

CHECK YOUR UNDERSTANDING.

EXAMPLE 1: For the following functions compute the derivative, $f'(x)$

$$\begin{array}{l} \text{a) } f(x) = x^7 \cdot \sin(x) \\ f'(x) = 7x^6 \cdot \sin x + x^7 \cdot \cos(x) \end{array} \left| \begin{array}{l} f = x^7 \quad g = \sin x \\ f' = 7x^6 \quad g' = \cos x \end{array} \right.$$

$$\begin{array}{l} \text{b) } f(x) = e^x \cdot \cos(x) \\ f'(x) = e^x \cos(x) - e^x \sin x \end{array} \left| \begin{array}{l} f = e^x \quad g = \cos x \\ f' = e^x \quad g' = -\sin(x) \end{array} \right.$$

$$\begin{array}{l} \text{c) } f(x) = 4^x \cdot \sqrt{x} = 4^x \cdot x^{1/2} \\ f'(x) = 4^x \ln 4 \cdot x^{1/2} + 4^x \cdot \frac{1}{2\sqrt{x}} \end{array} \left| \begin{array}{l} f = 4^x \quad g = x^{1/2} \\ f' = 4^x \ln 4 \quad g' = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} \end{array} \right.$$

EXAMPLE 2: For the following functions compute the both the first and second derivative.

$$\begin{array}{l} \text{a) } f(x) = x^6 \cdot \ln(x) \\ f'(x) = 6x^5 \cdot \ln x + \frac{1}{x} \cdot x^6 \\ = 6x^5 \cdot \ln x + x^5 \end{array} \left| \begin{array}{l} f = x^6 \quad g = \ln(x) \\ f' = 6x^5 \quad g' = \frac{1}{x} \end{array} \right. \begin{array}{l} \text{1st} \\ \text{2nd} \\ f = 6x^5 \quad g = \ln x \\ f' = 30x^4 \quad g' = \frac{1}{x} \end{array}$$

$$\begin{array}{l} f''(x) = 30x^4 \cdot \ln x + 6x^5 \cdot \frac{1}{x} + 5x^4 \\ = 30x^4 \ln x + 6x^4 + 5x^4 \\ = 30x^4 \ln x + 11x^4 \end{array}$$

$$\begin{array}{l} \text{b) } f(x) = \sin(x) \cdot e^x \\ f'(x) = \cos(x) \cdot e^x + \sin(x) \cdot e^x \\ f''(x) = [-\sin x \cdot e^x + \cos x \cdot e^x] + [\cos x \cdot e^x + \sin x \cdot e^x] \\ f''(x) = 2 \cos(x) \cdot e^x \end{array} \left| \begin{array}{l} f(x) = \sin x \quad g(x) = e^x \\ f'(x) = \cos x \quad g'(x) = e^x \\ f = \cos x \quad g = e^x \\ f' = -\sin x \quad g' = e^x \end{array} \right.$$

$$\begin{array}{l} \text{c) } f(x) = \ln(x) \cdot \cos(x) \\ f'(x) = \frac{1}{x} \cos x - \ln x \sin x \\ f''(x) = \left[-\frac{1}{x^2} \cos x - \frac{1}{x} \sin x \right] - \left[\frac{1}{x} \sin x + \ln x \cos x \right] \\ f''(x) = -\frac{1}{x^2} \cos x - \frac{2}{x} \sin x - \ln x \cos x \end{array} \left| \begin{array}{l} f = \frac{1}{x} \quad g = \cos x \\ f' = -\frac{1}{x^2} \quad g' = -\sin x \\ f = \ln x \quad g = \sin x \\ f' = \frac{1}{x} \quad g' = \cos x \end{array} \right.$$

EXAMPLE 3: For the following functions compute the derivative, $f'(x)$

$$\begin{aligned} \text{a) } f(x) &= \sqrt[3]{x^5} (6x^2 + 5x - 1) \\ &= x^{5/3} (6x^2 + 5x - 1) \\ &= 6x^{11/3} + 5x^{8/3} - x^{5/3} \\ f'(x) &= 22x^{8/3} + \frac{40}{3}x^{5/3} - \frac{5}{3}x^{2/3} \quad \text{or} \end{aligned}$$

$$\begin{aligned} f &= \sqrt[3]{x^5} = x^{5/3} & g &= (6x^2 + 5x - 1) \\ f' &= \frac{5}{3}x^{2/3} & g' &= 12x + 5 \\ f'(x) &= \frac{5}{3}x^{2/3} \cdot (6x^2 + 5x - 1) + x^{5/3} \cdot (12x + 5) \end{aligned}$$

$$\text{b) } f(x) = (\ln(6x)) \cdot (x^3 - x^2 + x)^2$$

$$\begin{aligned} f'(x) &= \frac{1}{x} \cdot (x^3 - x^2 + x)^2 + \\ &\quad \ln(6x) \cdot 2(x^3 - x^2 + x)(3x^2 - 2x) \end{aligned}$$

$$\begin{aligned} f &= \ln(6x) & g &= (x^3 - x^2 + x)^2 \\ f' &= \frac{6}{6x} = \frac{1}{x} & g' &= 2(x^3 - x^2 + x) \cdot (3x^2 - 2x) \\ z &= 6x & z &= x^3 - x^2 + x \\ \frac{dz}{dx} &= 6 & \frac{dz}{dx} &= 3x^2 - 2x \end{aligned}$$

$$\text{c) } f(x) = e^{x^2 + 2x - 1} \cdot (\ln(x^2))$$

$$\begin{aligned} f'(x) &= e^{x^2 + 2x - 1} (2x + 2) \ln(x^2) + \\ &\quad e^{x^2 + 2x - 1} \cdot \frac{2}{x} \end{aligned}$$

$$\begin{aligned} f &= e^{x^2 + 2x - 1} & g &= \ln(x^2) \\ f' &= e^{x^2 + 2x - 1} \cdot (2x + 2) & g' &= \frac{2x}{x^2} = \frac{2}{x} \\ z &= x^2 + 2x - 1 & z &= x^2 \\ \frac{dz}{dx} &= 2x + 2 & \frac{dz}{dx} &= 2x \end{aligned}$$

EXAMPLE 4: For the following functions compute the derivative, $f'(x)$

$$\begin{aligned} \text{a) } f(x) &= \frac{x^7}{\sin(x)} \\ f'(x) &= \frac{7x^6 \sin(x) - x^7 \cos(x)}{(\sin(x))^2} \end{aligned}$$

$$\begin{aligned} f &= x^7 & g &= \sin(x) \\ f' &= 7x^6 & g' &= \cos(x) \end{aligned}$$

$$\begin{aligned} \text{b) } f(x) &= \frac{e^x}{\cos(x)} \\ f'(x) &= \frac{e^x \cos(x) + e^x \sin(x)}{(\cos(x))^2} \end{aligned}$$

$$\begin{aligned} f &= e^x & g &= \cos(x) \\ f' &= e^x & g' &= -\sin(x) \end{aligned}$$

$$\text{c) } f(x) = \frac{4^x}{\sqrt{x}}$$

$$\begin{aligned} f'(x) &= 4^x \ln 4 \cdot x^{-1/2} - \frac{1}{2} x^{-3/2} \cdot 4^x \\ &= \frac{4^x \ln 4 \cdot x^{1/2} - \frac{1}{2} 4^x x^{-1/2}}{(x^{1/2})^2} \end{aligned}$$

$$\begin{aligned} f &= 4^x & g &= \sqrt{x} = x^{1/2} \\ f' &= 4^x \ln 4 & g' &= \frac{1}{2} x^{-1/2} \end{aligned}$$

Prod^{ct} $f = 6x^5 \cdot \ln x - x^5$ $g = (\ln x)^2$ → chain
 $f' = 30x^4 \cdot \ln x + 6x^4 \cdot \frac{1}{x} - 5x^4$ $g' = 2 \ln x \cdot \frac{1}{x}$
 $= 30x^4 \ln x + x^4$

EXAMPLE 5: For the following functions compute the both the first and second derivative.

a) $f(x) = \frac{x^6}{\ln(x)}$ $f = x^6$ $g = \ln x$
 $f' = 6x^5$ $g' = \frac{1}{x}$

$f'(x) = \frac{6x^5 \cdot \ln x - x^5}{(\ln x)^2}$ $f''(x) = \frac{(30x^4 \ln x + x^4)(\ln x)^2 - \ln x \cdot \frac{2}{x} \cdot (6x^5 \ln x - x^5)}{(\ln x)^4}$

b) $f(x) = \frac{\sin(x)}{e^x}$ $f = \sin x$ $g = e^x$

$f'(x) = \frac{\cos x e^x - \sin x e^x}{(e^x)^2}$ $f' = \cos x$ $g' = e^x$

$f''(x) = \frac{-2 \sin x e^x \cdot (e^x)^2 - 2(e^x)^2 [\cos x e^x - \sin x e^x]}{(e^x)^4}$
 $f' = -\sin x e^x + \cos x e^x - [\cos x e^x + \sin x e^x]$

c) $f(x) = \frac{\ln(x)}{\cos(x)}$ $f = \ln x$ $g = \cos x$
 $f' = \frac{1}{x}$ $g' = -\sin x$

$f'(x) = \frac{\frac{1}{x} \cdot \cos x + \ln x \sin x}{(\cos x)^2}$ $f''(x) = \frac{(-\frac{1}{x^2} \cos x - \frac{1}{x} \sin x + \frac{1}{x} \sin x + \ln x \cos x) \cos x^2 - \star}{(\cos x)^4}$

$\star = -2 \cos(x) \sin(x) \cdot [\frac{1}{x} \cos x + \ln x \sin x]$

EXAMPLE 6: For the following functions compute the derivative, $f'(x)$

a) $f(x) = \frac{x^2+7}{(x^3-3x+4)}$ $F = x^2+7$ $g = (x^3-3x+4)$
 $f' = 2x$ $g' = 3x^2-3$

$f'(x) = \frac{2x(x^3-3x+4) - (x^2+7)(3x^2-3)}{(x^3-3x+4)^2}$

b) $f(x) = \frac{12x^3-18x^2}{e^{6x}}$ $f = 12x^3-18x^2$ $g = e^{6x}$

$f'(x) = \frac{(36x^2-36x)e^{6x} - e^{6x} \cdot 6(12x^3-18x^2)}{(e^{6x})^2}$ $g' = e^{6x} \cdot 6$
 $f' = 36x^2-36x$ $z = 6x$
 $\frac{dz}{dx} = 6$

c) $f(x) = \frac{\ln(7x)}{(x^4+3x^3-4x^2)}$ $f = \ln(7x)$ $g = x^4+3x^3-4x^2$
 $f' = \frac{1}{7x} \cdot 7 = \frac{1}{x}$ $g' = 4x^3+9x^2-8x$

$f'(x) = \frac{\frac{1}{x}(x^4+3x^3-4x^2) - \ln(7x)(4x^3+9x^2-8x)}{(x^4+3x^3-4x^2)^2}$