CHECK YOUR UNDERSTANDING.

EXAMPLE 1: For the following functions compute the derivative, $f'(x)$, and $f'(1)$

a) $f(x) = e^{3x}$ $f'(x) = 3e^{3x}$ $f'(1) = 3e^3$

b) $f(x) = \ln(x^3 + 2x)$ $f'(x) = \frac{1}{x^3+2x} \cdot (3x^2+2) = \frac{3x^2+2}{x^3+2x}$

$f'(1) = 5/3$

c) $f(x) = e^{(x^3+2x)}$

$f'(x) = (3x^2+2) \cdot e^{(x^3+2x)}$ $f'(1) = 5e^3$

d) $f(x) = (x^3 + 2x)^{127}$

$f'(x) = 127(x^3+2x)^{126}$ $f'(1) = 127(3)^{126}$

e) $f(x) = (2^x - 5x^2)^7$

$f'(x) = 7(2^x - 5x^2)^6 \cdot (2^x \ln 2 - 10x)$

$f'(1) = 7(-3)^6(2\ln 2 - 10)$

EXAMPLE 2: Iodine-131 is a highly radioactive isotope that decays exponentially. The amount of Iodine, $I(t)$, in a sample after t days can be modelled by: $I(t) = 2^{-0.125t}$. Find the rate at which the Iodine-131 is decaying after 3 days.

$I'(t) = 2^{-0.125t} \cdot \ln(2) \cdot (-0.125)$

$I'(3) \approx$

EXAMPLE 3: For each of the following functions identify the inside and outside function (from composite functions) and then calculate the derivative of the original function noting that a,b, and c are constants:

$f(x) = g(\underbrace{h(x)}_{\text{inside}})$

a) $f(x) = (ax^2 + b)^c$ $g(x) = x^c$ $h(x) = ax^2 + b$ $f'(x) = c(ax^2+b)^{c-1} \cdot (2ax)$

b) $f(x) = be^{a^2x}$ $g(x) = be^x$ $h(x) = a^2x$ $f'(x) = be^{a^2x} \cdot (a^2)$

c) $f(x) = \ln(x^a + bx - c)$ $g(x) = \ln(x)$ $h(x) = x^a + bx - c$

$f'(x) = \frac{1}{x^a+bx-c} \cdot (ax^{a-1}+b) = \frac{(ax^{a-1}+b)}{x^a+bx-c}$

EXAMPLE 4: Use the information from the table below to answer the following questions:

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$	$f''(x)$
0	0	1	2	-1	4	-5	0
1	3	2	1	3	-2	-4	-4
2	1	0	3	-2	3	2	1
3	2	3	0	4	2	-3	2

a) Determine if $y = h(g(x))$ is increasing or decreasing at $x = 3$

$y' = h'(g(x)) \cdot g'(x)$

@3 $= h'(g(3)) \cdot g'(3)$

$= h'(3) \cdot 2 = -3 \cdot 2$

$= -6$ so decreasing

b) Find the equation of the tangent line to $y = f(g(x))$ at $x = 2$

$f(g(2)) = f(0) = 0$ $\quad f'(g(2)) \cdot g'(2)$

$(2,0)$ $\quad f'(0) \cdot 3 = -1 \cdot 3 = -1$

$y = -x + 2$

c) Find the slope of the tangent line to $y = e^{g(x)}$ at $x = 0$

$y' = e^{g(x)} \cdot g'(x)$

$= e^{g(0)} \cdot g'(0) = \boxed{e^1 \cdot 4}$

EXAMPLE 5: A ball at the end of an elastic band is oscillating up and down (see figure 1). Its height, given in feet, above the floor at time t, in seconds, is given by $h(t) = 4 + \sin(\frac{t}{2})$.

a) How fast is the ball traveling after 2 seconds? After 4 seconds? After 60 seconds?

velocity

$h'(t) = \cos(\frac{t}{2}) \cdot \frac{1}{2}$ $\quad h'(2) \approx$ $\quad h'(4) \approx$ $\quad h'(60) \approx$

b) Is the ball moving up or down after 2 seconds? After 4 seconds? After 60 seconds?

c) Is the vertical velocity of the ball ever equal to 0?

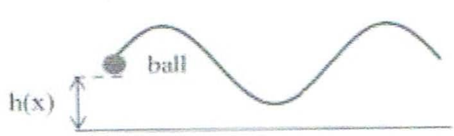

Fig. 1

$h'(t) = \frac{1}{2}\cos(t/2) = 0$

$\cos(x) = 0$ at $\pi/2$, $3\pi/2$, $5\pi/2$ etc

so $h'(t) = 0$ if $t = \pi, 3\pi, 5\pi$ etc.

EXAMPLE 6:

Consider the functions $f(x) = x^2$, $g(x) = e^x$, $h(x) = \sqrt{x-2}$ find the following:

Composition	New Composite Function	Derivative
$f(g(x))$	$(e^x)^2 = e^{2x}$	$z = 2x \quad \frac{dz}{dx} = 2$ $e^{2x} * 2$
$g(f(x))$	e^{x^2}	$z = x^2 \quad \frac{dz}{dx} = 2x$ $e^{2x} * 2x$
$h(f(x))$	$\sqrt{x^2 - 2} = (x^2 - 2)^{\frac{1}{2}}$	$z = x^2 - 2 \quad \frac{dz}{dx} = 2x$ $\frac{1}{2}(x^2 - 2)^{\frac{1}{2}} * 2x$
$h(g(x))$	$\sqrt{e^x - 2} = (e^x - 2)^{\frac{1}{2}}$	$z = e^x - 2 \quad \frac{dz}{dx} = e^x$ $\frac{1}{2}(e^x - 2)^{\frac{1}{2}} * e^x$
$h(h(x))$	$\sqrt{\sqrt{x-2} - 2} = \left((x-2)^{\frac{1}{2}} - 2\right)^{\frac{1}{2}}$	$z = \sqrt{x-2} - 2 \quad \frac{dz}{dx} = \frac{1}{2}(x-2)^{-\frac{1}{2}}$ $\frac{1}{2}((x-2)^{\frac{1}{2}} - 2)^{-\frac{1}{2}} * \frac{1}{2}(x-2)^{-\frac{1}{2}}$