General Relativity as a Quantum Field Theory

Course contents

I. Constructing GR through gauge invariance

Jan 20 . Intro comments, Higgs theory for gravity?, Role of Equivalence Principle, Energy and Momentum, Gauge Symmetries – QED, Yang Mills notation, Preview of construction, Lorentz Invariance notation

Jan 22 Review of philosophy, Local coordinate systems, Gauging Lorentz transformations, metric and tetrad, invariant volume, an invariant action for a scalar field, T_munu as source, equations of motion, covariant derivative, the connection.

Jan 29 Tensors, Aside: photon and lorentz invariance, metricity, derivation of connection, commutator of covariant derivatives and the Riemann curvature, Ricci scalar, properties of Riemann, Einstein Hilbert action, Einstein equation, FLRW exercise

Feb 3 Review, Cosmological constant, First order or Palatini formalism, Boundary term, Alternate form of action, Canonical vs symmetric energy momentum tensor, Angular momentum and spin current, Belinfante construction, 'Improved' energy momentum for scalars.

II. The weak field limit and tree level Feynman rules

Feb 5 Alternate form of Einstein Equation, weak field, expansion of the metric and curvatures, lack of a Green function, gauge invariance, harmonic gauge, gauge invariance for the scalar field, solution for a point mass, particle in a gravitational field, non-relativistic reduction, Einstein-Infield-Hoffman Hamiltonian and coordinate invariance, quantum effects in post-Newtonian limit.

Feb 10 Energy-momentum for gravity, Constructing GR from self-interactions(Deser), Gravitational waves, Transverse-traceless condition, Polarization tensor from photon polarizations, gravitons from second quantization, weak field QFT, graviton propagator, Feynman rule for matter coupling, triple graviton vertex, Newton potential from graviton exchanges.

III. Some advanced field theory techniques

Feb 12 Review of QED vacuum polarization, Vacuum polarization in gravity, Gaussian integrals for path integrals, Path integral for field, det O , exp Tr ln O, start background field method, Complex charged scalar, vacuum polarization via background field method.

Feb 19 Background field method continued, renormalization in QED, the Effective Action, Effective Lagrangians, Generalization, full renormalization, a non-trivial example – chiral perturbation theory, definition of heat kernel.

Feb 24 Background field exercises, Heat kernel in general, propagator from heat kernel, Example QED vacuum polarization, general heat kernel expansion and QFT divergences, Seeley DeWitt coefficients, Aside: Riemann normal coordinates, heat kernel coefficients for gravity, recall matter loop in gravity, reinterpret loop and divergence using heat kernel.

Feb 26 Heat kernel expansion, Compare and contrast methods, propagators in GR, heat kernel expansion of propagator, perturbative expansion. Ghost stories, constraining integrals

March 3 Ghost day, Constrained integrals, the Jacobian, gauge fixing in gauge theory, QED, covariant gauges, Fadeev Popov trick and ghosts, review of Yang Mills, ghosts in QCD.

IV. Path Integral Quantization of General Relativity

March 5 Background field expansion of connection, gauge invariance w.r.t background field, curvatures, Lagrangian, harmonic gauge fixing, the ghost Lagrangian, full path integral.

March 10 – part 1 One loop infinities – scalar field and perturbative matching, Gauss Bonnet identity, one loop divergences of gravity, pure gravity is one-loop finite, first indication of power counting, comments on quantizing gravity

V. Effective field theory of General Relativity

March 10 -part 2 Why do quantum calculations work?, uncertainty principle and locality, example – vacuum polarization, Appelquist Carazzone theorem.

March 12 EFT Organizing principles: #1 Locality, Effective Lagrangians, #2 energy expansion, integrating out a heavy field, #3 Loops renormalization, matching/measuring, the linear sigma model and its effective Lagrangian, renaming fields, equivalence of physical effects, Haag's theorem

March 24 Effective field theory as a full QFT, correspondence, tree-level matching to E^4, renormalize EFT, equivalence of finite parts of loops, matching the EFT to full theory, aside: imaginary parts of amplitudes, power counting, Weinberg theorem, matching vs measuring

March 26 Rules for EFT, GR as EFT, the Lagrangian ordered in energy, bounding the R^2 terms, quantize and renormalize, power counting, the Newtonian potential, non-analytic terms, quantum correction to Newton, infrared photons and gravitons, soft IR divergence cancellation.

March 31 Graviton-graviton scattering and IR effect, Reissner-Nordstrom metric and origin of classical terms, Bending of light and EP violation, (Lack of) running couplings in GR.

VI. Fermions, torsion, Holst

April 2 Non-local actions in GR, Limits of EFT –UV and extreme IR (End of EFT section) Back to the foundations: Lorentz transformation for fermions, SO(1,3) algebra, gauged coordinate transformations and tetrad, gauged Lorentz/spin transformations and the spin connection.

April 7 The two gauge symmetries with fermions, curvature from spin connection, relating spin connection and affine connection, equivalence of action formulations, introduction to torsion, normalization of torsion, properties, field strength for translation symmetry.

April 9 – part 1 Two field strengths, intro torsion phenomenology, decomposition of torsion tensor, mass term, but normalization, (but)^2 Hayashi-Shirafuji analysis, (but)^3 field redefinitions, minimal coupling to fermions, Holst action

VII. Anomalies

April 9 –part 2 Anomalies, 4 pathways, path integrals and current matrix elements, Noether current, path integral test for symmetries, Conditions: 1) invariance of Lagrangian 2) Invariance of P.I. measure

April 14 Scale invariance, the dilation current, virial current and the trace of the energy-momentum tensor, Fujikawa and the Jacobian of the scale transformation, heat kernel regularization, calculation of Jacobian, path integral test, the trace anomaly, application to QCD, sketch of Feynman diagram approach

April 16 IR derivations of anomalies, non-local actions, dispersion relations. Gravitational anomalies, scale, conformal symmetry, conformal scalar fields, Conformal anomaly, the Weyl tensor and conformal anomalies. The axial anomaly, heat kernel calculation, very brief phenomenology, the gravitational axial anomaly.

VIII. Hawking radiation

April 21 Plan for Unruh and Hawking, Sample project - instantaneous transition, Defining the vacuum, inverting to get creation operators, Bogoliubov transfromations and particle production, coherent states, a more general Bogioliubov transformation, condition for thermal distributions.

April 23 Accelerated observer, Rindler spacetime, conformal form of metric, light-cone coordinates, 2D conformal scalar field, quantization, Bogoliubov transformation, evaluation of Bogoliubov coefficients, Unruh temperature.

April 28 Hawking radiation, Schwarzschild metric, wave equation, 2D Kruskal coordinates, analogy to Rindler, Hawking temperature, field theory derivation, number density and greens functions, insensitivity to UV physics, tunneling argument, evaporation and BH endgame issues.