CHECK YOUR UNDERSTANDING.

EXAMPLE 1: For the following functions compute the derivative, $f'(x)$

a) $f(x) = x^7 * \sin(x)$

$f'(x) = 7x^6 \cdot \sin x + x^7 \cos(x)$

$f = x^7 \quad g = \sin x$

$f' = 7x^6 \quad g' = \cos x$

b) $f(x) = e^x * \cos(x)$

$f'(x) = e^x \cos(x) - e^x \sin x$

$f = e^x \quad g = \cos x$

$f' = e^x \quad g' = -\sin(x)$

c) $f(x) = 4^x * \sqrt{x} = 4^x \cdot x^{1/2}$

$f'(x) = 4^x \ln 4 \cdot x^{1/2} + 4^x \cdot \frac{1}{2\sqrt{x}}$

$f = 4^x \quad g = x^{1/2}$

$f' = 4^x \ln 4 \quad g' = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

EXAMPLE 2: For the following functions compute the both the first and second derivative.

a) $f(x) = x^6 * \ln(x)$

1st

$f = x^6 \quad g = \ln(x)$

$f' = 6x^5 \quad g' = \frac{1}{x}$

$f'(x) = 6x^5 \cdot \ln x + \frac{1}{x} \cdot x^6$

$= 6x^5 \cdot \ln x + x^5$

2nd

$f = 6x^5 \quad g = \ln x$

$f' = 30x^4 \quad g' = \frac{1}{x}$

$f''(x) = 30x^4 \cdot \ln x + 6x^5 \cdot \frac{1}{x} + 5x^4$

$= 30x^4 \ln x + 6x^4 + 5x^4$

$= 30x^4 \ln x + 11x^4$

b) $f(x) = \sin(x) * e^x$

$f(x) = \sin x \quad g(x) = e^x$

$f'(x) = \cos x \quad g'(x) = e^x$

$f'(x) = \cos(x) \cdot e^x + \sin(x) e^x$

$f = \cos x \quad g = e^x$

$f' = -\sin x \quad g' = e^x$

$f''(x) = [-\sin x \cdot e^x + \cos x e^x] + [\cos x \cdot e^x + \sin x \cdot e^x]$

$f''(x) = 2\cos(x) \cdot e^x$

c) $f(x) = \ln(x) * \cos(x)$

$f = \ln x \quad g = \cos x$

$f' = \frac{1}{x} \quad g' = -\sin x$

$f'(x) = \frac{1}{x} \cos x - \ln x \sin x$

$f = \frac{1}{x} \quad g = \cos x$

$f' = -\frac{1}{x^2} \quad g' = -\sin x$

$f = \ln x \quad g = \sin x$

$f' = \frac{1}{x} \quad g' = \cos x$

$f''(x) = \left[-\frac{1}{x^2} \cos x - \frac{1}{x} \sin x\right] - \left[\frac{1}{x} \sin x + \ln x \cos x\right]$

$f''(x) = -\frac{1}{x^2} \cos x - \frac{2}{x} \sin x - \ln x \cos x$

EXAMPLE 3: For the following functions compute the derivative, $f'(x)$

a) $f(x) = \sqrt[3]{x^5}\,(6x^2 + 5x - 1)$

$= x^{5/3}(6x^2+5x-1)$

$= 6x^{11/3} + 5x^{8/3} - x^{5/3}$

$f'(x) = 22x^{8/3} + \frac{40}{3}x^{5/3} - \frac{5}{3}x^{2/3}$ or

$f = \sqrt[3]{x^5} = x^{5/3}$ $\quad g = (6x^2+5x-1)$

$f' = \frac{5}{3}x^{2/3}$ $\quad g' = 12x+5$

$f'(x) = \frac{5}{3}x^{2/3}\cdot(6x^2+5x-1) + x^{5/3}\cdot(12x+5)$

b) $f(x) = \left(\ln(6x) * (x^3 - x^2 + x)\right)^2$

$$\boxed{f'(x) = \frac{1}{x}\cdot(x^3-x^2+x)^2 + \ln(6x)\cdot 2(x^3-x^2+x)(3x^2-2x)}$$

$f = \ln(6x)$ $\quad g = (x^3-x^2+x)^2$

$f' = \frac{6}{6x} = \frac{1}{x}$ $\quad g' = 2(x^3-x^2+x)\cdot(3x^2-2x)$

$z = 6x$ $\quad z = x^3 - x^2 + x$

$\frac{dz}{dx} = 6$ $\quad \frac{dz}{dx} = 3x^2 - 2x$

c) $f(x) = e^{x^2+2x-1} * (\ln(x^2))$

$$\boxed{f'(x) = e^{x^2+2x-1}(2x+2)\cdot\ln(x^2) + e^{x^2+2x-1}\cdot 2/x}$$

$f = e^{x^2+2x-1}$ $\quad g = \ln(x^2)$

$f' = e^{x^2+2x-1}\cdot(2x+2)$ $\quad g' = \frac{2x}{x^2} = \frac{2}{x}$

$z = x^2+2x-1$ $\quad z = x^2$

$dz/dx = 2x+2$ $\quad \frac{dz}{dx} = 2x$

EXAMPLE 4: For the following functions compute the derivative, $f'(x)$

a) $f(x) = \frac{x^7}{\sin(x)}$

$f'(x) = \frac{7x^6\sin x - x^7\cos x}{(\sin x)^2}$

$f = x^7$ $\quad g = \sin x$

$f' = 7x^6$ $\quad g' = \cos x$

b) $f(x) = \frac{e^x}{\cos(x)}$

$f'(x) = \frac{e^x\cos x + e^x\sin x}{(\cos x)^2}$

$f = e^x$ $\quad g = \cos x$

$f' = e^x$ $\quad g' = -\sin x$

c) $f(x) = \frac{4^x}{\sqrt{x}}$

$f = 4^x$ $\quad g = \sqrt{x} = x^{1/2}$

$f' = 4^x\ln 4$ $\quad g' = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{4^x\ln 4\cdot x^{1/2} - \frac{1}{2}x^{-1/2}\cdot 4^x}{(x^{1/2})^2}$

Product $f = 6x^5 \cdot \ln x - x^5$ $\quad g = (\ln x)^2$ chain

$f' = 30x^4 \cdot \ln x + 6x^4$ $\quad g' = 2\ln x \cdot \frac{1}{x}$

$= 30x^4 \ln x + x^4$

EXAMPLE 5: For the following functions compute the both the first and second derivative.

a) $f(x) = \frac{x^6}{\ln(x)}$ $\quad f = x^6 \quad g = \ln x$

$f' = 6x^5 \quad g' = \frac{1}{x}$

$$f'(x) = \frac{6x^5 \cdot \ln x - x^5}{(\ln x)^2} \qquad f''(x) = \frac{(30x^4 \ln x + x^4)(\ln x)^2 - \ln x \frac{2}{x} \cdot (6x^5 \ln x - x^5)}{(\ln x)^4}$$

b) $f(x) = \frac{\sin(x)}{e^x}$ $\quad f = \sin x \quad g = e^x$

$f' = \cos x \quad g' = e^x$

$$f'(x) = \frac{\cos x e^x - \sin x e^x}{(e^x)^2} \qquad f''(x) = \frac{-2\sin x e^x \cdot (e^x)^2 - 2(e^x)^2[\cos x e^x - \sin x e^x]}{(e^x)^4}$$

$f' = -\sin x e^x + \cos x e^x - [\cos x e^x + \sin x e^x]$

c) $f(x) = \frac{\ln(x)}{\cos(x)}$ $\quad f = \ln x \quad g = \cos x$

$f' = \frac{1}{x} \quad g' = -\sin x$

$$f'(x) = \frac{\frac{1}{x} \cdot \cos x + \ln x \sin x}{(\cos x)^2} \qquad f''(x) = \frac{\left(-\frac{1}{x^2}\cos x - \frac{1}{x}\sin x + \frac{1}{x}\sin x + \ln x \cos x\right)\cos x^2 - ☆}{(\cos x)^4}$$

$☆ = -2\cos(x)\sin(x) \cdot \left[\frac{1}{x}\cos x + \ln x \sin x\right]$

EXAMPLE 6: For the following functions compute the derivative, $f'(x)$

a) $f(x) = \frac{x^2+7}{(x^3-3x+4)}$ $\quad f = x^2 + 7 \quad g = (x^3 - 3x + 4)$

$f' = 2x \quad g' = 3x^2 - 3$

$$f'(x) = \frac{2x(x^3-3x+4) - (x^2+7)(3x^2-3)}{(x^3-3x+4)^2}$$

b) $f(x) = \frac{12x^3-18x^2}{e^{6x}}$ $\quad f = 12x^3 - 18x^2 \quad g = e^{6x}$

$f' = 36x^2 - 36x \quad g' = e^{6x} \cdot 6$

$z = 6x$

$\frac{dz}{dx} = 6$

$$f'(x) = \frac{(36x^2 - 36x)e^{6x} - e^{6x} \cdot 6(12x^3 - 18x^2)}{(e^{6x})^2}$$

c) $f(x) = \frac{\ln(7x)}{(x^4+3x^3-4x^2)}$ $\quad f = \ln(7x) \quad g = x^4 + 3x^3 - 4x^2$

$f' = \frac{1}{7x} \cdot 7 = \frac{1}{x} \quad g' = 4x^3 + 9x^2 - 8x$

$$f'(x) = \frac{\frac{1}{x}(x^4 + 3x^3 - 4x^2) - \ln(7x)(4x^3 + 9x^2 - 8x)}{(x^4 + 3x^3 - 4x^2)^2}$$