

GR as an EFT

Note Title

TRISEP

7/19/18

7/10/2018

Recall:

Main goal: How to think of gravity as part of our "core theory".

SM + GR

$$Z = \int [d\phi dA dy] \exp i \left[\int dx \sqrt{-g} \left[-\frac{1}{4} F^2 + \bar{\psi}_i \not{D}^2 \psi_i + 2 \not{\partial}^\mu \phi \not{\partial}_\mu \phi - V(\phi) - \Lambda + \frac{2}{R^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right] \right]$$

QM \supset GR (at ordinary energies)

~~one is not the correct problem, one would prefer to have a different theory.~~
But since I am among equally irrational men I won't be criticized I hope for the fact
that there is no possible, practical reason for making these calculations.

Need to overcome outdated claims:

“The existence of gravity clashes with our description of the rest of physics by quantum fields”

“Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.”

“Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.”

“The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory”

“Quantum mechanics and general relativity are incompatible”

EFT is the key

- 1) Identify low Energy d.o.f and symmetries
- 2) Most general \mathcal{L} , ordered in energy expansion
- 3) Quantize
- 4) Renormalize
- 5) Match / measure
- 6) Predictions from low energy propagators

$$S_g = \int d^4x \sqrt{-g} \left[-1 + \frac{2}{\kappa^2} R + C_1 R^2 + C_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]$$

$$S_m = S(\phi, g)$$

\uparrow
 $10^{-122} M_p^2$

$$R^2 = 32\pi G_N$$

$$C_i < 10^{+65}$$

First scalar

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\delta S_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{18\delta} \left[\frac{1}{\epsilon} \dots \right] [3R_{\mu\nu} R^{\mu\nu} - R^2] \leftarrow \sim O(\delta^4)$$

$$\overset{m}{\nearrow} \overset{C_m}{\nearrow} \sim O(E^4) \sim O(\delta^4)$$

$$\lambda \phi^4 \sim \text{loop} \sim O(\delta^4)$$

Gravity "simple"

't Hooft Veltman

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + K h_{\mu\nu}$$

Expansion

$$\boxed{\Gamma^{\lambda}_{\mu\nu}} = \boxed{\bar{\Gamma}^{\lambda}_{\mu\nu}} + \boxed{\underline{\Gamma}^{\lambda}_{\mu\nu}} + \boxed{\overline{\Gamma}^{\lambda}_{\mu\nu}}$$

$\hookrightarrow \mathcal{O}(h)$

$$\boxed{\underline{\Gamma}^{\lambda}_{\mu\nu}} = \frac{1}{2} \bar{g}^{\lambda\sigma} \left[\bar{D}_\mu h_{\nu\sigma} + \bar{D}_\nu h_{\mu\sigma} - \bar{D}_\sigma h_{\mu\nu} \right]$$

covariant w.r.t \bar{g}

$$\mathcal{L} = \frac{2\bar{g}R}{k^2} = \bar{g} \left[\frac{2}{k^2} \bar{R} + \frac{1}{2} D_\alpha h_{\mu\nu} \bar{D}^\alpha h^{\mu\nu} + \dots \right.$$

$$\left. h^\lambda \bar{R}_{\lambda\nu} h^{\mu\nu} + \dots \right]$$

Also $\mathcal{N}^a \rightarrow \mathcal{N}'^a = \mathcal{N}^a + \xi^a \quad \text{small}$

$$h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu$$

gauge invariance for h

Gauge fixing

$$\mathcal{S} = (\bar{D}^\mu h_{\mu\nu} - \frac{1}{2} \bar{D}_\nu h^\lambda_\lambda)$$

Recall

$$1 = \int S d\theta(x) \delta(f(A^\theta) - F(x)) \det \left(\frac{\partial f}{\partial \theta} \right)$$

Exponentiate

$$S dF e^{-i \frac{1}{2} \int S d^4x F^2(x)} \delta(f(A) - F(x)) = e^{-i \int S d^4x \frac{3}{2} f^2(A)}$$

Evaluate FP det with ghost fields

$$\det \frac{\partial f}{\partial \theta} = \int d\bar{c} d\bar{c} e^{i \int S d^4x \bar{c} \frac{\partial f}{\partial \theta} c}$$

Gravity

$$C_{\nu} = (\bar{D}_m h_{\mu\nu} - \frac{1}{2} \bar{D}_{\nu} h_{\mu}^{\mu})$$

$$\mathcal{L}_{GF} = \frac{1}{2} C_{\nu} C^{\nu}$$

$$\begin{aligned}\delta C_{\mu} &= \bar{D}_{\nu} (\bar{D}_{\nu} \xi_{\mu} + \bar{D}_{\mu} \xi_{\nu}) - \frac{2}{2} \bar{D}_{\mu} \bar{D}_{\nu} \xi^{\nu} \\ &= \bar{D}^2 \xi_{\mu\nu} + \bar{R}_{\mu\nu} \xi^{\nu}\end{aligned}$$

$$\frac{\delta C_{\mu}}{\delta \xi^{\nu}} = [\bar{D}^2 + \bar{R}_{\mu\nu}]$$

$$\mathcal{L}_{gh} = \bar{\gamma}^m \left[\underbrace{\bar{\partial}_{\mu\nu} \bar{D}^2 + \bar{R}_{\mu\nu}}_{\text{ghost field}} \right] \gamma^{\nu}$$

New Feynman rule (near flat)

$$h \quad h' - i [h_\alpha h_\mu \gamma_\nu] \dots$$

$$Z = \int [dh_{\mu\nu}] [\bar{d}\gamma_1 \bar{d}\bar{\gamma}_1] e^{i \int d^4x \sqrt{-g} [\mathcal{L}(h) + \mathcal{L}_{GF}(h) + \mathcal{L}_g(\gamma, \bar{\gamma})]}$$

$\ell H V$

$$\frac{1}{\Delta L} = \frac{1}{16\pi^2} \frac{1}{\epsilon} \left[\frac{1}{120} \bar{R}^2 + \frac{7}{120} \bar{R}_m \bar{R}^{md} \right]$$

($O(\epsilon^4)$)

$$m_m + m_m^{\epsilon^2}$$

First - non - predictions

$$\left[\frac{2}{k^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \right]$$

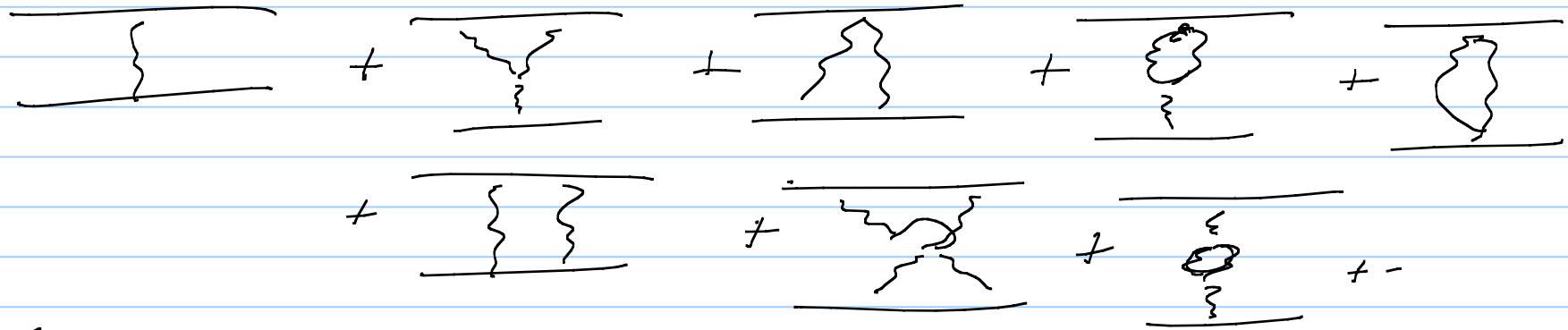
Only graviton $R_{\mu\nu} = 0 \Rightarrow$ no effect of c_1, c_2

Typical: $\underbrace{-}_{\text{}} + \underbrace{\text{}}_{\text{}} \hookrightarrow c_1 R^2 + \dots$

$$\left(\frac{k}{2} E \right)^2 \left[\frac{1}{g^2} + c_1 \underbrace{\frac{1}{g^2} g^4 \frac{1}{g^2}}_{\text{Const}} \right] -$$

$$\rightarrow V(r) = \text{usual} + c_1 \delta^4(r)$$

Newtonian potential



$\mathcal{M}(g)$

$$V(r) = \int_{(r_0)}^{d_g^3} e^{i\vec{q}\cdot\vec{r}} \mathcal{M}(g) \xleftarrow{N.R}$$

$$M = \frac{GMm}{g^2} \left[1 + aG(m+M)\sqrt{-g^2} + bGg^2 \ln g^2 + cGg^2 \right]$$

$$V(r) = -\frac{GMm}{r} \left[1 + a \frac{G(M+m)}{r c^2} + b \frac{Gk}{r^2 c^3} \right] + c \frac{GMm}{r^3} \delta(x)$$

↑
 classical ↑ quantum ↗
 go here

Calculate

$$V(r) = -\frac{GMm}{r} \left[1 + 3 \frac{G(m+M)}{r} + \frac{41}{10\pi^2} \frac{G\hbar}{r^2} \right],$$

↑
Classical

>

Unitarity techniques

Possibly Veltman

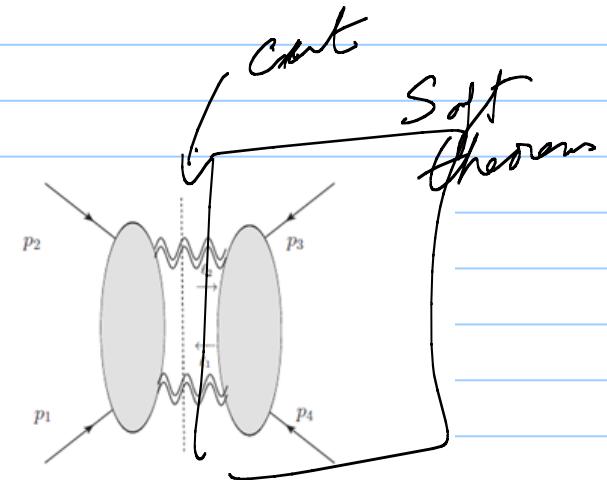
→ Box, triangle, bubbles

$$(GR) = (\gamma n)^2$$

Only physical D.O.F.

⇒ same answer

$\frac{4}{10\pi^2}$ is soft theorem - independent of spin



$$iM_0^{\text{tree}}(p_1, p_2, k_1^+, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{m^4 [k_1 k_2]^4}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1, p_2, k_1^-, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{\langle k_1 | p_1 | k_2 \rangle^2 \langle k_1 | p_2 | k_2 \rangle^2}{(k_1 \cdot p_1)(k_1 \cdot p_2)},$$

Massless off of Massive (Bending of light)

$$i\mathcal{M}_{[\phi(p_3)\phi(p_4)]}^{[\eta(p_1)\eta(p_2)]} \simeq \frac{\mathcal{N}^\eta}{\hbar} (M\omega)^2 \left[\frac{\kappa^2}{t} + \kappa^4 \frac{15}{512} \frac{M}{\sqrt{-t}} + \hbar\kappa^4 \frac{15}{512\pi^2} \right.$$

$$\times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4 \frac{bu^\eta}{(8\pi)^2} \log\left(\frac{-t}{\mu^2}\right)$$

$$+ \hbar\kappa^4 \frac{3}{128\pi^2} \log^2\left(\frac{-t}{\mu^2}\right)$$

$$\left. + \kappa^4 \frac{M\omega i}{8\pi t} \log\left(\frac{-t}{M^2}\right) \right], \quad (11)$$

not universal
 $(bu) \sim \frac{3}{4\pi} \text{ scalar}$
 $= \frac{-11}{120} \gamma$

Eikonal

$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4} \frac{G_N^2 M^2 \pi}{b^2} + \left(8bu^S + 9 - 48 \log \frac{b}{2b_0} \right) \frac{\hbar G_N^2 M}{\pi b^3} + \dots$$



Graviton - gravitons

$$\begin{aligned}
 \mathcal{A}^{1-loop}(++;--) &= -i \frac{\kappa^4}{30720\pi^2} (s^2 + t^2 + u^2) \\
 \mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3} \mathcal{A}^{1-loop}(++;--) \\
 \mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} \mathcal{A}^{tree}(++;++) \times (s t u) \quad (3) \\
 &\times \left[\frac{2}{\epsilon} \left(\frac{\ln(-u)}{st} + \frac{\ln(-t)}{su} + \frac{\ln(-s)}{tu} \right) + \frac{1}{s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right) \right. \\
 &\quad \left. + 2 \left(\frac{\ln(-u)\ln(-s)}{su} + \frac{\ln(-t)\ln(-s)}{tu} + \frac{\ln(-t)\ln(-s)}{ts} \right) \right]
 \end{aligned}$$

l R

where

$$\begin{aligned}
 f\left(\frac{-t}{s}, \frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)(2t^4+2t^3u-t^2u^2+2tu^3+2u^4)}{s^6} \left(\ln^2 \frac{t}{u} + \pi^2 \right) \\
 &\quad + \frac{(t-u)(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4)}{30s^5} \ln \frac{t}{u} \\
 &\quad + \frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \quad (4)
 \end{aligned}$$

Durfas
Norridge

No C_1, C_2

$\frac{1}{\mathcal{E}}$ canceled by soft radiation

$$\begin{aligned}
 & \left(\frac{d\sigma}{d\Omega} \right)_{tree} + \left(\frac{d\sigma}{d\Omega} \right)_{rad.} + \left(\frac{d\sigma}{d\Omega} \right)_{nonrad.} = & (29) \\
 & = \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2} \left\{ 1 + \frac{\kappa^2 s}{16\pi^2} \left[\ln \frac{-t}{s} \ln \frac{-u}{s} + \frac{tu}{2s^2} f \left(\frac{-t}{s}, \frac{-u}{s} \right) \right. \right. \\
 & \quad \left. \left. - \left(\frac{t}{s} \ln \frac{-t}{s} + \frac{u}{s} \ln \frac{-u}{s} \right) \left(3 \ln(2\pi^2) + \gamma + \ln \frac{s}{\Lambda^2} + \frac{\sum_{ij} \eta_i \eta_j \mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij} \eta_i \eta_j \mathcal{F}^{(0)}(\gamma_{ij})} \right) \right] \right\}.
 \end{aligned}$$

Beyond Scattering

$$QED \quad S = \int d^4x - \frac{1}{4} F^2 + \int d^4x A^a_y b_e^e F_{\mu\nu}^{ab} L(x-y) F^{\mu\nu}_{(y)}$$

$$L(x-y) = \langle x | \ln \Omega | y \rangle$$

$$S = \int d^4x \left[-\frac{1}{4} F^2 + b_e^e F_{\mu\nu} \ln \Omega F^{\mu\nu} \right]$$

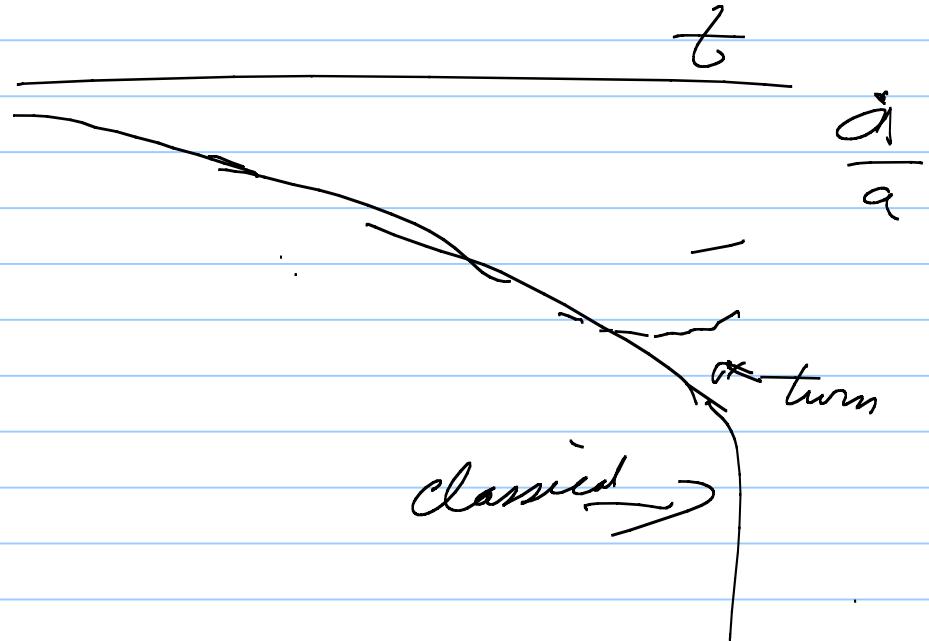
Barvinchko - Vilkovishky

- non local L

$$S_4 = \int d^4x F_g \left[C_1 R^2 + \alpha R \ln \Box R + \beta R_{\mu\nu} \ln \Box R^{\mu\nu} - \dots \right]$$

also $R^2 / D R \sim O(\delta^4)$

Example Collapsing FRW



Limits of EFT

UV

$$\mathcal{M} = \mathcal{M}_0 \left[1 + k_{c_i}^2 g^2 + k_g^2 \ln g^2 \right]$$

\sim

Falls apart

$$Gg^2 c_i \approx 1$$

IR limits to calculations

- Grav effects cumulate

$$\frac{1}{1 - \frac{2GM}{r}}$$

small curvatures large M

P.T. Sensitivity to very IR effects is non-trivial

Cliff B = horizon

“A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.”

Frank Wilczek
Physics Today
2002

$$Z = \int [d\phi dA dy] \exp i \int dx \sqrt{g} \left[-\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + \frac{1}{2} \phi \partial^\mu \phi - V(\phi) - \Lambda - \frac{2}{K^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]$$