

EPFL Lectures - Predictions of Quantum Gravity

03/2016

Recall EFT rules:

- 1) Most general \mathcal{L} , ordered by the energy expansion
- 2) Quantize
- 3) Renormalize
- 4) Match/measure
- 5) Predictions from low energy propagators

First - not predictions

$$S = \int d^4x \Gamma_g \left[-\frac{2}{\kappa^2} R + C_1 R^2 + C_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]$$

Typical effect: grav. potential

- treat as perturbation

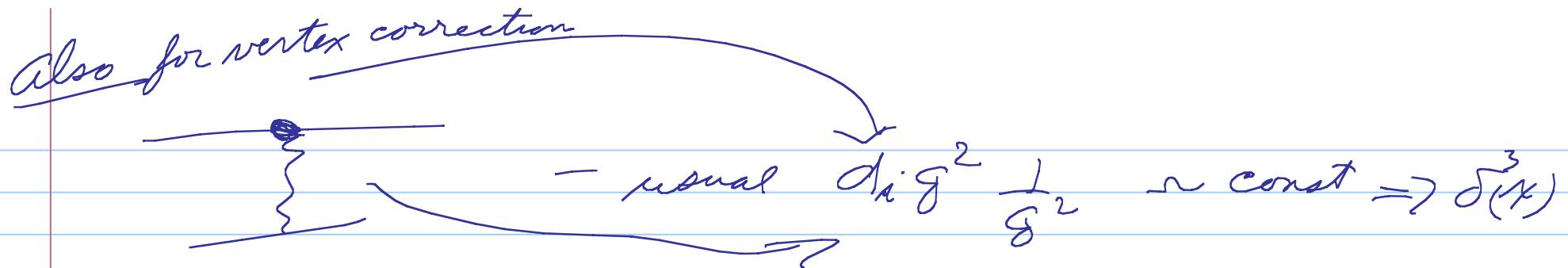
$$\underbrace{\quad}_{\text{---}} + \underbrace{\quad}_{\text{---}} =$$

$$\cancel{-} \underbrace{C_1 g^4}_{\text{---}}$$

$$\downarrow \begin{matrix} 3 \\ \delta W \end{matrix}$$

$$\left(\frac{X}{2}\right)^2 \left(\frac{E^2}{g^2}\right) + \left(\frac{R}{2}\right)^2 \left(\frac{E^2}{g^2}\right)^2 \times \frac{1}{g^2} \cancel{+ k C_1 g^4} \frac{1}{g^2} = K \frac{F^4}{4} \left[\frac{1}{g^2} + C_1 \rightarrow \text{const} \right]$$

To get $V(R)$: NR, F.T. $\frac{GMm}{r^2} + G \frac{M M'}{r^2} \delta^3(x) \leftarrow$
 r_{dir}



What can be predicted:

- effects distinct from $c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}$
- often non-analytic in momenta $\ln g^2$, $\sqrt{-g^2}$
- corresponds to non-local in position space - long distance massless particle

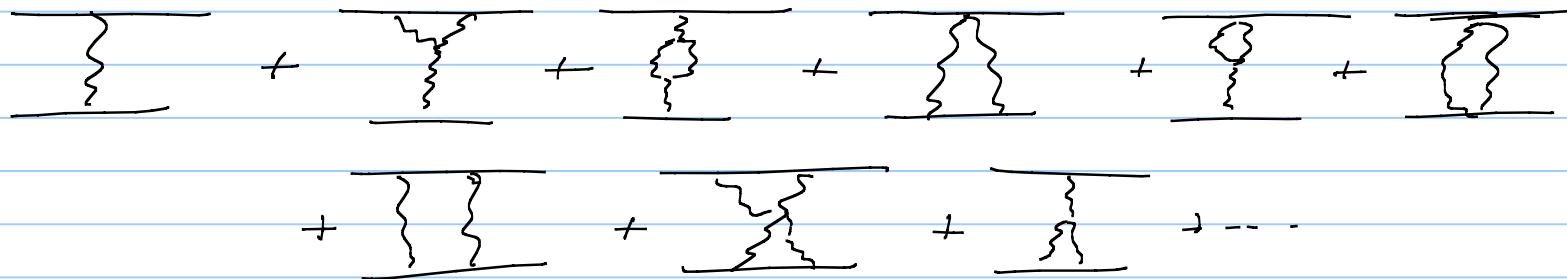
Logic:

Finite range effects can be Taylor expanded at low momenta

$$\text{ex } \ln[x(1-x)g^2 + m^2] \sim \ln m^2 + \frac{g^2}{m^2} \leftarrow \text{local}$$

$\ln g^2$ not local

Newtonian Potential



Scattering potential

non rel $(\frac{1}{2E_1}, \frac{1}{2E_2})$

$$V(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} \underline{\underline{m(q)}}$$

What to expect

$\delta^3(x)$

$$M = \frac{GMm}{r^2} \left[1 + aG(m+m)\sqrt{-g^2} + bGg^2 \ln(-g) + cGg^2 \right]$$

$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r} \quad \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2} \quad \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

$$V(r) = \frac{GMm}{r} \left[1 + a \frac{G(M+m)}{r} + b \frac{G}{r^2} \frac{t}{c^3} \right] + c \delta^3(x)$$

$\uparrow -$ \nwarrow \nearrow

classical
 $\frac{GM}{rc^2} \sim \frac{Gm^2}{mc^2}$

Calculation:

Q

BB-D-H, K-k

Know in advance that long distance parts are finite

$$\text{div} = \partial c_1 c_2 \sim \delta^3(x)$$

>

$$V(r) = -\frac{G m_1 m_2}{r} \left[1 + \frac{3}{2} \frac{G (M_1 + M_2)}{r} + \frac{41}{10 \pi^2} \frac{G \hbar}{r^2} \right]$$

"Low energy theorem" of quantum gravity

- must be true in any UV completion that gives GR at low E

Quantum Corrections

$$\frac{41}{10\pi^2} \frac{G}{r^2}$$

- tiny $G = \frac{1}{M_p^2} = l_p^{-2}$

$$l_p = \frac{\hbar c}{M_p} = \frac{0.2 \text{ GeV-fm}}{1.2 \times 10^{19} \text{ GeV}} \sim 10^{-20} \text{ fm}$$

- could have been calculated by Feynman (attempt by Radhovsky 1967)

- EFT logic valuable

Classical Correction: From loops

$$V(r) = -\frac{GM_1M_2}{r} \left[1 + 3 \frac{G(M_1 + M_2)}{rc^2} \right]$$

- agrees with Post-Newtonian expansion (in appropriate coordinates)

But - "loop expansion is to expansion" Not true

Gupta Radford
Iwasaki

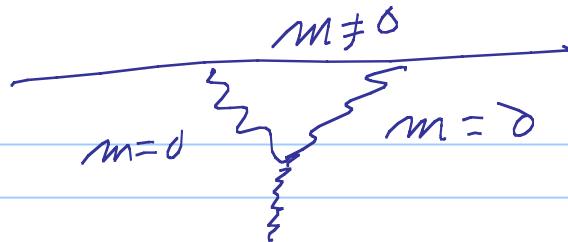
Proofs are wrong in that ✓ count this

$$\int d^4x \bar{\psi}(i\not{D} - m)\psi = \hbar \int d^4x \bar{\psi}(i\not{D} - mc^2 \frac{1}{\not{g}})\psi$$

← Hidden \hbar

$$M \quad \hbar \left[1 + \frac{mc^2}{\hbar} \frac{1}{\sqrt{g^{22}}} \right] \rightarrow \frac{1}{g^{22}} + 6\hbar \frac{mc^2}{\hbar} \frac{\sqrt{-g^{22}}}{g^{22}}$$

Specific calculations



all seem to involve 2 massless / 1 massive triangle

all " "

$$\sqrt{-g^{zz}}$$

} not thru
a.

Example Soon

Rosen-Nordström metric - Charged object [Explains classical corrections]

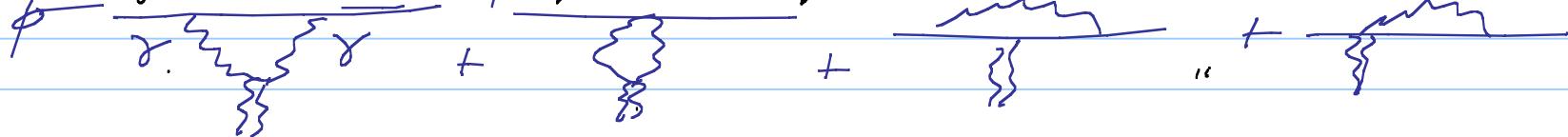
$$g_{00} = 1 - \frac{2GM}{r} + \frac{6\alpha}{r^2} + \dots + \frac{8}{3} \frac{6\alpha t}{MR}$$

$$g_{ij} = \delta_{ij} - \frac{2GM}{r} \delta_{ij} + \frac{6\alpha R_i R_j}{r^2} + \frac{4}{3} \frac{6\alpha t}{MR} (\delta_i^j - \delta_{ij})$$

harmonic gauge

$\alpha = \frac{e^2}{4\pi} \text{ here}$
(not)

Gravity classical here - photons loops



Pathway:

$$\Box h_{\mu\nu} = -8\pi G (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\lambda_\lambda)$$

$$h_{\mu\nu} = -16G \sum_{(2\pi)^3} \frac{\alpha^3}{(2\pi)^3} e^{iq\cdot x} \frac{1}{q^2} \left[T_{\mu\nu}(q) - \frac{1}{2} g_{\mu\nu} T^\lambda_\lambda(q) \right]$$

Matrix element:

$$\langle p' | T_{\nu\mu} | p \rangle = \frac{i(p-p')\gamma}{12E_2 E_1} \left[2 \sum_p P_p^+ F_1(g^2) - (g_{\mu\nu} g_{\rho\rho} - g_{\mu\rho} g_{\nu\rho}) F_2(g^2) \right]$$

Calculation:

$$F_1(g^2) = 1 + \frac{\alpha}{4\pi} \frac{q^2}{m^2} \left(\# + \frac{3}{4} \frac{m\pi^2}{1-g^2} + 2 \ln(-\sigma^2/m^2) \right)$$

classical

quantum

only

Recall HW problems from 2nd class

Rosen-Nordström metric

The RN metric describes the gravitational field around a charged, massive point particle.

- What is the $T_{\mu\nu}$ of the electromagnetic field around the charged particle?
- Use the Einstein eq in the weak field limit and the harmonic gauge to find the leading term for the metric due to this EM field
- Compare to the leading form of the RN metric in harmonic gauge:

$$g_{00} = 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} + \dots$$

$$g_{0i} = 0$$

$$g_{ij} = -\delta_{ij} - \delta_{ij} \frac{2Gm}{r} + G\alpha \frac{r_i r_j}{r^2} + \dots$$

Interpretation:

$$\overline{T_{\mu\nu}(r)} = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} T_{\mu\nu}(q)$$

Find $T_{00} = m \delta^3(r) + \frac{\alpha}{8\pi r^4} - \frac{\alpha t}{\pi m r^5}$

$$T_{ij} = \frac{-\alpha}{4\pi r^4} \left(\frac{r_i r_j}{r^2} - \frac{1}{2} \delta_{ij} \right) - \frac{\alpha t}{8\pi m r^5} \delta_{ij}$$

Recall $E+M$ $\overline{T_{\mu\nu}} = -F_{\mu\lambda} F^\lambda{}_\nu + \frac{1}{4} g_{\mu\nu} F^2$ $\leftarrow E$ field around particles

Using $\vec{E} = \frac{\alpha}{4\pi r^2} \hat{r}$ $\Rightarrow T_{00} = \frac{1}{2} E^2 = \frac{\alpha}{8\pi r^4}$

$$T_{ij} = -E_i E_j + \frac{1}{2} \delta_{ij} E^2 = \frac{-\alpha}{4\pi r^4} \left(\frac{r_i r_j}{r^2} - \frac{1}{2} \delta_{ij} \right)$$

$\Rightarrow \overline{\gamma \underbrace{E^2}_{\text{measures}} \gamma}$ measures classical E^2 around charged particle

Gravity as the square of a gauge theory

- on shell amplitude

- tree level

- origin KLT

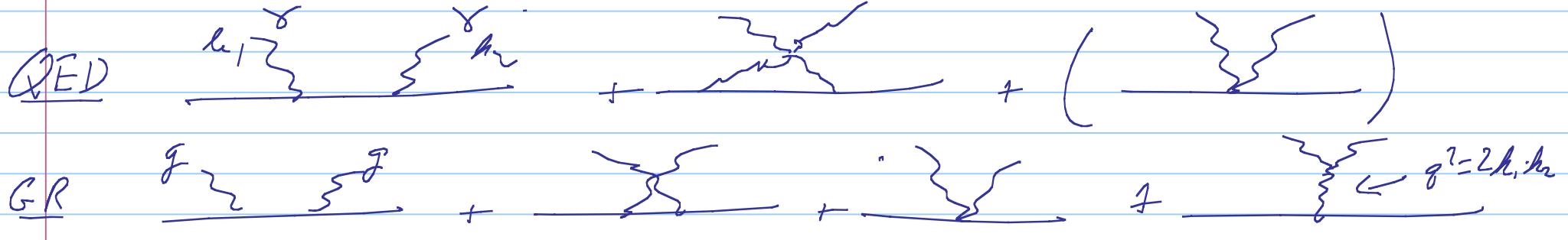


$$- E_{\mu\nu}(+2) = E_\mu(+1) \cdot E_\nu(+1)$$

- YM (color factors stripped)

Bern review
on webpage

Example : Compton amplitudes



$$M_F^{(S)}_{\text{grav}} = \frac{\kappa^2}{8\pi^2} \underbrace{\frac{p_1 \cdot h_1 p_2 \cdot h_2}{h_1 \cdot h_2}}_{\text{prefactor}} \times M_{EM}^{(d)} M_{EM}^{(S)}$$

Recall:

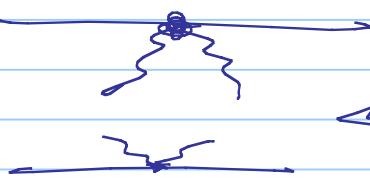
Feynman (1963) excerpt

Tree Thm.

This made me investigate the entire subject in great ~~detail~~ to find out what the trouble is. I discovered in the process two things. First, I discovered a number of theorems, which as far as I know are new, which relate closed loop diagrams and diagrams without closed loop diagrams (I shall call the latter diagrams "trees"). The unitarity relation which I have just been describing, is one connection between a closed loop diagram and a tree; but I found a whole lot of other ones, and this gives me more tests on my machinery. So let me just tell you a little bit about this theorem, which gives other rules. It is rather interesting. As a matter of fact, I proved that if you have a diagram with rings in it there are enough theorems altogether, so that you can express any diagram with circuits completely in terms of diagrams with trees and with all momenta for tree diagrams in physically attainable regions and on the mass shell. The demonstration is remarkably easy. There are several ways of demonstra-

Loops without loops

Tree theorem



← on shell

(no ghosts!)



= +
 ↗ nonphysical DOF ↗ cancelled these

2 Ways

1) Dispersion relation

$$\text{Amp}(g^2) = \frac{1}{\pi} \int_0^\infty dt e^{-imt} \frac{\text{Im Amp}(t)}{z - g^2 + i\epsilon}$$

✓ Tree Tree ← on shell
angular integral

2) Unitarity methods

- on shell states

- Passarino - Veltman - all 1 loop \Rightarrow scalar box, bubble, triangle -

- recogno by cuts

- programs

$$\begin{aligned} I_4(s, t) &= r_\Gamma \frac{1}{st} \left\{ \frac{2}{\epsilon^2} \left[(-s)^{-\epsilon} + (-t)^{-\epsilon} \right] - \ln^2 \left(\frac{-s}{-t} \right) - \pi^2 \right\}, \\ &= r_\Gamma \frac{1}{st} \left\{ \frac{4}{\epsilon^2} - \frac{2 \ln(-s) + 2 \ln(-t)}{\epsilon} + 2 \ln(-s) \ln(-t) - \pi^2 \right\} \end{aligned} \quad (3.5)$$

together with $I_4(s, u)$ and $I_4(t, u)$. Then the “triangle and bubble integrals”.

$$\begin{aligned} I_3(s) &= \frac{r_\Gamma}{\epsilon^2} (-s)^{-1-\epsilon} = -\frac{r_\Gamma}{s} \left(\frac{1}{\epsilon^2} - \frac{\ln(-s)}{\epsilon} + \frac{\ln^2(-s)}{2} \right) \\ I_2(s) &= \frac{r_\Gamma}{\epsilon(1-2\epsilon)} (-s)^{-\epsilon} = r_\Gamma \left(\frac{1}{\epsilon} - \ln(-s) + 2 \right) \end{aligned} \quad (3.6)$$

$$iM^{1\text{-loop}}|_{disc} = \int \frac{d^D \ell}{(2\pi)^D} \frac{\sum_{\lambda_1, \lambda_2} M_{\lambda_1 \lambda_2}^{\text{tree}}(p_1, p_2, -\ell_2^{\lambda_2}, \ell_1^{\lambda_1})(M_{\lambda_1 \lambda_2}^{\text{tree}}(p_3, p_4, \ell_2^{\lambda_2}, -\ell_1^{\lambda_1}))^*}{\ell_1^2 \ell_2^2} \Big|_{cut},$$

W

Simplest "calculations"

- Newtonian potential

- gravity = $(EM)^2$

- mult together

- use PV

- $M = \zeta I_2$

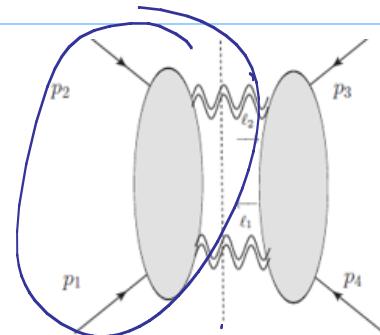
+ $C_2 I_3$

+ $C_4 I_4$

\rightarrow done

$$iM_0^{\text{tree}}(p_1, p_2, k_1^+, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{m^4 [k_1 k_2]^4}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1, p_2, k_1^-, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{\langle k_1 | p_1 | k_2 \rangle^2 \langle k_1 | p_2 | k_2 \rangle^2}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$



Soft Theorems at One Loop

Tree level soft theorem Low, Gross, Jackiw, Weinberg, Thirring

Compton QED — universal for soft momenta
GR

\Rightarrow Loops are universal also at low g^2

Trees \rightarrow loop

$$\left. \begin{array}{c} Q, L \\ \downarrow \quad \downarrow \\ + O(g^2) \end{array} \right\}$$

$V(R)$ universal + Spin dependent universal
GRFT

Light Bending at one loop

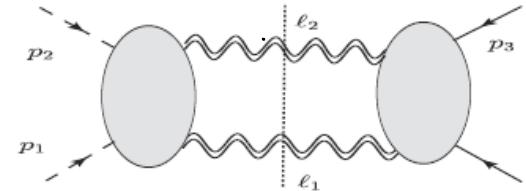
↙ classical

$$i\mathcal{M}_{[\phi(p_3)\phi(p_4)]}^{[\eta(p_1)\eta(p_2)]} \simeq \frac{\mathcal{N}^\eta}{\hbar} (M\omega)^2 \left[\frac{\kappa^2}{t} + \kappa^4 \frac{15}{512} \frac{M}{\sqrt{-t}} + \hbar\kappa^4 \frac{15}{512\pi^2} \right.$$

$$\times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4 \frac{bu^\eta}{(8\pi)^2} \log\left(\frac{-t}{\mu^2}\right)$$

$$+ \hbar\kappa^4 \frac{3}{128\pi^2} \log^2\left(\frac{-t}{\mu^2}\right)$$

$$\left. + \kappa^4 \frac{M\omega i}{8\pi t} \log\left(\frac{-t}{M^2}\right) \right], \quad (11)$$



$$b_{\text{u}}^0 = \frac{3}{40}$$

$$b_{\text{u}}^1 = -\frac{11}{120}$$

Not universal

also eikonal

$$\theta_\eta \simeq -\frac{b}{\omega} \int_{-\infty}^{+\infty} \frac{V'_\eta(b\sqrt{1+u^2})}{\sqrt{1+u^2}} du$$

$$\simeq \frac{4GM}{b} + \frac{15G^2M^2\pi}{4b^2} + \frac{8bu^\eta + 9 + 48\log\frac{b}{2r_0}}{\pi} \frac{G^2\hbar M}{b^3}$$

not universal

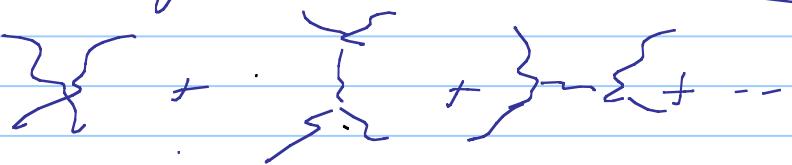
IR - term.

- no longer null geodesics

Graviton-graviton scattering - IR divergences

(Dunbar+Norridge)

$$A^{\text{tree}} = \frac{i}{4} k^2 \frac{\sqrt{G}}{2\pi} S^3$$



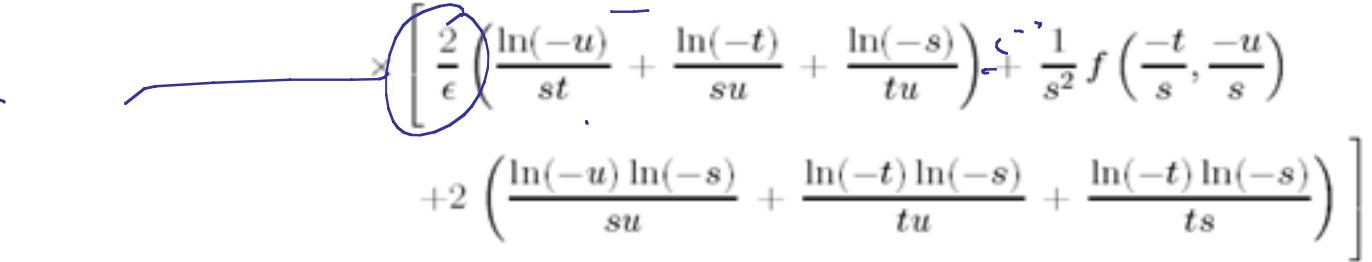
One loop
-finite

($\not{e} H V$ pure gravity finite)

$\Rightarrow c_i$ do not appear



$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i \frac{\kappa^4}{30720\pi^2} (s^2 + t^2 + u^2) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3} \mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} \mathcal{A}^{tree}(++;++) \times (s t u)
\end{aligned} \tag{3}$$



$$\left[\frac{2}{\epsilon} \left(\frac{\ln(-u)}{st} + \frac{\ln(-t)}{su} + \frac{\ln(-s)}{tu} \right) + \frac{1}{s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right) + 2 \left(\frac{\ln(-u) \ln(-s)}{su} + \frac{\ln(-t) \ln(-s)}{tu} + \frac{\ln(-t) \ln(-s)}{ts} \right) \right]$$

where

$$\begin{aligned}
f\left(\frac{-t}{s}, \frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)(2t^4 + 2t^3u - t^2u^2 + 2tu^3 + 2u^4)}{s^6} \left(\ln^2 \frac{t}{u} + \pi^2 \right) \\
&+ \frac{(t-u)(341t^4 + 1609t^3u + 2566t^2u^2 + 1609tu^3 + 341u^4)}{30s^5} \ln \frac{t}{u} \\
&+ \frac{1922t^4 + 9143t^3u + 14622t^2u^2 + 9143tu^3 + 1922u^4}{180s^4},
\end{aligned} \tag{4}$$



But IR divergence
Checked! Canceled!



$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \quad (29)$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2} \left\{ 1 + \frac{\kappa^2 s}{16\pi^2} \left[\ln \frac{-t}{s} \ln \frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right) \right. \right.$$
$$\left. \left. - \left(\frac{t}{s} \ln \frac{-t}{s} + \frac{u}{s} \ln \frac{-u}{s} \right) \left(3 \ln(2\pi^2) + \gamma + \ln \frac{s}{\Lambda^2} + \frac{\sum_{ij} \eta_i \eta_j \mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij} \eta_i \eta_j \mathcal{F}^{(0)}(\gamma_{ij})} \right) \right] \right\}.$$

Infrared Photons and Gravitons

Field theory I R divergences

Weenborg
on web page

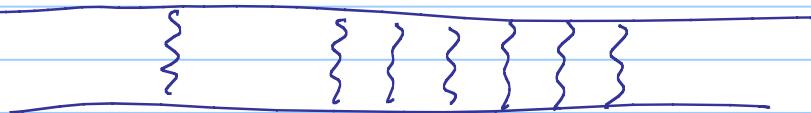
Gravity has only soft divergences

Soft emulsion

IR finiteness

Soft divergences cancel in QED + GR in the same way

Coulomb Phase and Newtons Phase



Photos

Gravitons

can be cured by using Coulombic wavefunction

Limits of the EFT

Most obvious - Expansion in the energy

$$M = M_0 \left[1 + K^2 c_i g^2 + K^2 g^2 \ln g^2 + K^4 g^4 + \dots \right]$$

\uparrow

EFT falls apart at M_P
 \Rightarrow some order

(or earlier)

Possible I.R. limits?

- limits on known techniques
- are there fundamental limits?

Gravitational effects build up

$$B_{1+} \sim \frac{1}{1 - \frac{2GM}{r}}$$

compare $r \rightarrow \infty$
 $\& r \rightarrow 2GM$

in same calc.

- fundamental limit?
- techniques?

\Rightarrow Extreme I.R. also interesting

"Integrated Curvature" as expansion parameter

Basic Summary

- QFT for GR
- renormalized
 - not interesting
- Shift focus to IR
 - prediction from $\Gamma g^2, \ln g^2 \leftarrow \text{EFT}$
- GR \cap QFT in EFT limit