

GRQFT conventions

As we proceed through the course, I will try to collect my conventions in this file:

Metric: $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$

Lorentz Transformations: $V'^{\mu} = \Lambda^{\mu}_{\nu} V^{\nu}$, $V'_{\mu} = \Lambda_{\mu}^{\nu} V_{\nu}$
 $\Lambda^{\mu}_{\rho} \Lambda^{\nu}_{\sigma} \eta_{\mu\nu} = \eta_{\rho\sigma}$, $\Lambda^{\mu}_{\rho} \Lambda_{\mu}^{\sigma} = \delta^{\sigma}_{\rho}$

Covariant derivative: $D_{\mu} A^{\lambda} = \partial_{\mu} A^{\lambda} + \Gamma_{\mu\nu}^{\lambda} A^{\nu}$
 $D_{\mu} A_{\nu} = \partial_{\mu} A_{\nu} - \Gamma_{\mu\nu}^{\lambda} A_{\lambda}$

Connection: $\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\sigma} (\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu})$

Riemann Curvature: $R_{\mu\nu\alpha}^{\beta} = \partial_{\mu} \Gamma_{\nu\alpha}^{\beta} - \partial_{\nu} \Gamma_{\mu\alpha}^{\beta} + \Gamma_{\mu\rho}^{\beta} \Gamma_{\nu\alpha}^{\rho} - \Gamma_{\nu\rho}^{\beta} \Gamma_{\mu\alpha}^{\rho}$
 $R_{\mu\nu\alpha\beta} = R_{\mu\nu\alpha}^{\gamma} g_{\gamma\beta} = R_{\alpha\beta\mu\nu} = R_{[\mu\nu][\alpha\beta]}$

Ricci: $R_{\nu\alpha} = R_{\mu\nu\alpha}^{\mu} = R_{(\nu\alpha)}$ $R = g^{\nu\alpha} R_{\nu\alpha}$

Einstein Hilbert action: $S = \int d^4x \sqrt{-g} \left[-\Lambda - \frac{2}{\kappa^2} R \right]$; $\kappa^2 = 32\pi G$

Einstein Equation: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$