CHECK YOUR UNDERSTANDING.

EXAMPLE 1: Compute the derivative, $f'(x)$, then find $f'(1)$ for the following functions:

a) $f(x) = x^2 + 3$ $\quad f'(x) = 2x + 0 \quad f'(1) = 2$

b) $f(x) = 7\sqrt{x} + \frac{5}{x^3}$ $\quad f'(x) = \frac{7}{2}x^{-1/2} - 15x^{-4} \quad f'(1) = -\frac{23}{2}$

$f(x) = 7x^{1/2} + 5x^{-3}$

c) $f(x) = 3x^5 + 2x^2 + \frac{1}{x}$ $\quad f'(x) = 15x^4 + 4x - x^{-2} \quad f'(1) = 18$

$f(x) = 3x^5 + 2x^2 + x^{-1}$

EXAMPLE 2: Find when $f'(x) = 0$ for the function $f(x) = 3x^2 + 12x + 4$

$f'(x) = 6x + 12$

$f'(x) = 0 = 6x + 12 \quad 6x = -12 \quad \boxed{x = -2}$

EXAMPLE 3: Suppose that the Revenue of a company can be modelled by $R(q) = -q^2 + 400q + 22500$. Find the rate of change of Revenue when $q = 100$.

derivative

$R'(q) = -2q + 400$

$R'(100) = -200 + 400 = 200$

EXAMPLE 4: Suppose the height of a ball, in feet, can be modeled by:

$$s(t) = 16 - (t - 4)^2$$

where time, t ,is measured in seconds. Find the instantaneous velocity at $t = 2$

Derivative

$S(t) = 16 - (t-4)^2 = 16 - [t^2 - 8t + 16] = -t^2 + 8t$

$S'(t) = -2t + 8$

$S'(t) = -2(2) + 8 = 4$

EXAMPLE 5: If $h(x) = \frac{4abx+c}{d}$ where $a, b, c, \&\ d$ are constants. Find $h'(x)$:

$$\frac{4abx + c}{d} = \boxed{\frac{4ab}{d}}x + \boxed{\frac{c}{d}}$$

CONSTANT

constant

$$h'(x) = \frac{4ab}{d} + 0$$

EXAMPLE 6: An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled downward and then released, it vibrates vertically. The equation of motion is

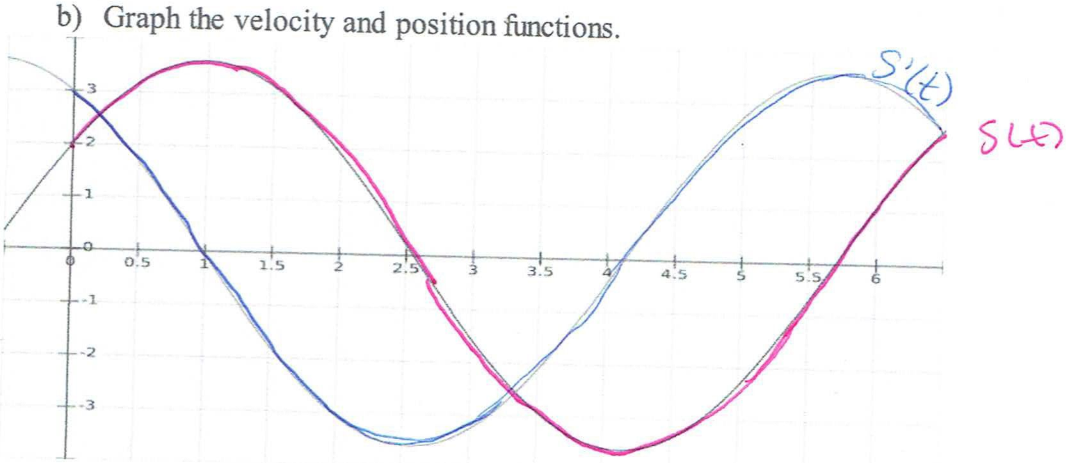
$$s(t) = 2\cos(t) + 3\sin(t)\,, \quad t \geq 0$$

where $s(t)$ is measured in centimeters and time t in seconds. (We take the positive direction to be downward.)

a) Find the velocity at time t. Be sure to include units.

$S'(t) = -2\sin(t) + 3\cos(t)\ \frac{cm}{sec}$

b) Graph the velocity and position functions.



$f'(x) = -3x^2 + 6x = -3x(x-2)$

$f'(x)$ — 0 + 0 —

-1 0 1 2 3

$f'(1) = +$

$f'(-1) = -$

$f'(3) = -$

EXAMPLE 7: For the function $f(x) = -x^3 + 3x^2 - 4$

a) Find the intervals where the function is increasing, decreasing.

Inc $(0,2)$ Dec $(-\infty, 0) + (2, \infty)$

b) Find the inflection points.

$f''(x) = -6x + 6 = 0$ $\boxed{x = 1}$

c) Find the intervals where the function is concave up, concave down.

$f''(x)$ + 0 —

-1 1

$f''(-1) = +$

$f''(2) = -$

d) Sketch the graph ccup $(-\infty, 1)$ ccdown $(1, \infty)$