

Photon  $M = M_0 e^{\frac{i}{4\pi} \frac{\ell_1 \ell_2}{\mu_{12}} \ln(\lambda)}$

$$\mu_{12} = |\vec{n}_1 - \vec{n}_2|$$

Graviton  $M = M_0 e^{-i \frac{G m_1 m_2 (1 + \beta_{12}^2)}{\beta_{12} [1 - \beta_{12}^2]^{\frac{3}{2}}} \ln \lambda}$

could be cured by using Coulombic wavefunctions