Practice Derivatives

1.) $f(x) = x^3 - 3x^2 + 5x$

$$\boxed{f'(x) = 3x^2 - 6x + 5}$$

2.) $f(x) = (x^2+4)(5x-1)$

$$= 5x^3 - x^2 + 20x - 4$$

$$\boxed{f'(x) = 15x^2 - 2x + 20}$$

3.) $f(x) = e^{(1+3x)^4}$

$$f'(x) = e^{(1+3x)^4} \cdot \frac{d}{dx}\left[(1+3x)^4\right]$$

$$= e^{(1+3x)^4}\left[4(1+3x)^3 \cdot 3\right]$$

$$\boxed{f'(x) = 12(1+3x)^3 e^{(1+3x)^4}}$$

4.) $f(z) = \ln(z^2+1)$

$$f'(z) = \frac{1}{z^2+1} \frac{d}{dz}\left[z^2+1\right]$$

$$\boxed{f'(z) = \frac{2z}{z^2+1}}$$

5.) $f(x) = (1+e^{5x})^{12}$

$$f'(x) = 12(1+e^{5x})^{11} \frac{d}{dx}\Big[\underbrace{1+e^{5x}}_{5e^{5x}}\Big]$$

$$\boxed{f'(x) = 60e^{5x}(1+e^{5x})^{11}}$$

6.) $f(x) = \ln(e^{-x} - x)$

$$f'(x) = \frac{1}{e^{-x} - x} \cdot \frac{d}{dx}\Big[\underbrace{e^{-x} - x}_{-e^{-x}-1}\Big]$$

$$\boxed{f'(x) = \frac{-e^{-x} - 1}{e^{-x} - x}}$$

7.) $f(x) = (\sin(4x))^5$

$$f'(x) = 5(\sin(4x))^4 \frac{d}{dx}\Big[\underbrace{\sin(4x)}_{4\cos(4x)}\Big]$$

$$\boxed{f'(x) = 20\cos(4x)(\sin(4x))^4}$$

8.) $f(x) = x^2(3x+1)^3$

$f = x^2$ $\quad + \quad$ $g = (3x+1)^3$

$f' = 2x$ $\quad\quad$ $g' = 3(3x+1)^2 \frac{d}{dx}[3x+1]$

$= 9(3x+1)^2$

$$\boxed{f'(x) = 9x^2(3x+1)^2 + 2x(3x+1)^3}$$

9.) $f(x) = 7x^3 \cdot e^{x^2}$

$f = 7x^3$ $\quad + \quad$ $g = e^{x^2}$

$f' = 21x^2$ $\quad\quad$ $g' = e^{x^2} \cdot \frac{d}{dx}[x^2] = 2xe^{x^2}$

$$\boxed{f'(x) = 14x^4e^{x^2} + 21x^2e^{x^2}}$$

10.) $f(x) = x^3 \cdot 2^x$

$f = x^3$ $\quad + \quad$ $g = 2^x$

$f' = 3x^2$ $\quad\quad$ $g' = 2^x \cdot \ln 2$

$$\boxed{f'(x) = x^3 \cdot 2^x \cdot \ln 2 + 3x^2 \cdot 2^x}$$

11.) $f(x) = \dfrac{x^2+5x+2}{x+3}$

low $= x+3$ — high $= x^2+5x+2$

d low $= 1$ — d high $= 2x+5$

$$f'(x) = \frac{(x+3)(2x+5) - (x^2+5x+2)}{(x+3)^2}$$

$$= \frac{2x^2 + \cancel{5x} + 6x + 15 - x^2 - \cancel{5x} - 2}{(x+3)^2}$$

$$\boxed{f'(x) = \frac{x^2+6x+13}{(x+3)^2}}$$

12.) $f(x) = \dfrac{1+x^2}{3+2x^2}$

low $= 3+2x^2$ — high $= 1+x^2$

d low $= 4x$ — d high $= 2x$

$$f'(x) = \frac{(3+2x^2)(2x) - (4x)(1+x2)}{(3+2x^2)^2} = \frac{6x + \cancel{4x^3} - 4x - \cancel{4x^3}}{(3+2x^2)^2}$$

$$\boxed{f'(x) = \frac{2x}{(3+2x^2)^2}}$$

13.) $f(x) = \cos(3x^3 \cdot 3^x)$

$$f'(x) = -\sin(3x^3 \cdot 3^x)\frac{d}{dx}[3x^3 \cdot 3^x]$$

$f = 3x^3$ $\quad$ + $\quad$ $g = 3^x$

$f' = 9x^2$ $\quad$ $\quad$ $g' = 3^x \ln 3$

$$\boxed{f'(x) = -\sin(3x^3 \cdot 3^x)(3x^3 \cdot 3^x \ln 3 + 9x^2 \cdot 3^x)}$$

14.) $f(x) = \sin\sqrt{e^x + 1}$

$f(x) = \sin((e^x+1)^{1/2})$

$$f'(x) = \cos((e^x+1)^{1/2})\frac{d}{dx}[(e^x+1)^{1/2}]$$

$\hookrightarrow \frac{1}{2}(e^x+1)^{-1/2}[e^x]$

$$\boxed{f'(x) = \frac{e^x \cos\sqrt{e^x+1}}{2\sqrt{e^x+1}}}$$

15.) $f(x) = 2x\cos(3x^2)$

$f = 2x$ + $g = \cos(3x^2)$

$f' = 2$ $g' = -\sin(3x^2)\cdot\frac{d}{dx}[3x^2]$

$= -6x\sin(3x^2)$

$$\boxed{f'(x) = -12x^2\sin(3x^2) + 2\cos(3x^2)}$$

16.) $f(x) = \dfrac{1+3e^{3x}}{e^{9x^4}}$

$low = e^{9x^4}$ − $high = 1+3e^{3x}$

$dlow = 36x^3\cdot e^{9x^4}$ $dhigh = 9e^{3x}$

$$\boxed{f'(x) = \frac{9e^{3x}\cdot e^{9x^4} - (1+3e^{3x})(36x^3e^{9x^4})}{\left(e^{9x^4}\right)^2}}$$

17.) $f(x) = \left(5x^3\ln(3x^2)\right)^5$

$f'(x) = 5\left(5x^3\ln(3x^2)\right)^4\frac{d}{dx}\left[5x^3\ln(3x^2)\right]$

$f = 5x^3$ + $g = \ln(3x^2)$

$f' = 15x^2$ $g' = \frac{1}{3x^2}\cdot 6x = \frac{2}{x}$

$f'(x) = 5\left(5x^3\ln(3x^2)\right)^4\left(5x^3\cdot\frac{2}{x} + 15x^2\ln(3x^2)\right)$

$$\boxed{f'(x) = 5\left(5x^3\ln(3x^2)\right)^4\left(10x^2 + 15x^2\ln(3x^2)\right)}$$

18.) $f(x) = (4x^3 - 6x^2)^8 (e^x + 3x)^4$

$f = (4x^3 - 6x^2)^8 \quad + \quad g = (e^x + 3x)^4$

$f' = 8(4x^3 - 6x^2)^7 (12x^2 - 12x) \quad g' = 4(e^x + 3x)^3 (e^x + 3)$

$$\boxed{f'(x) = 4(4x^3 - 6x^2)^8 (e^x + 3x)^3 (e^x + 3) + 8(4x^3 - 6x^2)^7 (12x^2 - 12x)(e^x + 3x)^4}$$

19.) $f(x) = \left(\dfrac{x + \cos(2x)}{4\sin(3x)}\right)^9$

$$f'(x) = 9\left(\frac{x + \cos(2x)}{4\sin(3x)}\right)^8 \frac{d}{dx}\left[\frac{x + \cos(2x)}{4\sin(3x)}\right]$$

low $= 4\sin(3x) \quad - \quad$ high $= x + \cos(2x)$

d low $= 12\cos(3x) \quad$ d high $= 1 - 2\sin(2x)$

$$\frac{4\sin(3x)(1 - 2\sin(2x)) - 12\cos(3x)(x + \cos(2x))}{(4\sin(3x))^2}$$

$$\boxed{f'(x) = 9\left(\frac{x + \cos(2x)}{4\sin(3x)}\right)^8 \left(\frac{4\sin(3x)(1 - 2\sin(2x) - 12\cos(3x)(x + \cos(2x))}{(4\sin(3x))^2}\right)}$$

20.) $f(x) = \cos\left(\frac{3x^3+1}{4x^6}\right)$

$$f'(x) = -\sin\left(\frac{3x^3+1}{4x^6}\right)\frac{d}{dx}\left[\frac{3x^3+1}{4x^6}\right]$$

low $= 4x^6$ high $= 3x^3+1$

d low $= 24x^5$ d high $= 9x^2$

$$= \frac{4x^6(9x^2) - (24x^5)(3x^3+1)}{(4x^6)^2}$$

$$= \frac{36x^8 - 72x^8 - 24x^5}{16x^{12}} = \frac{-36x^8 - 24x^5}{16x^{12}}$$

$$\boxed{f'(x) = -\sin\left(\frac{3x^3+1}{4x^6}\right)\left(\frac{-36x^8 - 24x^5}{16x^{12}}\right)}$$