

Modeling the atmosphere of Jupiter

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Reference: Turkington, Majda, Haven and DiBattista, (2001),
"Statistical equilibrium predictions of the jets and spots on Jupiter,"
Proc. Nat. Acad. Sci. **98**, pp. 12346–12350.

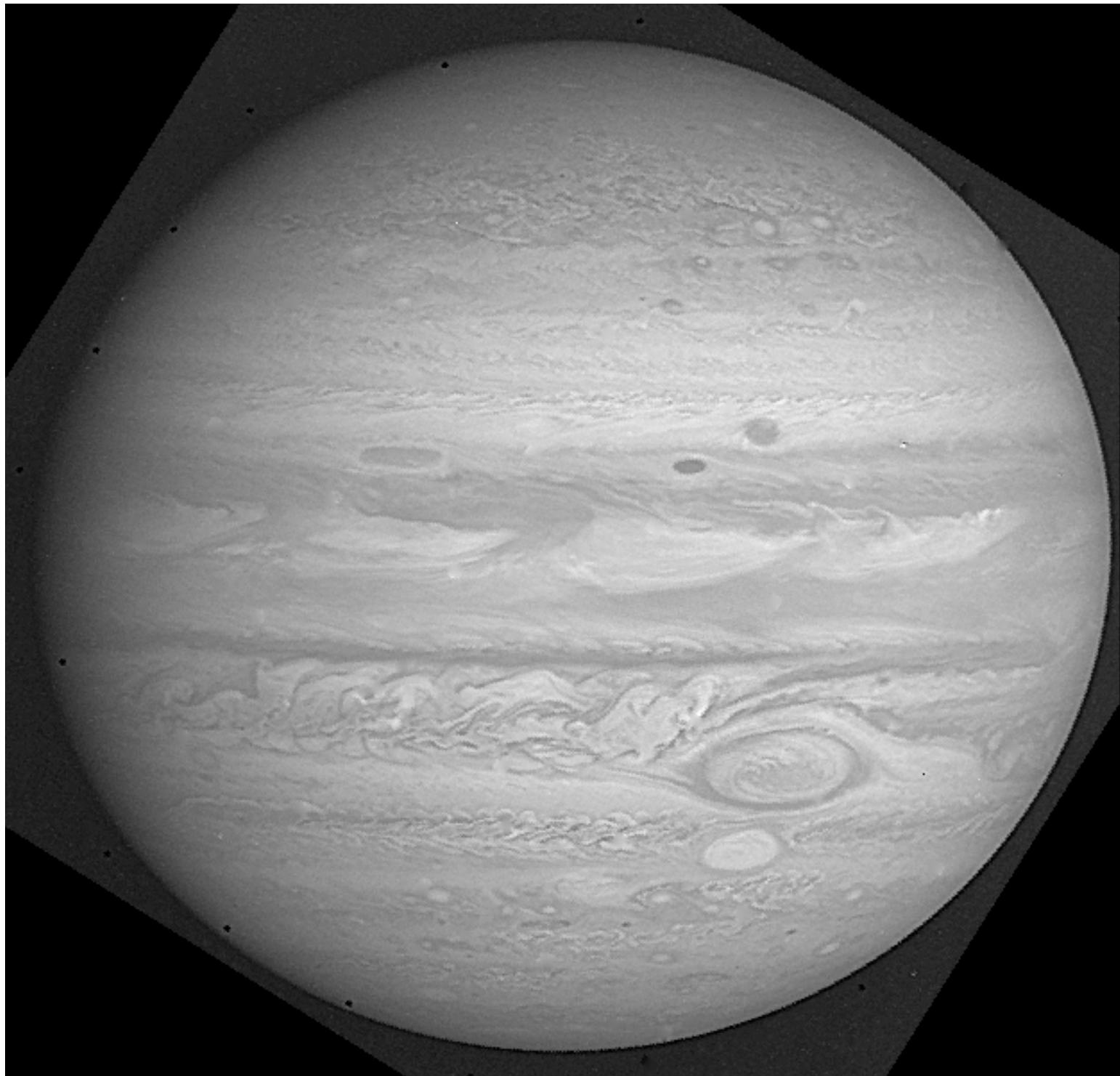
What we see:

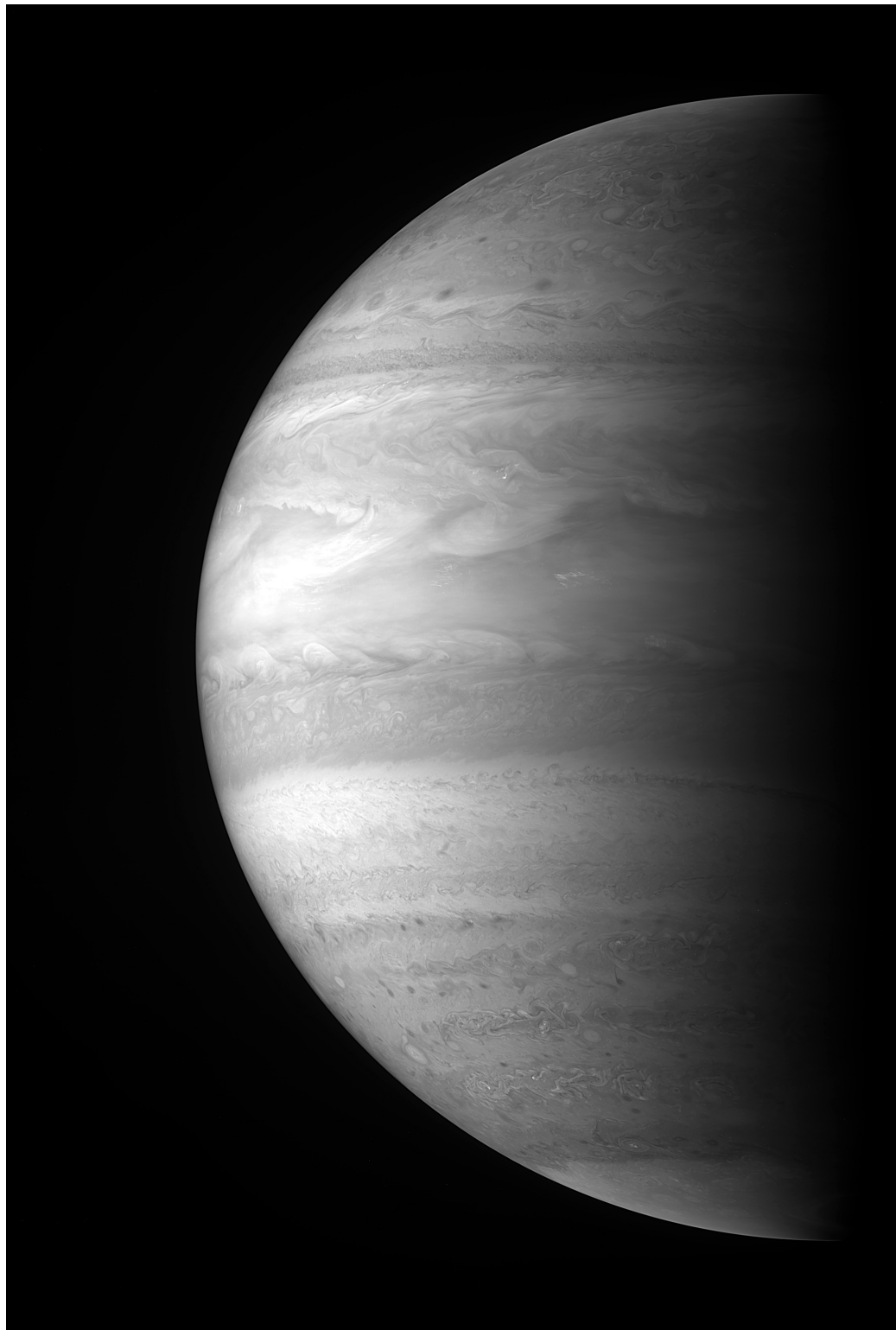
Horizontal features of the visible layer in the atmosphere

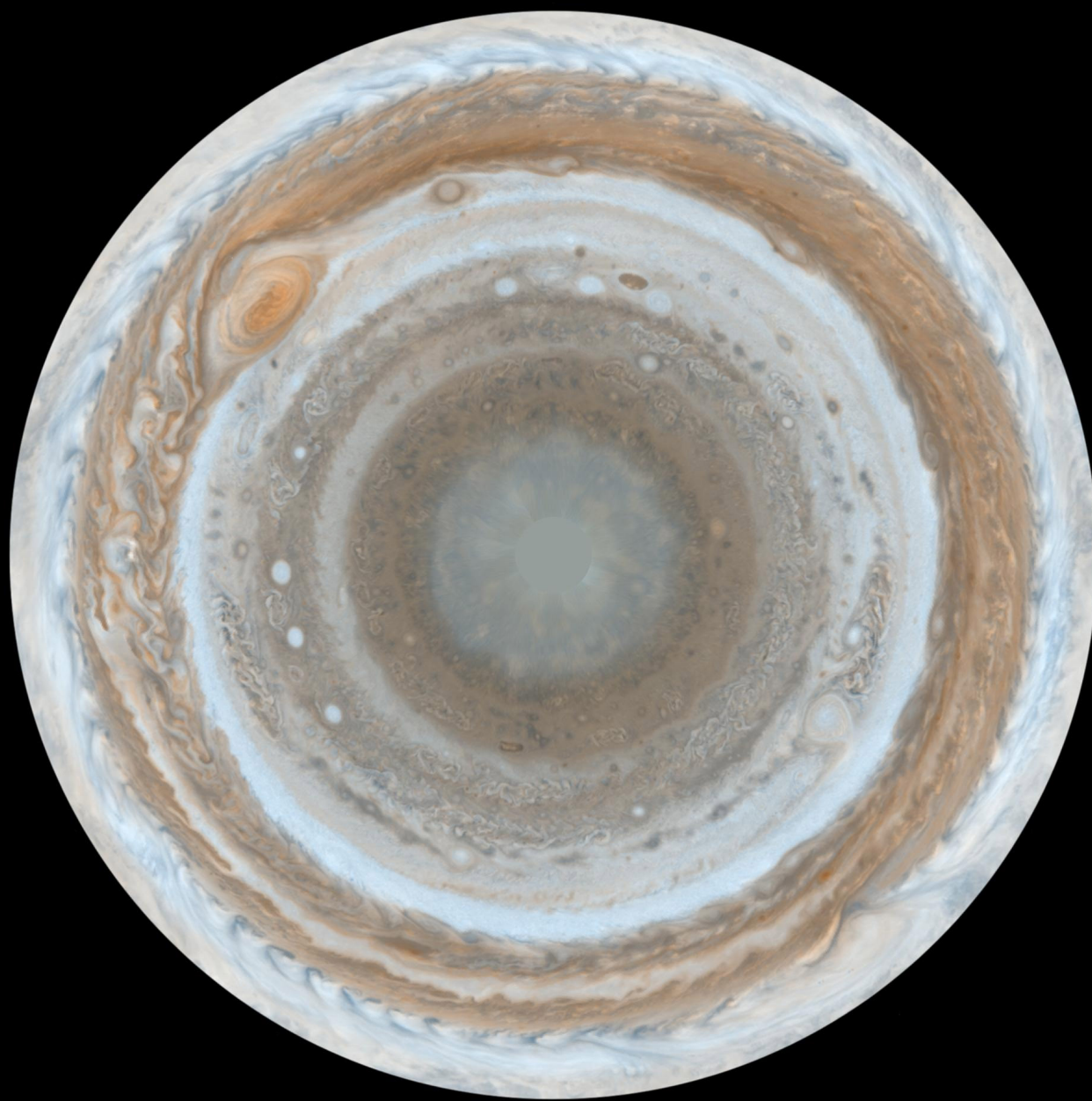
This layer is at the level of the cloud tops that form from thermal convection driven by the heat rising from Jupiter's interior.

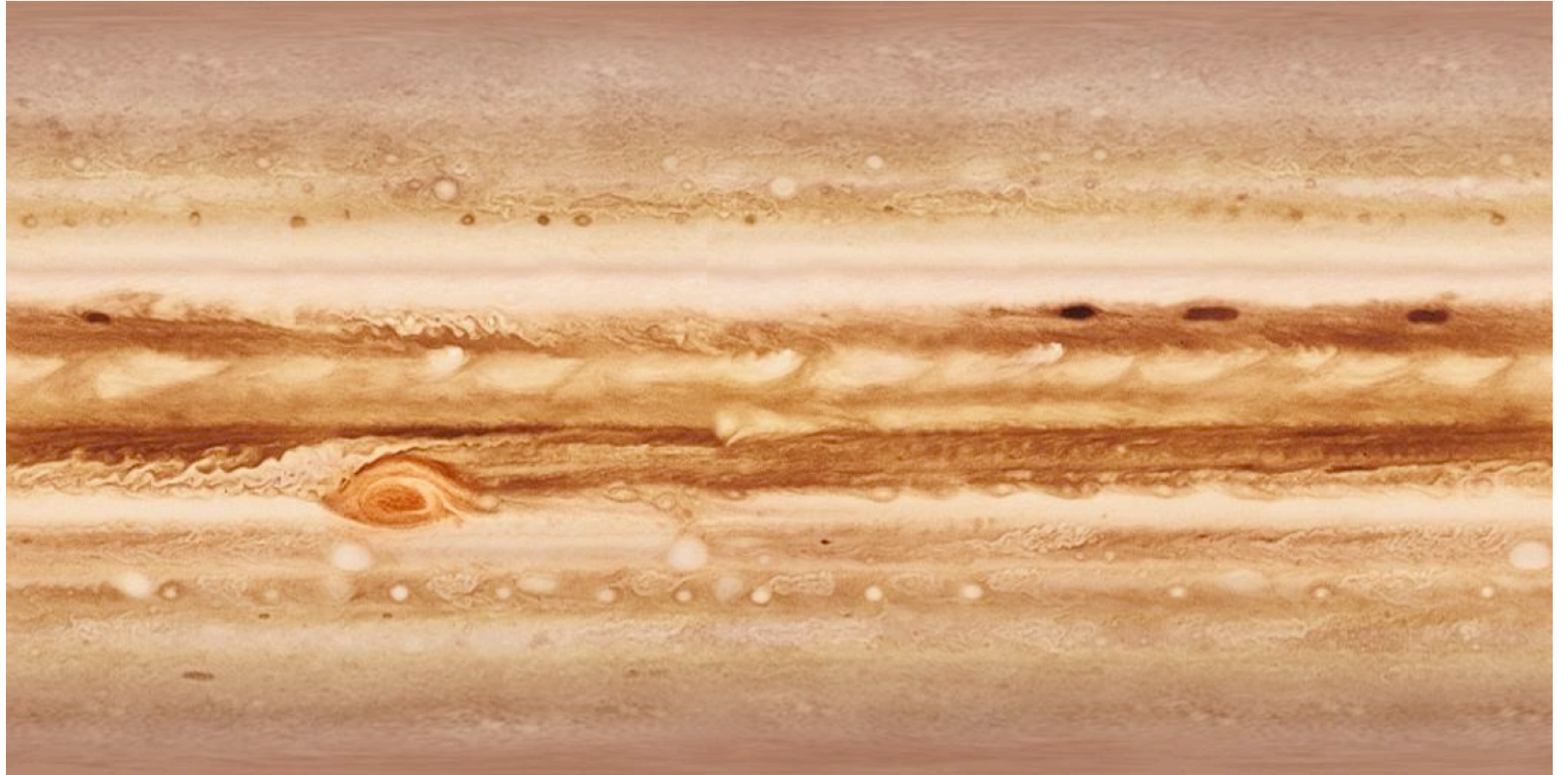
In this “weather layer” there are strong winds ($\sim 100 \text{ m s}^{-1}$):

1. alternating zonal jets (=west-east winds with shear)
2. embedded vortices (=spots that roll in the zonal winds)
3. turbulence (=irregular, chaotic fluctuations throughout)









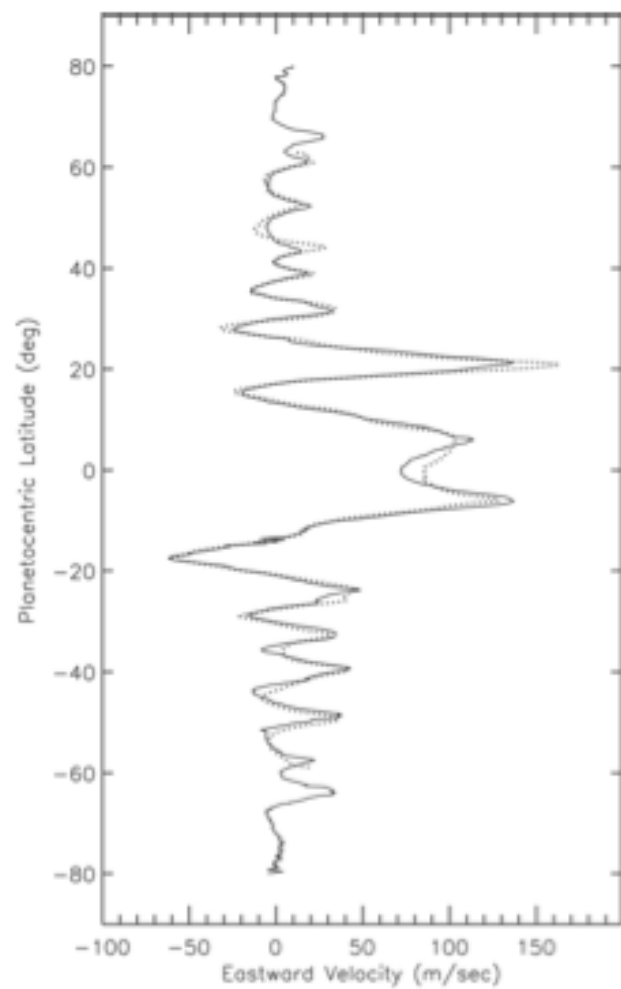


Figure 6.2. Zonal winds vs. latitude in 1979 and 2000. The dashed line is from *Voyager* (Limaye 1986) and the solid line is from *Cassini* (Porco et al. 2003).

What we only infer:

Vertical structure of the atmosphere

The heights of the clouds, the depth of the visible layer, and the density and temperature profiles in the vertical direction.

There are also strong flows beneath the visible layer, but little is known about them because they are invisible.

They influence the weather layer much like solid topography on Earth influences the flow of air in the troposphere.

How we model the winds in the weather layer: Geophysical Fluid Dynamics

Basic fact: Rotation is of primary significance.

Planetary rotation rate: $\Omega = 1.8 \times 10^{-4} \text{ s}^{-1}$

Rotation of GRS: Wind speed $V \sim 100 \text{ m s}^{-1}$, radius $L \sim 10^7 \text{ m}$

Relative vorticity: $\zeta = \mathbf{k} \cdot \nabla \times \mathbf{v} \sim V/L$.

$$\epsilon = \frac{V/L}{2\Omega} \sim \frac{1}{40} \ll 1 \quad \text{"Rossby number"}$$

Geostrophy: The principal balance of forces in the fluid is between Coriolis force and pressure gradient.

$$f = 2\Omega \sin(\text{latitude}) \quad \text{"Coriolis parameter"}$$

Vertical structure simplified to two homogeneous layers

Upper, visible, layer is shallow, lighter, and active.

Lower layer is deep, denser and passive.

Use equations of fluid dynamics to describe the upper layer, which has relative vorticity ζ and thickness h .

Study motion in a mid-latitude band.

Let x and y be longitude and latitude spatial coordinates.

The key scalar quantity in the model is

$$Q = \frac{\zeta + f(y)}{h} \quad \text{“Potential vorticity”}$$

Remarkable property: Q is transported by the fluid motion.

Reduced equations of motion

For small Rossby number $\epsilon \ll 1$, the motion (velocity, pressure, layer thickness) of the visible layer is determined by Q .

Q solves the “quasi-geostrophic equations”:

$$\frac{\partial Q}{\partial t} - \frac{\partial Q}{\partial x} \frac{\partial \psi}{\partial y} + \frac{\partial Q}{\partial y} \frac{\partial \psi}{\partial x} = 0,$$

$$Q = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \lambda^{-2}(\psi - \psi_2) + \beta y.$$

$Q(x, y, t)$ is the potential vorticity in active upper layer

$\psi(x, y, t)$ is the streamfunction for flow in active upper layer

$\psi_2(y)$ is the streamfunction for prescribed flow in lower layer

λ is the “deformation radius”: $\lambda = \sqrt{gD}/f$

β is the gradient of the Coriolis parameter: $\beta = \partial f / \partial y$

Model of typical midlatitude bands

Focus on two similar bands of Jupiter exhibiting different structures:

- Southern hemisphere band containing Great Red Spot and White Ovals (large anticyclones)
- Northern hemisphere band containing strong jets but no large vortices
- Both bands are bounded by eastward jets and have four zone or belt regions in which the jets alternate direction

Statistical mechanical model

Solving the nonlinear PDE for Q is extremely expensive.
Moreover, we want to understand the long-time, steady-state.

Instead suppose that $Q(x, y)$ is replaced by a “random field”
— imagine a large number of spinning fluid columns at regularly spaced sites over the mid-latitude band.

But these random spins are not completely independent because the PDE conserves total energy and total circulation:

$$\begin{aligned} E &= \frac{1}{2} \int (\psi_x^2 + \psi_y^2 + \lambda^{-2} \psi^2) dx dy \\ \Gamma &= \int Q dx dy \end{aligned}$$

Main idea: Characterize the persistent features of the visible layer as the most probable states of this conditioned random field.

Law of Large Numbers

Consider the averages of the random field Q over “medium-sized” subdomains (called “coarse-graining”).

This averaged potential vorticity \bar{Q} does not fluctuate — it is “coherent” and satisfies the time-independent PDE:

$$\bar{Q} = \frac{\partial^2 \bar{\psi}}{\partial x^2} + \frac{\partial^2 \bar{\psi}}{\partial y^2} - \lambda^{-2}(\bar{\psi} - \psi_2) + \beta y = G(\theta \bar{\psi} - \gamma)$$

θ is related to total energy, γ is related to total circulation, G is a nonlinear function derived from the *statistics* of the fluctuations of the random field Q .

Numerical problem: Find branches of solutions parameterized by θ and γ . This PDE for $\bar{\psi}$ is of “nonlinear eigenvalue type.”

Statistics of turbulent eddies in Jupiter's weather layer

Galileo mission data interpreted by Ingersoll et al. (2000) demonstrates that heat flow from Jupiter's interior energizes convection (with thunderstorms and lightning), which sustains a turbulent field of eddies.

The effect of this thermal forcing is to create a random field of Q having *anti-cyclonic skewness* due to cloud-top expansion of convective towers.

These general properties of the unresolved turbulence are modeled by a suitable probability distribution function which, in turn, determines G .

Main results

Computational strategy: Fix E , Γ from the observed zonal winds, and determine ψ_2 so that the equilibrium solution $\bar{\psi}$ coincides with the Limaye profile when the skewness is set to 0.

[Zonal flow in lower layer is inferred from Voyager data using this approach by Dowling (1995).]

Then compute branches of equilibrium solutions by increasing the skewness parameter toward negative (anti-cyclonic), which represents “turning up” the thermal forcing.

Southern hemisphere domain from $36.6^{\circ}S$ to $13.7^{\circ}S$

A large anticyclone emerges at the latitude of the Great Red Spot ($\approx 23^{\circ}S$), and a smaller anticyclone forms in the zone of the White Ovals ($\approx 32^{\circ}S$)

The zonally-averaged flow remains close to Limaye profile as skewness increases

Northern hemisphere domain from $23.1^{\circ}N$ to $42.5^{\circ}N$

No coherent vortices emerge in the zonal shear flow, and the branch of equilibrium states terminates in zonal flow

The zonal flow departs significantly from the Limaye profile as skewness increases

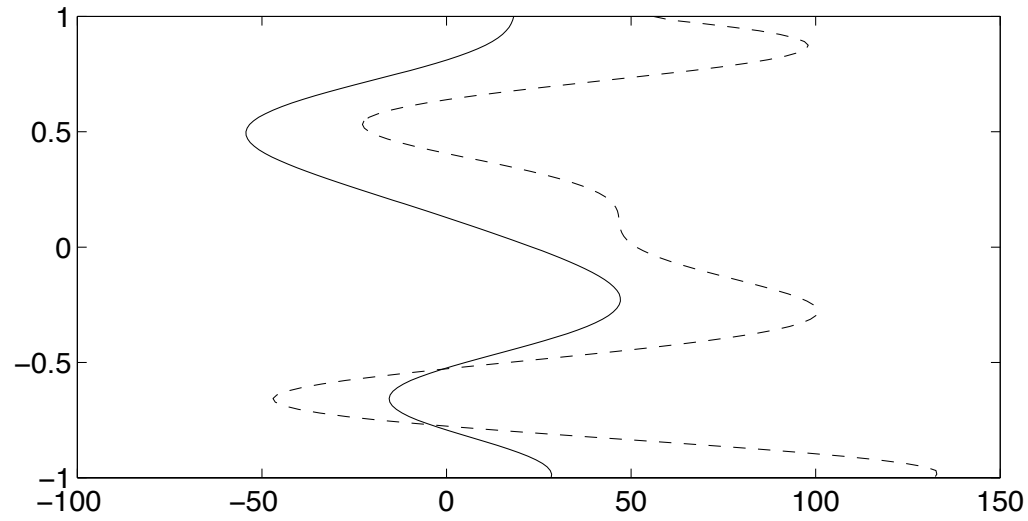


Figure 1: The Limaye zonal mean velocity profile for the southern hemisphere band from $36.6^{\circ}S$ and $13.7^{\circ}S$ (solid), and the zonal velocity profile for the lower layer (dashed) inferred by the Dowling procedure.

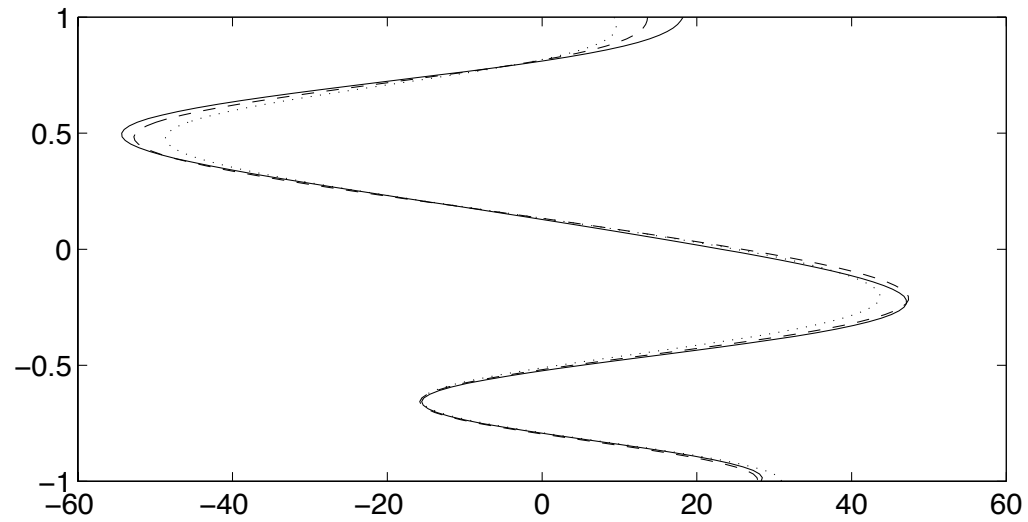


Figure 2: The Limaye profile (solid) in the southern hemisphere band, and the zonally averaged velocity profiles for the equilibrium states with $\epsilon = -0.02$ (dashed) and $\epsilon = -0.035$ (dotted).

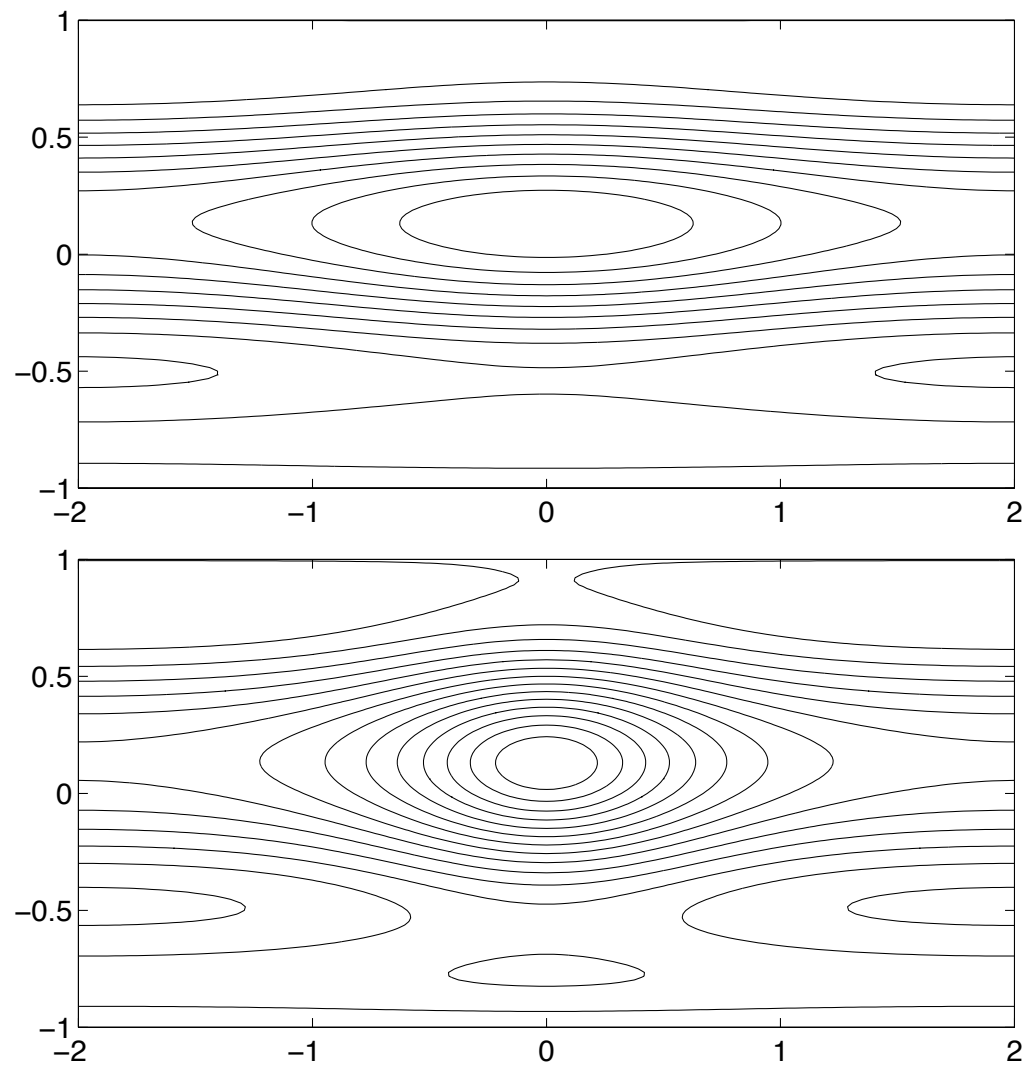


Figure 1: Mean streamline plots for the equilibrium states over the effective zonal topography in Fig. 1, with skewness parameter $\epsilon = -0.02$ (above) and $\epsilon = -0.035$ (below), and the same energy and circulation as the Limaye zonal flow. The length scale is $L = 14,000 \text{ km}$.

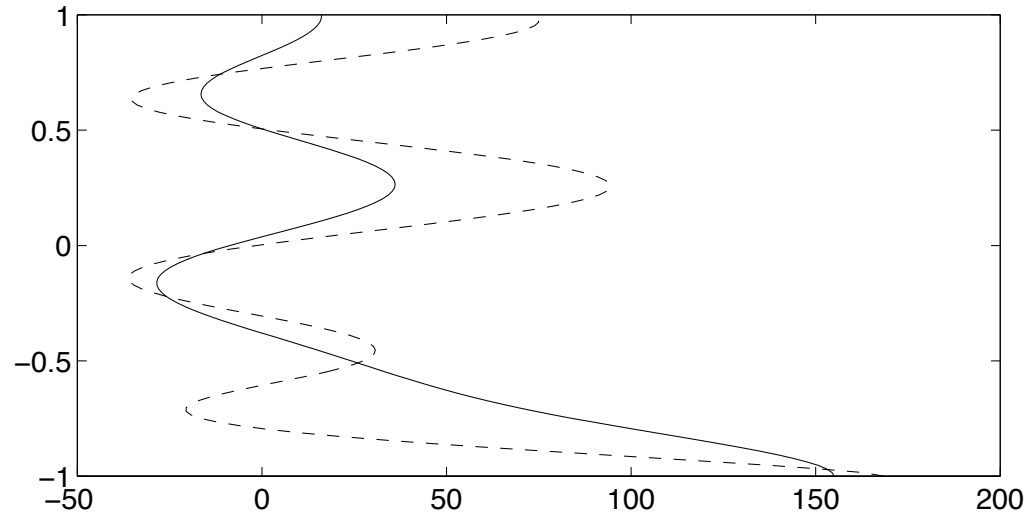


Figure 1: The Limaye zonal mean velocity profile for the northern hemisphere band from $23.1^\circ N$ to $42.5^\circ N$ (solid), and the zonal velocity profile for the lower layer (dashed) inferred by the Dowling procedure.

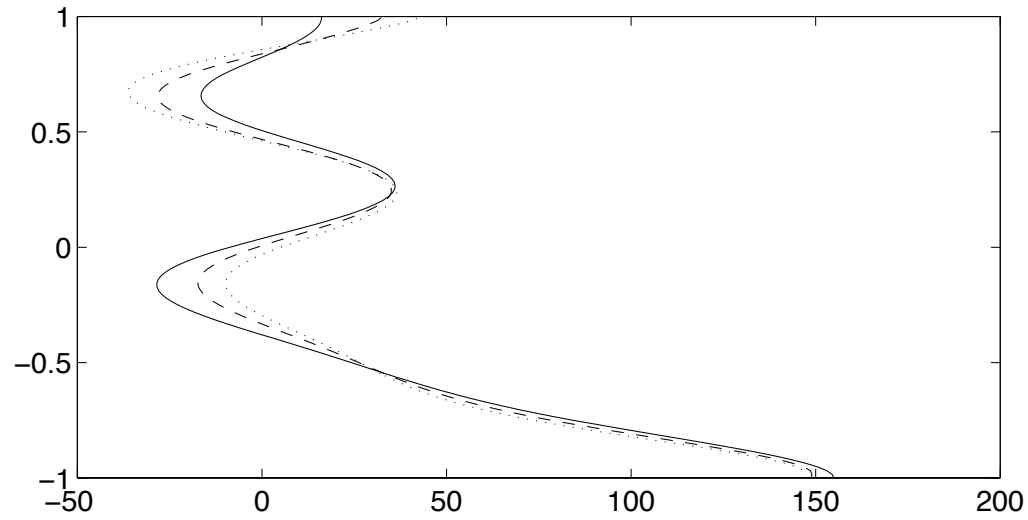


Figure 2: The Limaye profile (solid) in the northern hemisphere band, and the zonal velocity profiles for the equilibrium states with $\epsilon = -0.02$ (dashed) and $\epsilon = -0.032$ (dotted) and the same energy and circulation as the Limaye zonal flow.