Remembering trig formulas

Which trig formulas should you remember? Here's some guidance. Although you won't have the use of *Mathematica* for this exam, I've indicated how to find some using it.

1. Definitions of $\sin z$ and $\cos z$ in terms of power series.

2. Other trig and hyperbolic definitions are the same as in the real case, e.g., $\tan z = \sin z / \cos z$.

3. Many trig identities are the same as in the real case, e.g., $\sin^2 z + \cos^2 z = 1$.

4. Formulas for derivatives of trig and inverse trig functions are the same as in the real case.

5. You should certainly know the identity $\exp(i\,z) = \cos z + i \sin z$. But that's easy to remember because it just generalizes the formula you already know in the case when z is real.

6. Some other trig formulas follow from the one for $\exp(i\,z)$, provided you also use it to get first a formula for $\exp(-i\,z)$. For example: $\sin z = \frac{1}{2i}\left(e^{i\,z} - e^{-i\,z}\right)$. With *Mathematica*:

```
   TrigToExp[Cos[z]]
Exp[I z]/2 + Exp[-I z]/2
```

7. In *Mathematica* you can recover the definitions of sinh and cosh by `TrigToExp`. Thus:

```
   TrigToExp[Sinh[z]]
E^z/2 - E^(-z)/2
```

8. Some other formulas expressing trig functions in terms of functions of their real and imaginary parts are also recovered by using `ComplexExpand` in *Mathematica*. For example:

```
   ComplexExpand[Cosh[x + I y]]
Cos[y] Cosh[x] + I Sin[y] Sinh[x]
```

9. The "troublesome" formulas that express the inverse trig and inverse hyperbolic functions in terms of logs. These can be derived as in class or the text. If these come up in the exam, I'll give them to you. Again in *Mathematica*, `TrigToExp` comes to your aid. For example:

```
   TrigToExp[ArcSin[z]]
-I Log[I z + Sqrt[1 - z^2]]
```

10. To get the corresponding multi-valued functions, just replace the upper-case letters in function names with the corresponding lower-case characters so as to get "arc" and "log", etc.