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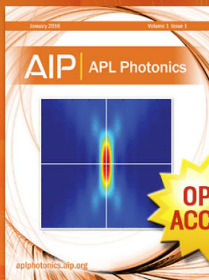
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Analysis of photon recycling using metallic photonic crystal

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We investigate a photon recycling scheme using two-dimensional metallic photonic crystals made of silver to improve the energy efficiency of an incandescent light source. A theoretical framework is presented to analyze the resultant photon-recycled lighting system. Calculation results show that the system can reach a maximum luminous efficiency of 125 lm/W, which is 8 times higher than that of a bare blackbody radiation at 2800 K. The color temperature of the system is calculated to be around 3500 K or below, while the color rendering index is between 68 and 90. These results suggest that a photon-recycled incandescent light source using metallic photonic crystals can be a viable alternative future lighting solution. © 2007 American Institute of Physics.

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I. INTRODUCTION

Incandescent light sources have long been the dominant lighting source for residential and portable applications. A study found that they still accounted for 90% of the residential lighting energy consumption in 2001.¹ However, because of their relatively poor energy efficiency and short lifetime, incandescent light sources are being replaced in many areas by other lighting technologies including fluorescent lamps, high-intensity discharge lamps, and recently, solid-state lighting devices such as light-emitting diodes (LEDs).²⁻⁵ Yet, incandescent light sources still possess certain crucial advantages, such as the warm white light of low color temperature they emit and their easiness to dim using inexpensive controls. They also have relatively inexpensive first-costs per lumen⁶ and long-established infrastructure. In addition, they do not contain hazardous materials such as mercury which is required for fluorescent lamps, nor do they need the complex thermal management scheme required for high-brightness LEDs.^{7,8} For these reasons, it is worthwhile to cast a new perspective on the long-proven incandescent light source as a viable alternative future lighting solution, especially when almost tenfold improvement in its luminous efficiency can possibly be made as will be shown in this article.

The inefficiency of incandescent lighting sources results from their large amount of infrared light emission leading to much input energy being wasted as heat rather than converted into useful visible light as shown in Fig. 1. If the infrared light can be recycled to generate visible light, the efficiency of incandescent light sources can be vastly improved.

To recycle the infrared light, a bandpass filter enclosing the filament can be employed. The filter transmits the useful visible light and returns the undesired infrared light back to the filament. The returned infrared light is reused to heat the

filament and it consequently reduces the amount of required input energy to maintain the temperature of the filament.

In this article, we show that the luminous efficiency of an incandescent light source can reach 487 lm/W through the incorporation of an ideal Gaussian-shaped bandpass filter which will be shown later. When a realistically designed two-dimensional (2D) metallic photonic crystal is applied as the filter, the luminous efficiency can still reach 125 lm/W at color temperatures of 3500 K. These numbers are comparable to those achieved in the state-of-the-art nitride LED systems,⁹ which currently demonstrated the best efficiency of any lighting device.

There have been efforts to accomplish the recycling process by means of dielectric-metal film stack^{10,11} or dielectric multilayered films.¹² While dielectric-metal film stack shows high reflectance beyond 2 μm and high transmittance in a visible region, its reflectance in near-infrared region ($\lambda = 0.8-2 \mu\text{m}$) is not high enough to fully reflect the infrared

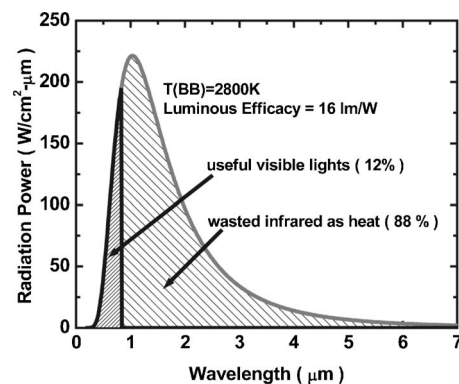


FIG. 1. The gray line is a blackbody radiation curve at 2800 K, which is a typical operating temperature of incandescent source at 100 W. The area shaded to the left is useful visible light (12%) and the area shaded to the right is wasted infrared as a form of heat (88%). The luminous efficiency is 16 lm/W. It is close to the typical luminous efficiency of a 100 W incandescent bulb, 17 lm/W.

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light. For the case of dielectric multilayer film, while it shows high reflectance in near-infrared region, its reflectance in mid-infrared region beyond $2 \mu\text{m}$ is low. In addition, it consists of two different materials having different thermal expansion coefficients. This mismatch in thermal expansion may cause thermal stress and cracks in the dielectric multilayer films.

To overcome these limitations we have chosen metal as a base material in our filter design. Metal materials can efficiently reflect all the infrared light. Metal materials are also more flexible and do not crack easily under temperature gradient.

The rest of this article is organized as follows: we first discuss a general theory of the photon recycling process in Sec. II. In Sec. III, we show the calculated optical properties of the metallic photonic crystal (MPC) and their performance in photon recycling using the theories discussed in Sec. II. In Sec. IV, we discuss the color qualities of the obtained results in Sec. III, such as correlated color temperature and color rendering index. Finally in Sec. V, we present some conclusions.

II. PHOTON RECYCLING PROCESS

Incandescent light sources emit blackbody radiation described by the Planck radiation formula, Stefan-Boltzmann law, and Wien's displacement law.¹³ The total radiation power (P_{BB}) from a blackbody at temperature T_b can be divided into two parts, visible (P_{vis}) and infrared (P_{infrared}) radiation:

$$\begin{aligned} P_{\text{BB}} &= \sigma T_b^4 A_b \\ &= P_{\text{vis}} + P_{\text{infrared}} \\ &= A_b \int_{\text{visible}} u(\lambda, T_b) d\lambda + A_b \int_{\text{infrared}} u(\lambda, T_b) d\lambda, \end{aligned} \quad (1)$$

where σ is the Stefan-Boltzmann constant, $u(\lambda, T_b)$ is radiation energy density from the blackbody, and A_b is the surface area of it. From Eq. (1), we can see that if we can maintain visible radiation and reduce infrared radiation effectively, a smaller amount of total power is needed to get the same amount of visible power and consequently the efficiency will increase. To illustrate this more precisely, we have employed an ideal system that has a spherical blackbody filament enclosed by a filter as schematically depicted in Fig. 2(a). We also assume that this system is enclosed by an enclosure that fully transmits all the radiation from the inside system and in which vacuum can be kept.

When the blackbody filament radiates its power to the filter, it is either transmitted, reflected, or absorbed by the filter. The transmitted power is either useful as visible light or wasted as infrared light. However, in our designed configuration, the reflected power is not wasted. It is completely absorbed by the filament and recycled. This is because the filament is a blackbody absorber, the filter system has a spherical symmetry, and the reflection is specular. The absorbed power by the filter increases its temperature (T_f) and makes it radiating. Some portion of this radiation from the

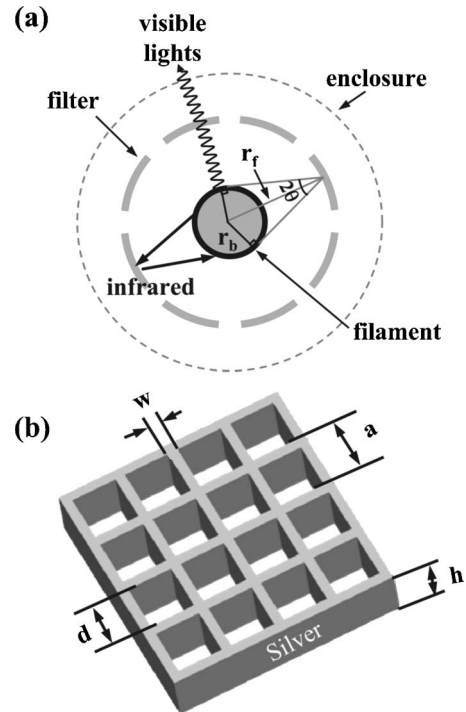


FIG. 2. (a) A schematic diagram of the ideal photon recycling system. A blackbody filament is enclosed by a filter and an ideal enclosure. The filter transmits visible light and reflects infrared. The reflected infrared is fully absorbed into the blackbody filament and recycled. The role of the ideal enclosure is to keep vacuum inside and it does not interact with the blackbody filament and the filter. T_b is the temperature of a blackbody and T_f is that of a filter. (b) A schematic of 2D metallic photonic crystal filter made of silver. a is pitch, d is the size of air opening, w is bar width, and h is thickness.

filter is also absorbed by the filament. Consequently, the power supplied to the filament has three terms, the electric power from an external source, the reflected optical power by the filter, and some part of the radiated optical power from the filter. From conservation of energy the power relation for the blackbody filament is, then

$$P_{\text{BB}} = A_b \int_{\text{all}} u(\lambda, T_b) d\lambda = P_{\text{Ext}} + P_{\text{Ref}} + P_{\text{Rad}}, \quad (2)$$

where P_{Ext} =electric power from an external source, P_{Ref} =power reflected by the filter, and P_{Rad} =part of power radiated by the filter that is absorbed by the filament. P_{Rad} can be estimated as follows. When the radiation from a spot on the filter falls into the filament, it will be absorbed by the filament, otherwise it will be reabsorbed by the filter. The solid angle covering the filament is $2\pi[1 - \cos \theta]$, where θ is half of the apex angle of the cone depicted in Fig. 2(a). Thus, the solid angle is $2\pi(1 - \sqrt{r_f^2 - r_b^2}/r_f)$, where r_b is the radius of the blackbody filament and r_f is the radius of the filter. The portion of the absorbed power by the filament can be calculated by dividing the solid angle of the filament with that of hemisphere, which is 2π . The power relation, Eq. (2) is, then

$$P_{\text{Ext}} = A_b \int_0^\infty u(\lambda, T_b) d\lambda - A_b \int_0^\infty \text{ref}_f(\lambda) u(\lambda, T_b) d\lambda - \left(1 - \frac{\sqrt{r_f^2 - r_b^2}}{r_f}\right) A_f \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_f) d\lambda, \quad (3)$$

where A_f is the surface area of the filter, $\text{ref}_f(\lambda)$ is the reflectance of the filter, $\text{abs}_f(\lambda)$ is the absorbance of the filter, r_b is the radius of the blackbody filament, and r_f is the radius of the filter.

From Eq. (3), the amount of external electric power that is required to maintain the filament at T_b can be calculated. To solve Eq. (3) we need to relate $\int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_f) d\lambda$ to $\int_0^\infty u(\lambda, T_b) d\lambda$. To do that, let us think of the power absorbed and radiated by the filter. The power absorbed by the filter comes from the blackbody filament and the filter itself. Thus, the power absorbed by the filter is

$$A_b \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_b) d\lambda + \frac{\sqrt{r_f^2 - r_b^2}}{r_f} A_f \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_f) d\lambda \quad (4)$$

and, by the Kirchhoff Law, the power emitted by the heated filter is

$$2 \times A_f \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_f) d\lambda. \quad (5)$$

The factor 2 is included in Eq. (5), because the filter has two sides. In steady state, Eqs. (4) and (5) are equal. Therefore, we have

$$A_f \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_f) d\lambda = \left(2 - \frac{\sqrt{r_f^2 - r_b^2}}{r_f}\right)^{-1} A_b \int_0^\infty \text{abs}_f(\lambda) u(\lambda, T_b) d\lambda. \quad (6)$$

By combining Eqs. (6) and (3), we get

$$P_{\text{Ext}} = A_b \int_0^\infty \left[1 - \text{ref}_f(\lambda) - \left(\frac{r_f - \sqrt{r_f^2 - r_b^2}}{2r_f - \sqrt{r_f^2 - r_b^2}}\right) \text{abs}_f(\lambda)\right] u(\lambda, T_b) d\lambda. \quad (7)$$

The temperature of the filter can also be calculated using Eq. (6). The total radiation power spectrum, $S(\lambda, T_b)$, through the ideal enclosure is the sum of the transmitted power through the filter and the outward radiated power from the heated filter

$$S(\lambda, T_b) d\lambda = [A_b \text{tr}_f(\lambda) u(\lambda, T_b) + A_f \text{abs}_f(\lambda) u(\lambda, T_f)] d\lambda. \quad (8)$$

As will be shown in Sec. III, the performance of a realistic system is limited by the absorption and the radiation loss of the second term.

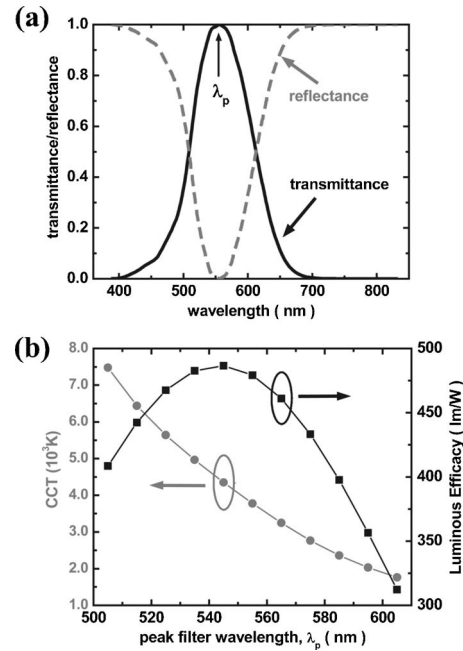


FIG. 3. (a) An ideal Gaussian filter with a fixed FWHM of 105 nm and $\lambda_p=555$ nm. The filter has no absorption. (b) The luminous efficiency (black squares) and the CCT (gray dots) of the ideal Gaussian filters as a function of peak filter position, λ_p .

The energy efficiency of a light source is evaluated by its luminous efficiency (LE), which is defined as

$$\text{LE} = \frac{683(\text{lm/W}) \int_0^\infty V(\lambda) S(\lambda, T_b) d\lambda}{\int_0^\infty S(\lambda, T_b) d\lambda} = \frac{683(\text{lm/W}) \int_0^\infty V(\lambda) S(\lambda, T_b) d\lambda}{P_{\text{Ext}}}, \quad (9)$$

where $V(\lambda)$ is the luminosity function¹⁴ and $683(\text{lm/W})V(\lambda)S(\lambda, T_b)$ is the luminous flux.

III. PHOTON RECYCLING USING IDEAL GAUSSIAN FILTER AND USING 2D METALLIC PHOTONIC CRYSTAL FILTER

Using the proposed photon recycling concept, we first examine an ideal filter with no absorption loss, i.e., $\text{abs}(\lambda) = 0$. It follows from Eq. (8) that the radiation power spectrum is simply given by: $A_b \text{tr}_f(\lambda) u_f(\lambda, T_b)$. As shown in Fig. 3(a), the filter has a peak transmission wavelength of $\lambda_p = 555$ nm which is the most sensitive wavelength to human eyes and a full width at half maximum (FWHM) of $\Delta\lambda_{\text{FWHM}} = 105$ nm. The filter is also assumed to have a Gaussian spectral line shape, which mimics the luminous function, $V(\lambda)$.¹⁴ Using this ideal filter, we obtain a luminous efficiency of 475 lm/W. We have also varied the filter's peak wavelength, λ_p , from 505 nm to 605 nm to find the maximum efficiency. The calculated luminous efficiency as a function of λ_p is shown as the square dots in Fig. 3(b). At $\lambda_p = 545$ nm a maximum efficiency of 487 lm/W is pre-

dicted. This corresponds to a near perfect matching of the filtered power spectrum $S(\lambda, T_b)$ to $V(\lambda)$.¹⁴ As λ_p is increased, the spectral matching is not as good and the luminous efficiency decreases from 487 to 312 lm/W at $\lambda_p = 605$ nm. Also shown in Fig. 3(b) is the calculated correlated color temperature (CCT) (Ref. 14) as a function of λ_p for this system. It shows that CCT decreases monotonically from 7500 K to 2000 K as λ_p varies from 505 nm to 605 nm. At maximum efficiency, the CCT is 4400 K.

To implement this idea, a 2D MPC filter is chosen to recycle the otherwise wasted infrared photons. Silver is selected as the metallic material, because it has a low intrinsic absorption in the visible and near infrared wavelengths. The property of low absorption is the key to simultaneously achieving a high filter transmittance in the visible and a high filter reflectance in the infrared. To calculate the optical properties of the MPC filter such as reflectance, absorbance, and transmittance, the transfer matrix method (TMM) (Ref. 15) and realistic refractive index values¹⁶ are used.

We select three different configurations of the MPC filters to examine. To select them we consider waveguide cut-off wavelength which is twice the air opening dimension. The selected configurations are $a_1=350$ nm and $d_1=250$ nm, $a_2=400$ nm and $d_2=300$ nm, and $a_3=500$ nm and $d_3=400$ nm, respectively. Here a is the lattice constant and d is the size of the air opening as depicted in Fig. 2(b). The thickness of the MPC filters is $h=500$ nm. The calculated optical properties of the three filters are shown in Figs. 4(a)–4(c), respectively. All three filters exhibit a high reflectance (the solid curve) in the infrared and a high transmission band (the dotted curve) in the visible. In Fig. 4(a), the transmission line shape, $tr_f(\lambda)$, follows closely the Gaussian function, which is consistent with that of the ideal filter. Also, the absorbance spectrum (the dashed curve), $abs_f(\lambda)$, has a finite value of $\sim 20\%$ in the visible and $\sim 5\%$ in the infrared. This absorption heats up the filter and contributes to heat loss. These computed curves, $tr_f(\lambda)$ and $abs_f(\lambda)$, are used as the input parameters for calculating the radiation power spectrum $S(\lambda, T_b)$, the luminous flux, and the luminous efficiency. In the calculation, the temperature of the blackbody filament is assumed to be $T_b=2800$ K. The ratio of the radius of the filament, r_b , and the radius of the filter, r_f , is 1:12.

In Fig. 5, we show, for each of the three filters, the filtered power spectrum, the luminous flux, along with a blackbody radiation curve at $T_b=2800$ K. In Fig. 5(a), the luminous flux curve (the red solid curve) follows closely the filtered power spectrum (the black solid curve). This is because there is a good matching between the filtered spectrum and the luminous function, $V(\lambda)$. For this filter configuration, the calculated luminous efficiency and flux per unit area of the filament are 125 lm/W and 3591 lm/cm², respectively. In Fig. 5(b), $S(\lambda, T_b)$ starts to deviate from the luminous flux curve. As a result, the calculated luminous efficiency is slightly lower (105 lm/W). However, the luminous flux increases to 4443 lm/cm². In Fig. 5(c), the deviation becomes larger and the corresponding luminous efficiency and flux are 60 lm/W and 4403 lm/cm². The calculated temperatures of

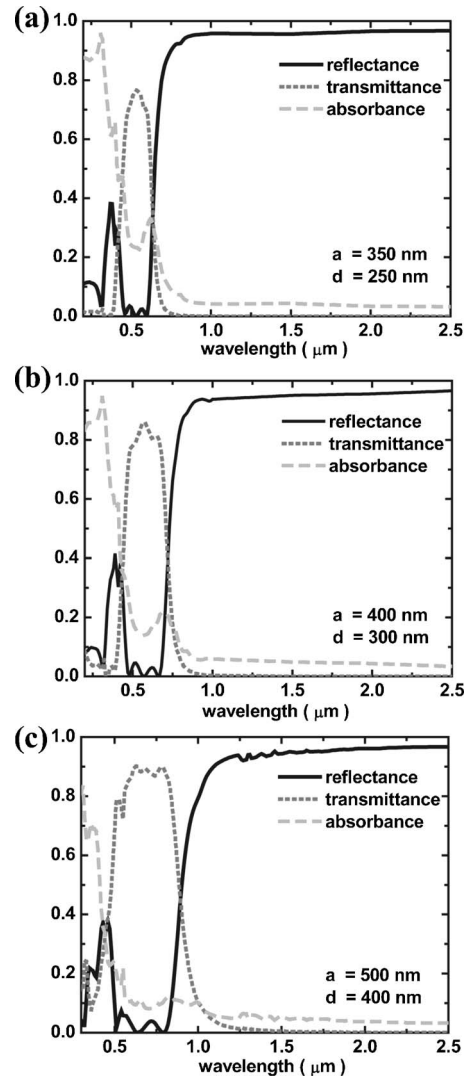


FIG. 4. The optical properties of the 2D MPC filters. (a) $a_1=350$ nm and $d_1=250$ nm, (b) $a_2=400$ nm and $d_2=300$ nm, and (c) $a_3=500$ nm and $d_3=400$ nm, respectively. The thickness, h , is 500 nm.

the filters are 951 K, 955 K, and 955 K for $a=350$ nm, 400 nm, and 500 nm, respectively. These values are lower than the melting temperature of silver, 1234.9 K.

The discrepancy in the luminous efficiency between the ideal filter and the realistic MPC filter is due to the finite absorbance of the MPC filter. Particularly, the second term in Eq. (8) contributes to an infrared loss as the filter's radiation is centered at $\lambda \sim 3$ μm . For our specific filters, this absorption loss consumes a significant portion of the total input power, 40%–67%, and hence reduces the luminous efficiency. To achieve a much higher efficiency of >200 –400 lm/w, the material loss must be overcome. However, comparing with the luminous efficiency of the blackbody at 2800 K, 16 lm/W, the proposed MPC filters can still improve the luminous efficiency by up to 8 times.

IV. COLOR QUALITY OF THE MPC-FILTERED LIGHT

For general purpose illumination, not only high efficiency but also the color quality is important in evaluating a light source. The color quality of a light source can be char-

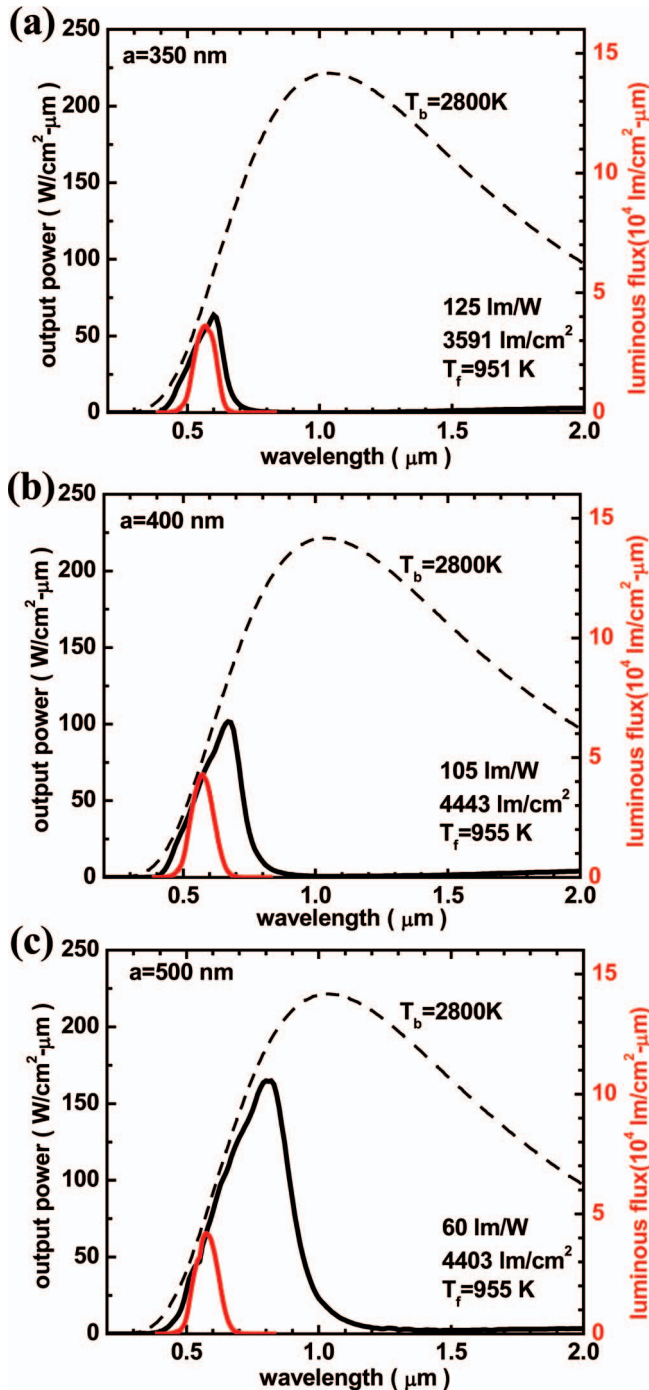


FIG. 5. (Color) The incandescent light source output spectra after MPC-filtering. (a) $a_1=350$ nm, (b) $a_2=400$ nm, and (c) $a_3=500$ nm, respectively. The black solid curves are output spectra after filtering and the black dashed curve is a blackbody radiation at 2800 K, the assumed temperature of the incandescent filament. The red solid curves are luminous flux. T_f is the temperature of the filter.

acterized by three parameters, namely, correlated color temperature (CCT), color chromaticity, and color rendering index (CRI). To calculate these quantities we use the method described in Ref. 14.

CCT is a way to assign a color temperature to a color near but not on the Planckian locus. CCT is also generally used to categorize color tone. If CCT is lower than 3300 K, the color is categorized as warm tone and if CCT is higher than 5300 K, the color is categorized as cool tone. The cal-

TABLE I. The calculated luminous efficiency, CCT, and CRI of the lights after filtering by MPC.

Lattice constant (nm)	Luminous efficiency (lm/W)	CCT (K)	CRI
350	125	3547	68
400	105	2749	89
500	60	2474	90

culated CCTs for the filtered lights are 3547 K, 2749 K, and 2474 K for MPC with $a=350$ nm, 400 nm, and 500 nm, respectively. These results are tabulated in Table I. The calculated CCTs imply that the filtered lights are warm tone or close to it.

The color coordinates, a measure of color chromaticity, are calculated to be $x=0.4235$, $y=0.4467$ for $a_1=350$ nm, $x=0.4670$, $y=0.4300$ for $a_2=400$ nm, and $x=0.4906$, $y=0.4328$ for $a_3=500$ nm and plotted in Fig. 6. For comparison, the color coordinates of a blackbody at 2800 K are also plotted.

CRI is a measure of the ability of a light source to reproduce the true color of objects. CRI has a range between 0 and 100, with 0 indicating minimum and 100 indicating maximum color rendering capability. For example, the CRI of a blackbody radiation source is 100 and that of a standard fluorescent lamp is around 60. The CRI's of MPC filtered lights are calculated to be 68, 89, and 90 for $a=350$ nm, 400 nm, and 500 nm, respectively, and these are also tabulated in Table I. These calculation results show that though the MPC-filtered light has a lower CRI value than that of the blackbody radiation, it is still higher than that of a fluorescent lamp.

V. CONCLUSION

In conclusion, we have investigated the concept of photon recycling for improving incandescent light emission. A

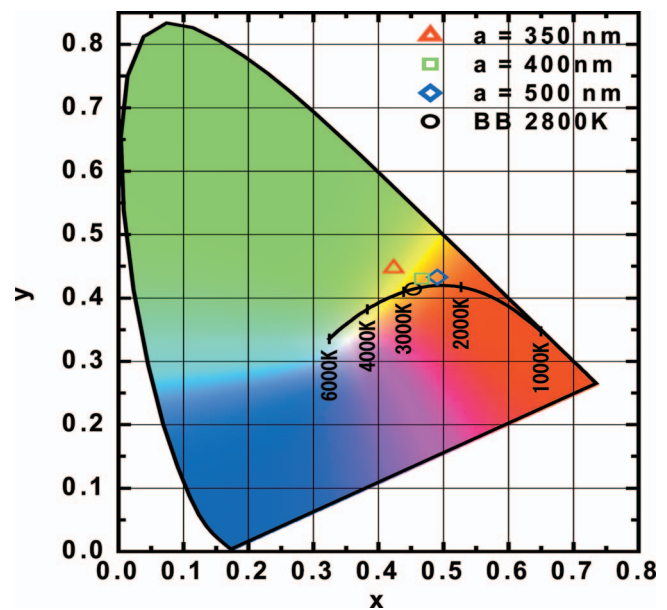


FIG. 6. (Color) The chromaticity color coordinates. The coordinates of three MPC-filtered lights and a blackbody at 2800 K are plotted.

maximum luminous efficiency of 475 lm/W is predicted for a filament at a temperature of 2800 K and using an ideal Gaussian-shape filter. We have also introduced a realistic 2D silver photonic crystal filter for photon recycling. A luminous efficiency of 125 lm/W is calculated, which is 8 times bigger than that of a traditional blackbody filament. The CCTs and the CRIs of the MPC-filtered lights are also calculated. The CCT for our configurations ranges from 2427 K to 3547 K, implying the filtered lights are warm tone. The CRIs are in the range of 68–90. These results show that the performance of photon recycled incandescent source using MPC can be comparable to the current most efficient lighting devices.

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