Developing Methods for Measuring the Pion Polarizability

A DISSERTATION PRESENTED
BY
ANDREW SCHICK
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Thesis advisor: Rory Miskimen Andrew Schick

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Abstract

The purpose of this thesis is to construct and test some of the fundamental parameters of a multiwire proportional chamber (MWPC), *and* to understand the context in which the MWPC will be used with regards to measuring the polarizability of the pion. Thus, the thesis will be divided into two large sections, the first providing theoretical background to understand the importance of measuring the polarizability of the pion, the second documenting both the lab methods and what was measured during my stay in MENP.

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Listing of figures

For my father.

Acknowledgments

THIS THESIS OWES GREAT DEBTS to many people, both in the Physics department and out of it. First and foremost, I would like to thank Professor Miskimen for his thoughtful guidance during all phases of this project. Second, thanks to Professor David Kawall for serving on the adjudicating committee for this thesis. Elton Smith, David Lawrence, and the rest of the team at Jefferson Laboratory deserve recognition for their creativity, support, and vision. My gracious thanks go to all the members the MENP research group, past and present; the science presented here is a product of our combined efforts. In particular I would like to thank Christian Haughwout and Robert Johnston for their unparalleled technical expertise. Finally, thanks to Sean McGrath for his tireless graphical and editorial support, for without him this thesis would surely not be what it is.

Part I

Theory

1 Introduction

1.1 Goals of this Thesis

I hope to achieve two goals with my thesis. On one hand, the thesis is explicitly about testing gas compositions inside a multiwire proportional chamber (MWPC) prototype. This project alone is ambitious enough to constitute an honors thesis. However, the context surrounding the ”why” of the project is of utmost importance, and it is my personal goal to create a document for use in my lab to help demystify for any newly acquired undergraduates the importance of their work with respect to the bigger picture. Thus the thesis is divided into three parts, two pedagogical, one documenting results (which also serves as a collection of labs new students can perform.)

1.2 A Note For New Students

The primary intention of this thesis beyond satisfying a graduation requirement is to illuminate for new students joining the MENP lab both the relevance of their work in regards to physics at large and to guide them in developing useful lab skills. In writing this I've discovered it's impossible to write a paragraph about standard model physics in which every physics word that appears in the paragraph has already been rigorously defined. The issue with self study in upper level physics is that usually there's some sort of nuance in notation or assumed familiarity with a concept that bars undergraduate physicists from understanding the information that follows. Even as recently as the beginning of the semester of which this thesis is submitted there were things that stopped me from fully grasping the material that I now think of as entirely obvious and essential. I have tried my best to spell things out that I remember not fully comprehending when I began my thesis work. Learning physics is an iterative process, and many physics words appear in this thesis before I explain them in detail because without using them, there would be nothing to talk about. For a really in-depth survey of the field of particle physics, I recommend taking Physics 556 (Particles and Nuclei) which covers nearly everything presented in the theoretical part of this thesis.

2

The Pion Polarizability

2.1 Statement of Purpose

The Medium Energy Nuclear Physics lab under Professor Rory Miskimen at UMass Amherst has been tasked with building an array of Multiwire Proportional Chambers (MWPC) to be used by the Jefferson Lab in Newport News, Virginia in a precision test of the polarizability of the pion. Electromagnetic polarizabilities are a measure of an object's deformation when subjected to electromagnetic fields. To measure this deformation, one must measure scattering angle cross sections which requires tracking the trajectories of particles. This is the role that the MWPCs perform.

A pion is a combination of a quark and an anti-quark. They are held together by the strong nuclear force. Quarks carry charge. By subjecting them to an electromagnetic field of known strength and measuring how much the two body system deforms we can test the strength of the strong nuclear force binding the two quarks. The polarizability for a pion is not easy to measure–you can't just collect a bunch of pions and bombard them with charged particles. Unlike the proton, pions decay with the π^+ and π^- decaying at 2.6×10^{-8} and the π^0 decaying at 8.4×10^{-17}. More inventive methods have to be performed. As outlined in Miskimen et al's letter of intent included in the back, there are 3 methods by which the pion polarizability can be measured:

1. Radiative pion photoproduction
$\gamma p \rightarrow \gamma' \pi^+ n$
2. Primakoff effect of scattering a high energy pion in the Coulomb field of a heavy nucleus
$\pi A \rightarrow \pi' \gamma A$
3. $\gamma\gamma \rightarrow \pi^+\pi^-$

Both the COMPASS collaboration and our experiment will use method 3. Experiments using the first two methods have yielded results not in agreement with the theoretical predictions of ChPT. COMPASS has recently completed their experiment and did obtain results in agreement with ChPT. The COMPASS collaboration measured the pion polarizability by shooting a beam of pions at a target comprised of nickel. When the pions were within two particle lengths of the nucleus of the nickel nucleus they were deformed and sent on different trajectories emitting a photon. This sounds as though its something separate from $\gamma\gamma \rightarrow \pi^+\pi^-$ but by crossing symmetry we see this involves the same energy as $\gamma\pi \rightarrow \gamma\pi$.

Why measure the pion polarizability? Quantum theories of nuclear physics start by making assumptions about the symmetries of nature that then imply values for measurable quantities. Electromagnetic polarizabilities are one such possible implied quantity. Theories are local approximations. By this I mean when you model dropping your pencil from a height h you don't include gravitational effects from the moon in your considerations. But working within the unintuitive realm of quantum physics, it isn't always obvious what effects you can omit. Experimental tests of your theory are the chief means by which to check that your assumptions of symmetries and the relevant pieces on the stage were correct. The current theory which has made successful predictions of low energy Quantum Chromodynamics (the theory of nuclear interactions) is known as Chiral Perturbation theory (ChPT). Recent experiments of the pion polarizability have yielded measurements that don't agree with the predictions of ChPT, though these experiments have been riddled with uncertainties and suffer from having few data points. A precision test is required to settle whether ChPT is the effective field theory for low energy QCD.

2.2 Wait, What?

If nothing in the previous section made sense to you, or if it seemed far too general to be of use in demystifying the purpose of the MENP lab's work don't worry. A large portion of this thesis is intended to contextualize the goals of this lab in the big picture of physics.

The first document you should read when beginning work in this lab is the letter of intent [1] from Miskimen et al to the Jefferson labs advisory committee which I have included in the back of this thesis. This letter was *not* written to be understood by undergraduates–do not think that you are in over your head if the letter seems unintelligible to you. If you haven't taken the particles and nuclei class offered here at UMass, odds are it will be one of your first times seeing a majority of the physics words in that letter. I will now quote a few sentences from that letter to point out some words we'll have to define before we can even grasp what's being discussed.

"The charged **pion polarizability** ranks among the most important tests of **low-energy QCD** presently unresolved by experiment. Analogous to precision measurements of $\pi^{\circ} \leftarrow \gamma\gamma$ that tests **intrinsic odd-parity (anomalous)** sector of QCD, the **pion polarizability** tests the **even-parity** sector of **QCD**.

"**Hadron** polarizabilities are best measured in **Compton scattering** experiments, where in the case of **nucleon** polarizabilities one looks for a deviation of the **cross section** from the prediction of **Compton scattering** from a structureless **Dirac particle**."

"Because a free pion **target** doesn't exist, the measurements to date of the charged pion polarizability have been plagued by experimental and theoretical uncertainties." [1]

The words in bold inspire some guiding questions for discussion of the physics behind the experiment: What are pions? What is polarizability? What is QCD, and why is low-energy QCD something that needs to be tested separately from QCD at large? What is an intrinsic property of a particle? What is parity? What are hadrons, nucleons, and dirac particles? What are targets, compton scatterings and cross sections?

There's no quick way to learn what all of these things mean if you don't already know them. You could search for them on Wikipedia, but each article then requires you to read 5 more articles to figure out what the original was saying. It's far more effective to present each concept as natural extensions of the things that come before them, starting with a very rough picture of the physical interactions of the universe, then the math language utilized by the theories we use to describe those interactions, then the theoretical predictions of measurements, and then how we measure those things in the lab, defining these words as they come up. Let us first begin with the Standard Model of physics.

3

A Brief Introduction to the Standard Model

The Standard Model is the collection of theories that describe the electromagnetic, weak nuclear, and strong nuclear interactions between quantum particles.

3.1 Particles

For as long as recorded history (and likely before then as well) humanity has wished to answer "What are the fundamental constituents of the world we live in?" The past century however has been unlike any of those before us with regards to this question because for the first time ever we're trying to answer it from the angle of "well, let's go and look!" Physics doesn't seek just to look at what things are made of but to also figure out the rules of "the game". General relativity seeks to answer questions such as "how much does matter curve spacetime?" but *particle physics* is the cutting edge of asking "what is matter made of?"

Perhaps one of the most illuminating introductions to the field is the first chapter of Griffiths' book "Introduction to Elementary Particles"[3] which should be required reading for any new undergraduate students working in the MENP lab here at UMass Amherst. This chapter covers in non-confusing language the birth and progress of the entire field to where it stands today. I will now give a hugely condensed summary of the chapter here.

Particle physics begins with the discovery of the electron by J.J. Thomson in 1897. It was known that these were charged particles with extremely small mass and it was also known there was some other object called an atom (Chemistry by this time was already a very active field of research). Electrons were identified to be constituents of matter. However, atoms were known to be electrically neutral and to have much more mass so electrons were thought to be paired with positively charged analogues in a plum pudding paste. Rutherford demonstrated that this was not so–the atom was mostly empty space with a very small and very heavy center with positive charge. This center was called the nucleus and the nucleus of hydrogen was given the name proton. Niels Bohr proposed electrons orbit protons like planets and this gave a decent prediction of the spectral lines of hydrogen. However it made no sense that if helium were two electrons and two protons that it would weigh 4 times as much as hydrogen and lithium weighs 7 times as much. In 1932 Chadwick discovered the neutron and resolved this mass issue. These three particles can be thought of as the embodiment of the classical period in elementary particle physics.

Happening concurrently with these developments was the discovery of the Photon. There was this great problem in physics at the turn of the century called the ultraviolet catastrophe where statistical mechanics (which had provided excellent predictions for all of the thermal physics questions up to that point) was producing nonsensical answers when trying to explain the experimental data for the electromagnetic radiation emitted by a hot object. i.e. the blackbody spectrum. Planck realized he could fit the data accurately if he made the assumption that electromagnetic radiation is quantized. Planck assumed the quantization was due to a peculiarity in the emission process. It was Einstein in 1905 who took it much further by making the assumption that quantization was inherent in the electromagnetic field and then used that to explain the photoelectric effect.

Next the question was posed "what holds the nucleus together?" Protons should be repelling each other – how come they are content to stay in the nucleus? Yukawa proposed in 1934 that like protons and electrons being held together by the electromagnetic field with the photon as the quanta of the field, there should be a nuclear field that keep neutrons and protons together. He then went on to explore the question of what the quanta of this nuclear field would be. This nuclear quanta came to be called the meson, because its predicted mass was between the electron and the proton (meson meaning middle-weight). Two candidates for Yukawa's meson were found in cosmic rays, the pion and the muon, and the pion turned out to be Yukawa's particle.

The next development in particle physics came from Dirac imposing special relativity onto the quantum mechanics that had been developed during this time. The results of this suggested that there existed "antiparticles". Shortly after positrons were discovered. Next it was observed that Muons had decay products. This gave rise to the discovery of the neutrinos.

In 1947 a picture of cosmic rays passing in a cloud chamber revealed that some new particle was decaying into a π^+ and π^- indicating a neutral particle with twice the mass of a pion. Brown and her collaborators deemed it the kaon.

Many more particles were to be discovered. The rest of the chapter deals with the confusion of making sense of them all which for the purpose of this summary we need not include. What should be said however is that the first big breakthrough in categorizing particles came in 1961 when Gell-man produced the analogue of the periodic table of elements for particle physics: the Eightfold Way.

The rest of the chapter isn't so much important (the details of the ensuing 30 years of trying to make sense of the particles) as is the results which we are left with today which I will simply skip to.

It was put forward that protons, neutrons, kaons, pions, and a whole host of other particles were made up of entities named quarks. The particles made up of quarks interacted with each other through the strong nuclear force. These particles were grouped under the heading of "Hadron". Quarks carry a second type of charge, known as color. There are three different types of color charge: red, green, and blue. All observable particles are chromatically neutral. Thus you can have two types of hadrons, one's composed of 1 red, 1 green, 1 blue colored quark (Baryons) or 1 colored quark with its same colored antiquark (mesons). The theory which describes the interactions of quarks is known as quantum chromodynamics or QCD. Other particles like electrons, muons, and neutrinos which did not have nuclear interactions were grouped on their own. These are known as leptons. Both hadrons and leptons have three generations. The first generation quarks are up and down, 2nd are charm and strange, 3rd are top and bottom or truth and beauty.

Lepton	Charge	Mass
e^-	$-1e$	0.51 MeV
μ^-	$-1e$	105.65 MeV
τ^-	$-1e$	1777.03 MeV
ν_e	0	$<$ 3 eV
ν_μ	0	$<$ 0.19 MeV
ν_τ	0	$<$ 18.2 MeV

Table 3.1.1: Lepton Properties

Flavor	Charge	Mass
u	2/3 e	$2.3^{+0.7}_{-0.5}$ MeV/c^2
d	-1/3 e	$4.8^{+0.7}_{-0.3}$ MeV/c^2
c	2/3 e	1.275 ± 0.025 GeV/c^2
s	-1/3 e	95 ± 5 MeV/c^2
t	2/3 e	173.5 ± 0.6 GeV/c^2
b	- 1/3 e	4.18 ± 0.03 GeV/c^2

Table 3.1.2: Quark Properties.

The pion is a meson. It is composed of a quark and anti-quark pair.

3.2 Fundamental Interactions

As best as we can tell as of December 2015, there are only four fundamental forces in the universe. They are gravity, the electromagnetic, the weak nuclear, and the strong nuclear forces. Forces are actually a redundant concept, which will become apparent in the course of this thesis as we develop the machinery necessary to understand why performing a precision measurement of the pion's polarizability is a big deal. What classical physics interprets as forces are really consequences of conservation of four-momentum and quantum mechanical facts such as the Pauli exclusion principle (I will make these concepts precise in later chapters.) It's more appropriate to refer to them as four fundamental interactions. You may wonder: "t tells us that they are distinct interactions?"

Interactions are characterized by a number of things: what bosons mediate their interactions, their relative strengths to one another at certain length scales (this is a consequence of the prior fact) what conserved charges they contain (electromagnetism has its binary electric charge, whereas the strong nuclear force

Table 3.2.1: The Four Fundamental Interactions

Property/Interaction	Gravitation	Weak	Electromagnetic	Strong	
		(Electroweak)		Fundamental	Residual
Acts on:	Mass - Energy	Flavor	Electric Charge	Color Charge	Atomic Nuclei
Particles Experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles Mediating:	Graviton (hypothesized)	W / Z bosons	Photon	Gluons	Mesons
Strength at scale of quarks:	10^{-41}	10^{-4}	1	60	N/A
Strength at scale of protons:	10^{-36}	10^{-7}	1	N/A	20

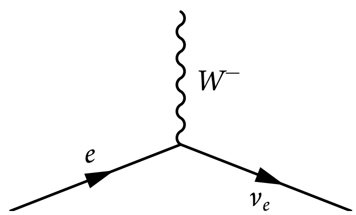

Figure 3.2.1: Charged Weak Current Vertex.

has its three color charges).

Each fundamental interaction can be interpreted as current of some conserved charge. Electromagnetism has its current of electric charge. Strong forces have color currents. Particles, though intuitive are actually some what ill defined. You *could* interpret the weak force as a current of what makes a particle the particle it is. It is at the heart of particles decaying into other particles.

So, to revisit the first sentence of this chapter: The Standard Model of particle physics is the collection of theories that describe the electromagnetic, weak, and strong interactions. Gravity does not yet have a quantum theory and general relativity is *not* included in the Standard Model. The theory that describes electromagnetic interactions between leptons and photons is Quantum Electrodynamics (QED). The theory that describes strong interactions between quarks and gluons is Quantum Chromodynamics (QCD). The theory that describes weak interactions between is Glashow Weinberg Shalom (GWS) weak interactions which describes the interactions between all leptons and baryons and the W and Z bosons. Given that the pion is a meson composed of a quark and anti-quark pair bound together by the strong nuclear force, how much the pion deforms under electromagnetic forces should be predicted by the theory of QCD. But QCD is a much more complicated theory than QED, so it will be fruitful to first use QED to introduce a few key concepts before attempting to describe QCD.

3.3 Feynman Diagrams, QED, QCD

If you have taken Quantum Mechanics already, you may remember that the starting point is to write the Hamiltonian (the total energy of the system) $H = E_{total} = T + V = \mathbf{p}^2/2m + V$ and then make the canonical substitution of the momentum vector for the momentum operator $\mathbf{p} \to i\hbar\nabla$. Then operating this quantity on the wave function yields the familiar Schrodinger's equation, from which the rest of quantum mechanics is derived. An alternative and equally valid starting point is the Lagrangian rather than the Hamiltonian, which is $L = T - V$ and again making the canonical substitutions. By performing path integrals across fields you can also build quantum mechanics. This is actually how QED is built, but the mathematics which describes it is beyond the scope of this thesis. Richard Feynman, who pioneered much of QED, also boiled down the mathematics of the path integrals into pictures and it is through this lens that we will approach QED and QCD.

All QED Feynman diagrams make are in essence composed of this simple vertex.

These diagrams can be used to calculate observable quantities for particle interactions such as decay rates of particles and scattering cross sections. Each line of the diagram is actually a pictorial representation of math. External lines actually are a factor of...each vertex stands for one factor of the electromagnetic coupling constant $\alpha_{EM} = \frac{e^2}{4\pi\varepsilon_0\hbar c} = 1/137$. The coupling constant determines the strength of the electromagnetic force on an electron. The multiplicative total of

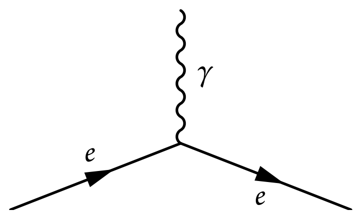

Figure 3.3.1: The fundamental vertex for QED.

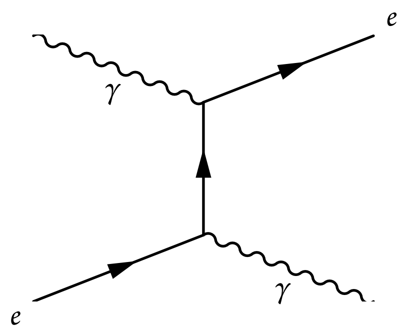

Figure 3.3.2: Feynman diagram representation of Compton scattering.

all of the factors the Feynman calculus instructs you to add for parts of the picture is actually a probability for some type of event such as a particle decay or a particle to scatter off at some solid angle to occur. Thus, if every vertex adds a factor of $1/137$, diagrams that have many complicated vertexes are actually suppressed and we need not consider them.

This whole procedure for extracting a probability from a picture, and the rules by which factors get added in at vertexes for outgoing lines, etc. is referred to as the Feynman Calculus.

QED is the most successful theory of the standard model, making very precise confirmed predictions. Subsequent field theories have been intentionally modeled on it. The analogous set up of QCD is two quarks and a gluon. $q \to q + g$. The forces between two quarks are mediated by the exchange of gluons in the same way that photons are exchanged between electrons and positrons. In QED the photon did not carry any electric charge, and so things like $e \to \gamma + \gamma$ were forbidden by conservation of charge (and mass). But since

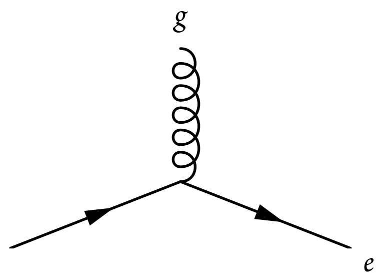

Figure 3.3.3: The fundamental vertex for QCD.

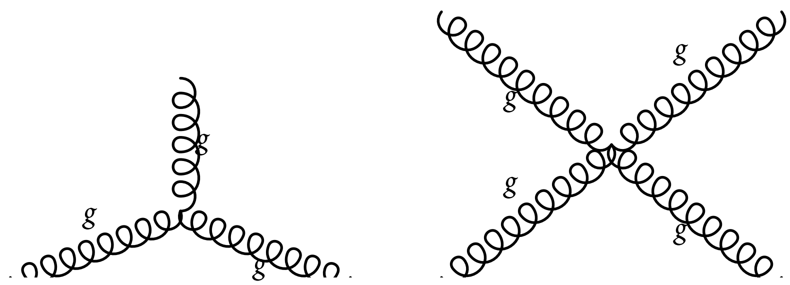

Figure 3.3.4: Glueballs.

gluons themselves carry color charge they can couple to each other and you can have entities known as glueballs.

Unlike the coupling constant for electromagnetism, the coupling constant for QCD is actually slightly larger than 1, making infinitely more complex diagrams the most probable decays/scatterings.

At first everyone thought that the Feynman calculus would just not be useful for describing nuclear reactions until it was discovered that coupling constants are in fact, not constants. They vary depending on how much space is in between them because of a phenomenon called screening. The coupling constant for QCD gets smaller at close distances (like those of quark spacings) and larger at for big distances. Thus, the methodology of QED can be used for QCD provided the lengths between the interacting particles are sufficiently small. However the relevant distances for the nuclear physics we're concerned with in our lab takes place at large distances.

3.4 Natural Units, Energy Scales

To illustrate the relation between differences in high energy/low energy nuclear physics and of the differences in coupling constants at various lengths, I will now introduce an "industry standard" in particle physics: the use of natural units. Many equations in relativity and quantum mechanics are full of c's and $\hbar$'s. But value these constants have are merely products of the arbitrary divisions of space and time that we've set for ourselves. There is nothing special about the actual distance of a meter. We can redefine our system of units so that the speed of light and planck's constant are simply unitless and equal to 1. There are many different natural unit systems used for various types of physics, but the official Natural Units for particle physics is $c = \hbar = k_b = 1$.

In these units, Einstein's famous equation becomes: $E = mc^2 \implies E = m$ Energy and mass now have the same units. They are equivalent. Recall the Planck relation $E = hv = hc/\lambda \rightarrow E = 1/\lambda$ implying that energy and mass have units of inverse length. Thus high energy physics $\implies$ small distances, the regime where the Feynman calculus approach can be applied to QCD. Low energy physics $\implies$ large distances, the regime where the coupling constants favor infinite complexity and other theoretical models must be used. The overarching purpose of measuring the pion polarizability is to verify if we have made a correct modeling of low energy QCD.

4
Linear Algebra

If we have any hope of discussing the nature of reality we must all first agree on what we're talking about where and when. Thus we need coordinate systems. Linear algebra is the mathematics that describes coordinate transformation that relate one coordinate system to another. Linear algebra is the essential math for both relativity and quantum mechanics, two subsets of physics that are undeniably prerequisites for understanding anything about Quantum Chromodynamics and what we are working on in this lab. This chapter will touch upon the basic concepts of linear algebra and the two subsequent chapters will make immediate use of them in describing quantum mechanics and special relativity.

A more precise mathematical definition of linear algebra is "Linear algebra is the study of linear maps on finite-dimensional vector spaces." [**?**]

The idea of studying a linear operator by restricting it to small subspaces leads to eigenvectors.

F is used because **R** and **C** are examples of what are called **fields**.

4.1 Vectors and Vector Spaces

Physics uses mathematical objects to describe reality. One can make a mathematical object by using a symbol to represent a set of "rules" or collection of things that behave certain ways. Let me give some examples. Sets are one such mathematical object, composed of (not necessarily mathematical) objects. More precisely: a set is a collection of distinct objects. The collection of the numbers {4,5,8} is a set. The collection of the numbers {4, 5, 4, 5, 5, 8} is the same set. Sets only care about the distinct objects in them. Another mathematical object is a list. A list of length n is an ordered collection of n objects separated by commas and surrounded by parentheses. A list of length n looks like this: $(x_1, \ldots, x_n)$. A list of length 2 is an ordered pair and a list of length 3 is an ordered triple. For $j \in \{1, \ldots, n\}$ we say that x_j is the j^{th} coordinate.

Lists differ from sets in two ways: in lists, order matters and repetitions are allowed, whereas in sets, order and repetitions are irrelevant. The lists (3,5) and (5,3) are not equal, but the sets {3,5} and {5,3} are equal.

Both vectors and coordinate points are lists, but when we think of (x_1, x_2) not as a point but as an arrow starting at the origin and ending at (x_1, x_2) we refer to it as a vector and denote it by $\vec{r} = (x_1, x_2)$. Vectors can be decomposed into components and have a certain length given by the Pythagorean theorem.

An example of a vector space is $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$ which in English reads "the set $\mathbb{R}^2$ is the ordered points (x, y) such that x and y are members of the set of real numbers." Therefore, elements of $\mathbb{R}^2$ are two dimensional vectors. One not need restrict the concept of a vector to real numbers; you can validly construct the vector space $\mathbb{C}^n$ which is the collection of all possible ordered complex numbers $(z_1, z_2, .., z_n)$.

A vector space is not simply a set of vectors–it has other requirements. For example it must contain the zero vector $\vec{0}$. And any linear combination of elements in the vector space produces a vector that is still in the vector space. This is referred to as "closed under scalar multiplication and addition."

4.2 Linear Transformations, Matrices

It's presupposed that if you are reading this you have a familiarity with what functions are. Functions associate an element that's in one set to an element that's in another.

Theorem 1 *Let X and Y be sets. A function f from $X \rightarrow Y$ is denoted by $f{:}X \rightarrow Y$ and is a rule that assigns to each element $x \in X$ a unique element $f(x) \in Y$.*

Most all of the math done in first year calculus involve functions that map elements in $\mathbb{R}$ to $\mathbb{R}$. In an analogous way, you can create functions that map vectors to other vectors, i.e. elements from a vector space to another vector space. A function of a vector $f(\mathbf{r})$ is called linear if $f(\mathbf{r}_1 + \mathbf{r}_2) = f(\mathbf{r}_1) + f(\mathbf{r}_2)$ and $f(a\mathbf{r}) = af(\mathbf{r})$ where a is a scalar.

You can imagine linear functions that act on vectors as coordinate transformations. For instance, a vector function that preserves the magnitude of the vector it acts on can be thought of as a rotation. You can imagine the result of performing a mapping in different ways. One way is to imagine dragging the head of the arrow to the new point while keeping the coordinate axes the same. On the other hand, you could imagine it as the vector standing still while the coordinate axes themselves rotate and stretch until the vector now points at the vector element it was mapped to. In the case of rotations, the first is called active rotations, and the second passive rotations. One of the most important steps for understanding relativity is to become comfortable with representing a change of coordinate systems by that of a matrix acting on a vector.

We can represent a vector in component notation in the following way, where i,j,k represent the unit vectors.

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = x'\mathbf{i}' + y'\mathbf{j}' + z'\mathbf{k}' \tag{4.1}$$

Notice that the vector is the same vector before and after changing the coordinate systems. Lets now perform a series of projections of **r** onto the primed coordinate system's axes by taking dot products with unit vectors.

$$\begin{aligned} \mathbf{r}\cdot\mathbf{i}' = x(\mathbf{i}\cdot\mathbf{i}') \quad &+y(\mathbf{j}\cdot\mathbf{i}') \quad +z(\mathbf{k}\cdot\mathbf{i}') \quad = x'(\mathbf{i}'\cdot\mathbf{i}') + \cancel{y'(\mathbf{j}'\cdot\mathbf{i}')} + \cancel{z'(\mathbf{k}'\cdot\mathbf{i}')} \\ \downarrow \qquad\qquad &\downarrow \qquad\qquad \downarrow \qquad = x' \\ |1||1|\cos\theta_1 \qquad &\cos\theta_2 \qquad \cos\theta_3 \end{aligned} \tag{4.2}$$

The cosine of the angle between two unit vectors is just some number, which we can represent with some algebraic symbol. The table below represents the cosines of the angles between unit vectors.

Table 4.2.1: My caption

	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
$\mathbf{i}'$	a_1	b_1	c_1
$\mathbf{j}'$	a_2	b_2	b_3
$\mathbf{k}'$	a_3	b_2	b_3

Thus we can rewrite (**??**) as

$$\mathbf{r}\cdot\mathbf{i}' = x' = a_1 x + b_1 y + c_1 z \tag{4.3}$$

r projected onto the other unit vectors is then

$$\begin{aligned}\mathbf{r}\cdot\mathbf{j}' &= y' = a_2 + b_2 + c_2 z \\ \mathbf{r}\cdot\mathbf{k}' &= z' = a_3 x + b_3 y + c_1 z\end{aligned} \tag{4.4}$$

We can actually now make use of the rules of matrix multiplication to tidy up what we've written here. Let:

$$\mathbf{A} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \rightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{4.5}$$

Or more succinctly:

$$\mathbf{r}' = \mathbf{A}\mathbf{r} \tag{4.6}$$

Thus we see how the description of a vector in one coordinate system can be related to another by just a matrix. When we talk about special relativity, we'll use matrices to relate one observer's frame of reference to another's through a matrix.

4.3 Operators, Eigenvectors, and Eigenvalues

If you've already taken quantum mechanics, you may recall that all observable values we can know about a system are obtained by "operators" acting upon a wave function, and the predicted measurable result is indicated by the eigenvalue of the eigenstate $|\psi\rangle$. In strictly mathematical terms, an operator is a linear mapping from a vector space to itself. That is to say, rather than connecting one object in a bag by a string to an object in another bag, the string connects the object to another object in the same bag. The eigenvectors of a linear operator are the representative basis vectors that get mapped to themselves (eigen is German for self). That is to say, eigenvectors are invariant elements for some linear operator–the operator doesn't change them to be a linear combination of some other basis vectors that define the vector space. This is not to say that the operator cannot scale the basis vector to be larger in magnitude. The amount by

Table 4.4.1: Symmetries and conservation laws.

Symmetry		**Conservation Law**
Translation in time	$\leftrightarrow$	Energy
Translation in space	$\leftrightarrow$	Momentum
Rotation	$\leftrightarrow$	Angular Momentum
Gauge Transformation	$\leftrightarrow$	Charge

which an eigenvector is scaled by an operator is called the eigenvalue. If the operator leaves a basis vector *truly* invariant, the eigenvalue is just one.

4.4 Symmetries and Groups

Perhaps the most important proof in classical physics was demonstrated by the mathematician Emmy Noether. Her theorem proves that whenever there is a symmetry there is an associated conservation law.

The mathematical description of symmetry lies within a field called **group theory**.

Griffiths defines a symmetry in the following way: "[A symmetry] is an operation on a system that leaves it invariant -that carries it into a configuration indistinguishable from the original one. Consider the equilateral triangle. It is carried into itself by a clockwise rotation through 120° (R_+), and by a counterclockwise rotation through 120° (R_-), by flipping it about its vertical axis" or one of the new vertical axes after a 120° rotation (R_a). Doing nothing to the triangle also leaves it invariant (I for identity operation). "Then we could combine operations–for example rotate clockwise through 240° degrees. But that's the same as rotating counter clockwise by 120° (i.e. $R_+^2 = R_-$)."

The set of all symmetry operations on a particular system has very similar properties to vector spaces. They are:

1. Closure: If R_i and R_j are in the set, then the product R_iR_j (meaning first perform R_i and then R_j) is also in the set; that is $R_k = R_iR_j$.

2. Identity operation: There is an operation in the set that maps members of the set to themselves

3. Inverse Operation: there's a way to undo a symmetry operation. For example in the case of rotations, you can rotate $\pi/2$ clock-wise, and then rotate $\pi/2$ counter-clockwise to return to where you started.

4. Associativity: $(R_i R_j) R_k = R_i (R_j R_k)$

Sets with these properties are referred to as groups. We could impose further properties, such as commutativity. If symmetry operations commute, we call the group Abelian. Every group has a matrix representation. For instance, rotation of an xy coordinate axis by any θ is a symmetry operation. We represent this group by the standard rotation matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{4.7}$$

Rotation matrices are orthogonal, which is to say that their transposes are their inverse transformations. $O^T O = \mathbb{I}$ Classifying groups based on properties of their matrices is displayed in the following tables. The prefix U stands for unitary; O for orthogonal; S for special which means the matrix has determinant 1.

Table 4.4.2: Important symmetry groups.

Group name	Dimension	Matrices in Group
$U(n)$	$n \times n$	unitary
$SU(n)$	$n \times n$	unitary, determinant 1
$O(n)$	$n \times n$	Orthogonal
$SO(n)$	$n \times n$	Orthogonal, determinant 1

5

Special Relativity

5.1 Space-time and Special Relativity

Special relativity emerged out of the desire to have the laws of physics be invariant between inertial reference frames. When it was shown that Maxwell's equations were not invariant under a Galilean transformation, they were assumed to be incorrect. When they were modified to be invariant under Galilean transformations they implied new phenomena that were not found to be in nature. The Maxwell's equations were the correct description of reality, and reluctantly the physics community accepted that it was Newton's laws that had to be modified to preserve the type of invariance implied by Maxwell's equations.

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= 4\pi\rho && (i) \\
\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial B}{\partial t} &= 0 && (ii) \\
\nabla \cdot \mathbf{B} &= 0 && (iii) \\
\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c}\mathbf{J} && (iv)
\end{aligned}
\tag{5.1}
$$

But implementing this type of rule for leaving vectors invariant under inertial reference frame transformations had implications about time. In the Newtonian sense, time was an absolute thing, i.e. quantities in nature are functions of $(x(t), y(t), z(t)$ that varied as an absolute universal clock ticked away. But what Einstein showed in his theory was that time was a local thing. Different inertial reference frames have their own times requiring quantities in nature be functions of (t,x,y,z). To correctly go between inertial frames you introduce a new rule for coordinate transformations called the Lorentz transformation.

$$
\begin{aligned}
x' &= \gamma(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \gamma\left(t - \frac{v}{c^2}x\right) \\
\gamma &\equiv \frac{1}{\sqrt{1 - v^2/c^2}}
\end{aligned}
\tag{5.2}
$$

As we saw in chapter 4, transformations between reference frames can be represented by matrices which greatly reduce the amount that has to be written. In the next section we will introduce a new type of notation that similarly simplifies notating coordinate transformations in special relativity.

5.2 Four Vector Notation

Let us adopt the following notational convention.

$$x^0 = ct, \qquad x^1 = x, \qquad x^2 = y, \qquad x^3 = z \tag{5.3}$$

The Lorentz transformation can then be written as

$$\begin{aligned} x^{0'} &= \gamma(x^0 - \beta x^1) \\ x^{1'} &= \gamma(x^1 - \beta x^0) \\ x^{2'} &= x^2 \\ x^{3'} &= x^3 \\ \beta &= \frac{v}{c} \end{aligned} \tag{5.4}$$

If μ takes on values 0,1,2,3

$$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} \tag{5.5}$$

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.6}$$

$$x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\nu} \tag{5.7}$$

So in an analogous way to chapter 5, a coordinate transformation gets represented with a matrix. The relativistic invariant quantity similar to the length of a vector in euclidean geometry is:

$$S^2 \equiv (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x^{0'})^2 - (x^{1'})^2 - (x^{2'})^2 - (x^{3'})^2 \tag{5.8}$$

S is analogous to arc length interval in geometry. It is an interval in space-time.

5.3 Einstein Summation Convention

Einstein summation convention tells us to sum over repeated indexes. The infinitesimal invariant space time interval can be written in this notation as

$$dS^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = g_{\mu\nu} dx^\nu dx^\mu \tag{5.9}$$

where $g_{\mu\nu}$ is called the space-time metric. In special relativity which treats space-time as flat

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{5.10}$$

In general relativity where space-time is curved, the metric takes on more interesting values besides just ones. Whether an index appears on as a subscript or a superscript is a matter of whether it has a factor of the metric attached to it. For instance $x_\mu = g_{\mu\nu} x^\nu$. If you see a Greek letter upper index multiplied by something that has that same Greek letter as a lower index, they "contract" and cancel. In particular $g_{\mu\nu} g^{\mu\nu} = \delta^{mu}_{mu} = 1 + (-1)^2 + (-1)^2 + (-1)^2 = 4$.

5.4 Relativistic Invariances

We can define the four velocity as $U^\mu = \frac{dx^\mu}{d\tau}$ where τ is the proper time. Four momentum then is just the four velocity times mass. The conserved quantity in relativistic kinematics is:

$$p^\mu p_\mu - m^2 c^2 = 0 \tag{5.11}$$

5.5 Maxwell's Equations Revisited

Let us make another notation convention:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \tag{5.12}$$

This is sometimes referred to as the four divergence, which is apparently deserving of the name when we translate it back into familiar Cartesian notation:

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}, \qquad \partial_1 = \frac{\partial}{\partial x}, \qquad \partial_2 = \frac{\partial}{\partial y}, \qquad \partial_3 = \frac{\partial}{\partial z} \tag{5.13}$$

Let us now see if we can tidy up Maxwell's equations with our all the new notations we've adopted. Here they are again without our four vector notation:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi\rho \quad &(i) \\ \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial B}{\partial t} &= 0 \quad &(ii) \\ \nabla \cdot \mathbf{B} &= 0 \quad &(iii) \\ \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c}\mathbf{J} \quad &(iv) \end{aligned} \tag{5.14}$$

To make use of four-vectors, let us define the following two objects. The first of which is the tensor $F^{\mu\nu}$.

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \tag{5.15}$$

$F^{\mu\nu}$ is referred to as the Electromagnetic field strength tensor. Notice that the EM field strength tensor is both antisymmetric ($F^{\mu\nu} = -F^{\nu\mu}$) and traceless (all 0's down the diagonal). The second object we will need to define combines charge density and current density (charge density on the move) into one four-current J^μ

$$J^\mu = (c\rho, \mathbf{J}) \tag{5.16}$$

If we take the four divergence of the EM field strength tensor, we find:

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu \tag{5.17}$$

Let's take a second derivative of this expression

$$\partial_\nu \partial_\mu F^{\mu\nu} = \frac{4\pi}{c} \partial_\nu J^\nu \tag{5.18}$$

The two lower indexes μ and ν acting on F's two upper μ and ν has the effect of taking the trace of $F^{\mu\nu}$, but $F^{\mu\nu}$ is traceless implying the left hand side of the equation is zero. Dividing out $\frac{4\pi}{c}$ we find that:

$$\partial_\mu J^\mu = 0 \tag{5.19}$$

Notice I have substituted ν for μ here. In equations with only one index, what you *call* that index is entirely arbitrary. $\partial_\mu J^\mu = 0$ is entirely equivalent to $\partial_\nu J^\nu = 0$.

If you've already taken an upper level EM course, you may recall that the magnetic field $\mathbf{B}$ is derivable from a magnetic potential $\mathbf{A}$.

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{5.20}$$

Rewriting equation (ii) of (5.14) in terms of $\mathbf{A}$:

$$\nabla \times \left(\mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \tag{5.21}$$

You can think of the divergence operator as counting up all the flow thats going in the radial direction and the curl as counting up all the flow thats tangential to the radial direction. Therefore if you take the curl of a divergence you'll get zero, since the divergence got rid of everything that was tangential. Recall that the electric field can be written as the gradient of a potential V. Since

$$\nabla \times (-\nabla V) = 0 \tag{5.22}$$

and (5.21) holds, we can infer that

$$\mathbf{E} = -\nabla V - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}. \tag{5.23}$$

Thus, eq. (5.21) becomes

$$\nabla \times \left(-\nabla V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \right) = \nabla \times (-\nabla V) = 0 \tag{5.24}$$

This equation can be further simplified if we introduce another four vector quantity:

$$A^\mu = (V, \mathbf{A}) \tag{5.25}$$

We can now define the EM strength tensor in the following way (check for yourself!)

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{5.26}$$

Plugging this into eq. (5.18) yields:

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = \frac{4\pi}{c} J^\nu \tag{5.27}$$

Further simplifications in notation can be defined in the following way:

$$\Box A^\mu = \frac{4\pi}{c} J^\mu \tag{5.28}$$

where the box operator (called the D'Alembertian) is

$$\Box \equiv \partial^\mu \partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{5.29}$$

It is left now as an exercise to the reader that this equation reproduces Maxwell's equations.

6
What do we measure?

BEFORE WE DISCUSS ANY THEORIES beyond classical electrodynamics it will be fruitful to first have a discussion of what quantities we would like our theories to predict for the experimentalists to measure in the lab.

6.1 Decay Rates

A theory is a good theory if it makes more predictions than the measurements it needs.

One quantum observable is a decay rate Γ. A decay is a process that has no memory. The probability that a particle decays is constant in time.
$|\langle f|i\rangle|^2 = constant\ P(t, t+dt) = \Gamma dt \rightarrow \Gamma$ is constant.

If at time t I have N particles, then at the time $t + dt$ I have $N + dN$ particles.

$dN = -(\Gamma dt)N$ (divide by N & integrate) $\rightarrow N(t) = N(t_\circ)\,\mathrm{e}^{-\Gamma(t-t_\circ)}$

Thus we can measure Γ. A particle X can decay through different channels:

Decay Mode	Decay Rate
$X \rightarrow A + B + C$	$\Gamma_{X\rightarrow A+B+C}$
$X \rightarrow M + N$	$\Gamma_{X\rightarrow M+N}$
$X \rightarrow Y + Z$	$\Gamma_{X\rightarrow Y+Z}$

The total decay rate is just the sum of the partial ones.

$\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_{(i)}$ **Total Decay Rate**

$\text{BR}(X \rightarrow A + B + C) = \frac{\Gamma_{X\rightarrow A+B+C}}{\Gamma_{\text{tot}}}$ **The Branching Ratio**

6.2 Cross Sections

Imagine you have an array of paint ball guns aimed a white wall of area A. Inside the white wall is a detector that makes a beep every time a paint ball hits it. Hovering in front of the wall are N number of identical metal spheres which present an apparent area of $\sigma = \pi r^2$ called the "effective cross-section." The ratio of the white wall covered by the spheres is $N\sigma/A$. You turn on your array of paint ball guns, keeping track of how many paint balls are fired and how many hit the wall. If you shoot n_{shot} paint balls, and hear n_{beep} beeps then the fraction of paint balls that got stopped by the spheres is $n_{stop} = (n_{shot} - n_{beep})/n_{shot}$ which for a sufficient number of paint balls shot should be equal to the ratio of the wall's area covered by the spheres. $N\sigma/A = (n_{shot} - n_{beep})/n_{shot}$. Rearranging this equation we find that the effective cross section is

$$\sigma = \frac{A(n_{shot} - n_{beep})}{N\, n_{shot}} \tag{6.1}$$

and from this we now know the area and radius of one of our metal spheres. The whole procedure utilizes probabilities to measure distances.

This is a bit of a naive picture of cross sections. In actual particle physics, it isn't a simple matter of whether the metaphorical paint ball hits the target in front of it. The "target" spheres in particle physics interact with the projectiles fired at them, scattering them off at various angles. The angles that incoming particles get scattered off at is a function of the type of interaction between the target and the incoming particle, the momentum of the incoming particle, and the distance the incoming particle would have missed the exact center of the target (the impact parameter). For example, say you have a electron scattering off of a proton target. The proton is much more massive than the electron, and for all of our purposes we can treat the proton as a stationary object before and after the electron scatters off of it. The closer the fired electron comes to the target proton, the greater electromagnetic force will deflect the electron, scattering it off at a sharper angle.

7

Relativistic Quantum Mechanics

7.1 The Dirac Equation

To properly describe reality, we need to pair quantum mechanics with special relativity. The Schrodinger equation describes nonrelativistic quantum mechanics. When we include relativity, considering also the particles spin we obtain the following equivalents to Schrodinger's wave equation: For spin 0 relativistic quantum there is the Kelin–Gordon equation; Spin 1/2 is Dirac equation; Spin 1 is the Proca equation.

One way to derive the Schrodinger eq is to start with the classical Hamiltonian, the invariant total energy of the system.

$$\frac{p^2}{2m} + V = E \tag{7.1}$$

and then make the canonical substitution

$$\mathbf{p} \to -i\hbar\nabla, \qquad E \to i\hbar\frac{\partial}{\partial t}. \tag{7.2}$$

Replacing momentum with the momentum operator, and then acting this equation onto the quantum state vector Ψ.

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{7.3}$$

To make a field equation that's relativistic, rather than start with a classical invariant like the Hamiltonian, we pick a relativistic invariant such as:

$$p^\mu p_\mu - m^2c^2 = 0 \tag{7.4}$$

Making an equivalent canonical substitution for four momentum:

$$p_\mu \to i\hbar\partial_\mu \tag{7.5}$$

and recalling that

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \tag{7.6}$$

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}, \qquad \partial_1 = \frac{\partial}{\partial x}, \qquad \partial_2 = \frac{\partial}{\partial y}, \qquad \partial_3 = \frac{\partial}{\partial z} \tag{7.7}$$

we can act this now quantized invariant quantity onto our quantum state vector to get another equation:

$$-\hbar^2\partial^\mu\partial_m u\psi - m^2c^2\psi = 0. \tag{Klein-Gordon Equation}$$

The Klein-Gordon equation is referred to as a scalar field equation. Rewriting in our familiar Cartesian coordinates:

$$-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi = \left(\frac{mc}{\hbar}\right)^2\psi \tag{7.8}$$

One issue with this equation is that it is not reconcilable with the Born statistical interpretation of quantum mechanics. This is due to the fact that it has two time derivatives as opposed to the one in the Schrodinger equation. So Dirac set out to find a new equation that was first order in time. The result of his efforts was:

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi - mc\psi = 0 \tag{Dirac Equation}$$

where

$$\gamma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{1} & 0 \end{pmatrix} \tag{7.9}$$

and

$$\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{7.10}$$

8
Chirality

In geometry, an object is chiral (possesses chirality) if it is asymmetric in a way that makes it not identical to its mirror image. In physics, a chiral phenomenon is something that doesn't look the same if you watch it happen in a mirror. For this reason, chirality is deeply related to parity.

8.1 Parity

A parity transformation in 3 dimensions is given by:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \tag{8.1}$$

Using a similar line of reasoning to Newton and Einstein–that the laws of physics should somehow be invariant in different inertial reference frames–physicists assumed the laws of physics should also not care whether you're observing them through a mirror or not. It was Chien-Shiung Wu who demonstrated in 1956 that nature doesn't play by those rules when she showed that in the beta decay of cobalt 60, almost all of the electrons are emitted in the direction opposite to nuclear spin. Spin is arbitrarily defined by the right hand rule: curl your fingers around in the direction the object is spinning and point your thumb up. The direction of your thumb is the direction of the spin. By looking at the cobalt in the mirror, the object will spin in the opposite direction giving an opposite spin vector, but the electrons will still be emitted in the same direction, which now is in the *same* direction as the nuclear spin. This parity violation is our first taste of a chiral phenomenon (something that isn't the same as its mirror image).

A variable is called **even** if under a parity transformation it is invariant. A variable is called **odd** if under parity transformation its sign is flipped.

8.2 Helicity

We are familiar at this point with the spin of a particle. But for the sake of brief review, the total angular momentum $\vec{J} = \vec{S} + \vec{L}$ where $\vec{S}$ is this spin and $\vec{L}$ is the orbital angular momentum. Helicity is defined as the projection of $\vec{J}$ onto the linear momentum $\vec{p}$. But $\vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{p} \cdot \vec{L} = 0$. Therefore $\vec{p} \cdot \vec{J} = \vec{p} \cdot \vec{S}$. Thus helicity is the projection of a particle's spin onto its momentum. If a particle's spin is in the direction of its momentum we say that it is a right-handed particle (or right-handed helicity). If it points opposite we call it left-handed.

Imagine there is a spin-up electron traveling at some velocity $\vec{v}$ with right-handed helicity. Because an electron has mass, it cannot exceed the speed of light so you decide to put on your rocket powered roller skates and race the electron. As you overtake the electron it now appears to have a momentum going backwards in your rest frame, so now $\vec{p} \cdot \vec{S}$ picks up a minus sign in $\vec{p}$ and the helicity has flipped from right-handed to left-handed, simply by traveling faster than it. Thus, helicity is *not* an intrinsic property of massive particles since it changes depending on what frame of reference you're looking at them in.

However, if we repeat the same scenario with a massless particle which always travels at the speed of light its helicity will always be the same. There is no reference frame in which we could overtake a photon. So what we have now is that helicity is sometimes an intrinsic property and sometimes not. Wouldn't it be nice if there was some quantity that was intrinsic to all particles? A property that reduced to helicity when mass is added to considerations?

8.3 Chirality

A spin 1/2 fermion is an asymmetric particle. If you rotate it 360 degrees you pick up a minus sign. Let $|\Psi\rangle$ be the state of the fermion. Chirality is defined by operating on $|\Psi\rangle$ with the projection operators $(1-\gamma^5)/2$ and $(1+\gamma^5)/2$ where γ^5 is one of the dirac matrices which solve the Dirac equation discussed in the previous chapter. The eigenvalues of γ^5 are ± 1 and therefore $|\Psi\rangle$ has left or right chirality components in an analogous way to the spin of a particle. To be less obscure at cost of accuracy, chirality can be thought of the orientation of the rotation a fermion goes through to pick up its minus sign after its 360 degree rotation. (Like rotating through the complex unit circle, but with 2π only getting you to -1.) The γ^5 operator is identical to the helicity operator for massless fields.

If a theory is asymmetric in its chiralities it is called a chiral theory. The electroweak theory is an example of a chiral theory. The weak interaction only affects left-handed fermions $\Rightarrow$ parity is not conserved $\Rightarrow$ electroweak theory is a chiral theory. This is to be contrasted with a vector theory, such as QCD where left and right-handed chiral particles behave identically. If right-chiral and left-chiral particles behave in the same way, we say there is a chiral symmetry.

Part II

Experiment

9

Tools of the Trade

9.1 Triggers

Particle physics would be impossible without triggers. Due to the sheer bulk of events happening at any one given moment, indiscriminate recording of reality would both be intractable to analyze, as well as impossible to store on your lab's computer (about 2TB of information would be created each second!). A trigger is in essence an "on" switch, which only fires when certain conditions relevant to your experiment have been met. This allows you to only record events of interest and reduces the amount of data you collect to about 20MB/s. How you set up your trigger is experiment dependent–one trigger certainly does not fit all applications.

9.2 Cosmic Rays

Cosmic rays are high energy particles from outer space. There are two classes of cosmic rays: primary cosmic rays, which are mostly protons or alpha particles originating outside the solar system, and secondary cosmic rays, which are created when the primaries interact with particles in the atmosphere. "Cosmic ray" is a bit of a misnomer. When they were originally discovered by Victor Hess in 1912, they were believed to be electromagnetic in origin. However, cosmic rays are not "rays" or even one type of radiation; "cosmic rays" is a blanket term for the various charged particles that either arrive or are created by those arriving in our atmosphere. If you are inside, perhaps on the third floor of a fifteen story building (like our lab in LGRT), odds are all of the cosmic rays available for a trigger are muons. The muon is a lepton, meaning it is a spin-1/2 particle, obeys the Pauli exclusion principle, and does not interact with the strong force. It has a very long life time relative to other cosmic rays, a low interaction rate, and deposits very little energy when it passes through materials. It is for these reasons most every (non-noise) event the trigger detects will be a passing muon. Though not all cosmic rays we detect will be muons, they will most always be minimum ionizing particles, and one can ensure that they are detecting cosmic rays and not a radioactive source someone left about by measuring the energy deposited in a scintillator during a triggered event.

9.3 Scintillators and Photomultiplier Tubes

A scintillator is defined to be any material that produces a pulse of light shortly after the passage of a particle.[4] The size of the pulse is a measure of ionization energy deposited by a passing particle. There are two main categories of scintillators: organic and inorganic. Within each category are different materials, all having their own particular strengths and weaknesses. When constructing a trigger, it is best to choose a scintillator with a good time resolution (neglecting spatial resolution) since our only desire is to trim the amount of data we collect.

Among organic scintillators, the fastest response times can be obtained with liquid scintillators, but often the most elegant solution is to use a plastic scintillator which is easier to work with but still has fast response times. When a charged particle passes through the scintillator, it loses a fraction of its energy and excites atoms within the scintillator. When the atoms deexcite, light is produced. In a plastic scintillator this light is transported to a photomultiplier tube (PMT) by total internal reflection using a light guide, usually also made of plastic.

Once light reaches the PMT, it is converted to an electrical pulse through the photoelectric effect. At the interface between the PMT and scintillator is a photocathode coated with a material with a low work function to ensure electron emission in the PMT.[4] A relevant consideration of a PMT is its quantum efficiency: the number of emitted electrons per incident photon. This allows you to know the incident radiant power from the photoelectric current. After emission, electrons are accelerated by a high voltage through the PMT creating more free electrons. The current is then read out at the anode of the PMT. No PMT is totally free of noise, and noise usually manifests itself in the form of dark current and afterpulsing. Dark current is created by thermally emitted electrons from the photocathode, whereas afterpulsing arises when an ion makes it back to the photocathode and initiates a second electric pulse not correlated with any new event in the scintillator.

The scintillator and the PMT working together can be used to generate a signal for fast electronics, which then analyze the analog input and check if it matches your criteria for recording the information from an event.

9.4 Fast Electronics

Fast electronics modules are used to decide if the signal from a detector matches the event you want to trigger on. They are called "fast" because they are able to process pulses at 100 MHz. Two essential fast electronics are used throughout our lab work: a discriminator and a coincidence unit. Discriminators convert analog signals from the detectors to digital. The output of a discriminator can be only one of two possible states: zero or one (TRUE or FALSE; ON or OFF), and voltage levels have been arbitrarily chosen (often 5V for on, 0V for off) to represent these logic states. Most discriminators have two settings: threshold and width. The threshold is the voltage level at which it decides it should switch from OFF to ON. Once the discriminator determines that it should turn ON, the width governs the duration for which it outputs its ON signal before it switches OFF again. A coincidence unit takes input from discriminators. It performs further logical operations, such as AND, NAND, OR, etcetera.

10
Multiwire Proportional Chamber

The multiwire proportional chamber (MWPC) was developed at CERN in 1968 by Georges Charpak. Charpak, along with colleague Fabio Sauli have produced multiple sources detailing MWPCs. Two sources are here cited for this review: Principles Of Operation of Multiwire Proportional and Drift Chambers by Sauli [5], and Multiwire Proportional Chambers and Drift Chambers by Charpak and Sauli. [6] These works are also supplemented by Fernow's[4] chapter on proportional counters in his Introduction to Experimental Methods in Particle Physics.

Fernow gives the most concise introduction to proportional counters I have come upon:

"Proportional Chambers are particle detectors consisting essentially of a container of gas subjected to an electric field. A passing particle can leave a trail of electrons and ions in the gas. The charged particle debris are collected at the chamber electrodes and in the process provide a convenient electrical signal, indicating the passage of the particle. The detector operates as a proportional chamber when the applied electric field is large enough so that the accelerated electrons cause secondary ionization, yet small enough so that the output pulse is still proportional to the number of primary ion pairs. Multiwire proportional chambers are widely used for particle tracking and for triggering."

To this day, the MWPC remains one of the most reliable methods of detecting particles. However, before we go into further detail about *multiwire* proportional chambers we will first describe a single wire proportional chamber (as Charpak has done in his writeup.) Imagine you have one conducting wire stretched down the axis of a conducting cylinder, and the cylinder is filled with an ionizable gas. A voltage is applied between the chamber walls at ground and the wire. As an ionizing particle passes through the cylinder, it liberates electrons from the gas atoms which then drift towards the wire. Once electrons have drifted to the near vicinity of the wire, they have picked up enough kinetic energy to knock electrons out of other atoms creating an avalanching effect close to the wire. The positive gas ions (referred to as holes) drift away from the wire all the way to the metal casing. This process creates a pulse on the wire which then can be read out as a current.

The jump from a single wire to multiwire is essentially putting many of these single wire's next to each other inside the metal casing. With the wires in an array, measuring the current pulses tells us about the particles path with accuracy.

As quoted from Charpak[6] these physical phenomena control the properties of the detector:

1. The energy loss distribution of the radiation being detected. While in most applications no useful information is required from the pulse heights, there are cases of growing importance where it is necessary to have a response proportional to the energy losses: transition radiation detectors, identification of particles in the relativistic rise region, etc.

2. The drift of electrons and ions in gases under the influence of moderate electric fields, where they do not experience ionizing collisions.

3. The multiplication of electrons in short-range avalanches produced by the very intense fields in the vicinity of the anode wires.

4. The propagation of discharges over large distances, mediated mainly by photons emitted by the atoms excited in the avalanche process.

5. The electrostatic properties of multiwire structures which control the charge distribution on the different electrodes and the relative intensity of the electric field

6. The charge distributions induced on the different electrodes by the motion of the liberated ions.

In our design, we will also be adding field wires (naming the wires in a standard MWPC sense wires) held at ground potential interspersed between the sense wires to minimize the low electric field regions and to narrow the drift time distribution of ions to achieve a more time-sensitive detector.

10.1 Gas Gain and Gas Mixtures

The functionality of the detector depends heavily on the gas used to fill it. When a particle passes through it ionizes the gas. The argon once excited can only return to the ground state through a radiative process, which will produce a photoelectron from the cathode. This photoelectron then can again be accelerated to create another avalanche, but this avalanche wont correspond to a passing particle. To combat the effects from secondary emission we introduce polyatomic molecules to the mixture, which have more rotational and vibrational degrees of freedom allowing for many more modes to non-radiatively return to the ground state. This type of behavior is common among most hydrocarbons and alcohols and of several inorganic compounds such as freon, CO_2 and BF_3.

The molecules can dissipate the excess energy through elastic collisions or by breaking apart.

These additive polyatomic chemicals serve as quenchers, and they change entirely the operation of a counter. They offer good photon absorption and help to remove complications from secondary emissions.

Part III

Lab Exercises

11
Characterizing a PMT

In this chapter we present a tutorial for educational laboratories and novices on building an optimized cosmic ray trigger for particle physics experiments using scintillators, photomultiplier tubes, and NIM electronics. We discuss what results indicate the trigger has been properly set up and is ready to be used in an experiment. Our particular triggering system uses a dual faced long plastic scintillator paddle with two photomultiplier tubes on either end, an amplifier, discriminator, and coincidence module. We characterize each photomultiplier tube and create an energy loss spectrum to verify our cosmic ray signal. Once set up, our apparatus can be easily modified for interesting experiments, such as using a spark chamber to measure the mass of a muon, or analyzing ion drift times in a multi-wire proportional counter.

11.1 Methods

11.1.1 Circuit Logic

To create the circuit logic that builds the trigger, we used an approximately 2 meter scintillator paddle, two photomultiplier tubes (PMTs), a Power Designs Inc. model 1570 High Voltage supply for PMT A, an Ortec 456 0-3KV High Voltage supply for PMT B, Nuclear Instrumentation Modules (NIM) plugged into a NIM bin to do the actual logic, an EG&G/Ortec Model DB463 Delay Box to delay the raw signal from the PMTs until it matches up with the signal that had to pass through the circuit logic, and a Tektronix TDS 2042B oscilloscope to display the two PMT signals when the trigger from the NIM modules tells the scope to fire. The scintillator created light when a cosmic ray passes through it, which then was turned into an electric signal via the photoelectric effect in the PMTs. The signals from each of the PMTs were then carried via two equal-length BNC cables to a LeCroy model 612A 12 channel PM amplifier. This amplifier only multiplied voltage–it **did not integrate** the signal. This is important to note here because if the signal had been integrated, determining the energy deposited in the scintillator by the cosmic ray would have been a matter of counting peak heights of captured waveforms on the oscilloscope. Integration of the signal was left to post data collection analysis. On the amplifier, for every input there are two outputs. One of the outputs was sent to a delay box to be read out later on the oscilloscope; the other output was sent to a LAS model 621AL quad discriminator which converts the analog signal into a digital one. On the discriminator, two settings were adjusted: the voltage threshold (i.e. the height of the signal below which it doesn't bother sending an ON signal) was set to .35 V for each PMT, and the time width (duration of the ON pulse) was set to 50 ns for each PMT input during the voltage sweep. When performing the integrals of the signals, the time width was decreased to 15 ns to help further clean up our signal. 15 ns was the time it takes for an information of an event happening at the furthest edge of the scintillator to reach the PMT at the other end. This was calculated using the speed of light c in a vacuum, the index of refraction of a plastic scintillator $n = 1.58$ and the length of the scintillator $L = 193$ cm. Since the photons are guided to the PMTs via total internal reflection, an assumption was made that they are bouncing at 45° angles towards the PMTs and a factor of $\frac{1}{\sqrt{2}}$ was added to the speed.

$$\begin{aligned} t &= \frac{Ln}{c}\sqrt{2} \\ &= \frac{193 \text{ cm}}{c} 1.58\sqrt{2} \\ &= 14.4 \text{ ns} \end{aligned}$$

This calculation was rounded up to 15 ns as an extra precaution.

From the discriminator box, each (now digital) PMT signal is sent to a EG&G Model C314NL Major Coincidence module. Only when the coincidence module receives the ON signal from PMT A and PMT B within their 50 ns second windows does it produce an ON signal. This ON signal is then sent two places: an Ortec model 778 Dual Counter (though only one of its inputs were used), and the Tektronix oscilloscope to be used as a trigger for the scope.

All the electrical information between PMTs, NIM modules, delay boxes and the oscilloscope were carried via BNC RG58 cables, and LIMO connectors which require 50 Ω termination. Incorrectly terminated connections will produce feedback in your signal.

11.1.2 Voltage Sweep

First both high voltage supplies for each tube were set to their approximate optimal voltages. These values were determined by observing where each PMT signal on the scope looked most like a characteristic cosmic ray signal. These approximate optimal values were 1250 V for tube B, and 1370 for tube A.

Tube B was held fixed at its approximate optimal value of 1250 V while a 500 V

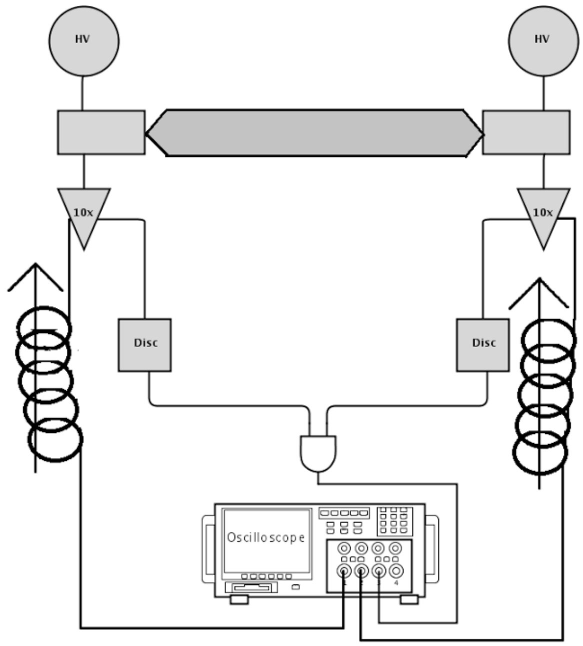

Figure 11.1.1: Circuit to create trigger logic.

sweep was performed on tube A. Then tube A was set to its approximate optimal value of 1370 V while a 500 V sweep was performed on tube B.

To collect data, we synchronized a 5 minute timer with the Ortec counter module. The number of events read out on the counter at the end of 5 minutes was then divided by the number of seconds in 5 minutes to give the trigger frequency in Hz. A 5 minute trial was performed every 20 volts for tube A, and every 10 volts for tube B starting 250 volts below their approximate optimal values.

11.1.3 Energy Loss

To determine the energy deposited in the scintillator by a passing cosmic ray, we integrated the first peak of captured waveforms from the oscilloscope produced by PMT A. This calculation was repeated for many waveform captures for the purpose of creating an energy loss distribution. FIG. 11.1.2 shows one such waveform an integration was performed on. The integral value was then plotted on a energy vs. count plot.

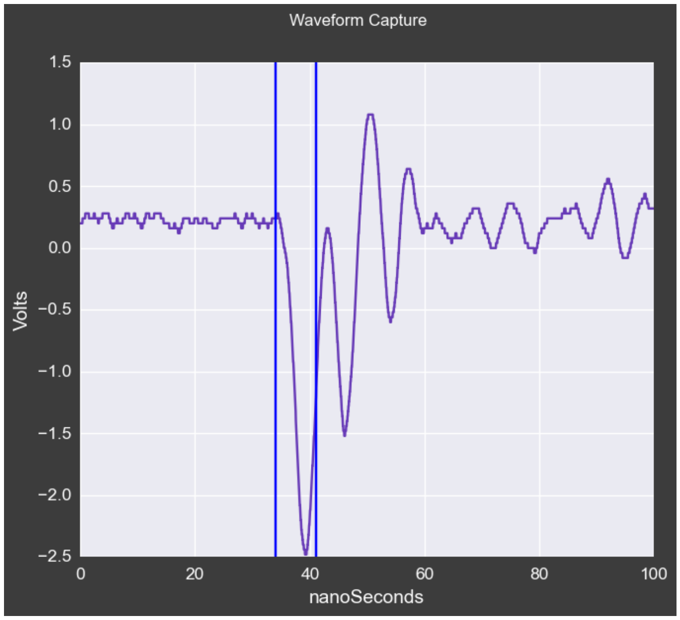

Figure 11.1.2: Sample waveform capture. The vertical lines represent the timing window the integration was performed on. The subsequent reflections were a product of noise and incorrect terminations on the BNCs inputting into our oscilloscope, which were later fixed.

11.2 Results

11.2.1 Optimized PMT Voltages

Figs. 11.2.1 and 11.2.2 show the results of sweeping a 500 volt range on one PMT while holding the other at its approximate optimal voltage. In both figures, the left axis is the trigger rate in Hz of the coincidence NIM module.

Fig. 11.2.1 clearly has a change in slope beginning at about 1290 V, and noise inside the PMT begins to have a profound effect on the coincidence trigger rate beyond 1450 V.

Fig. 11.2.2 shows that tube B's growth has changed to a new rate by 1250 V. PMT noise is not significant for tube B in the voltage range swept out.

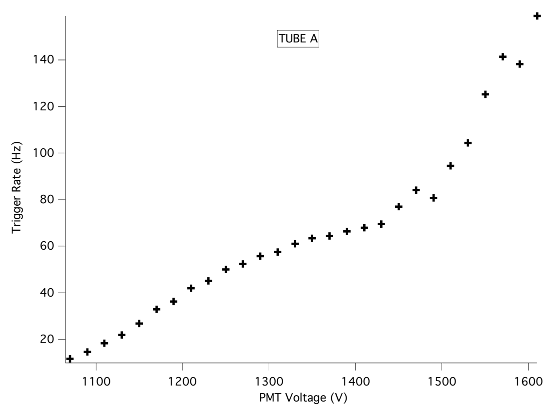

Figure 11.2.1: Trigger rate vs. PMT voltage, Tube A with Tube B fixed at 1250 V

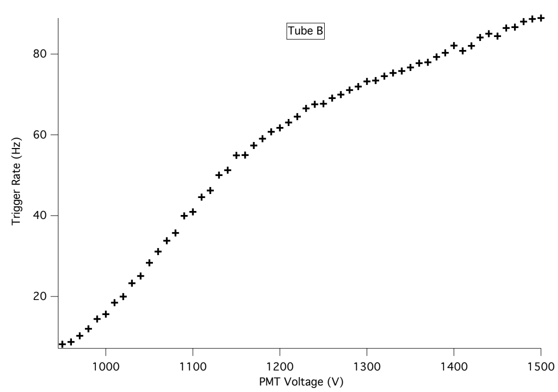

Figure 11.2.2: Trigger rate vs. PMT voltage for Tube B with Tube A held at 1370 V.

PMT	Optimal Voltage
Tube A	1390 V
Tube B	1350 V

Table 11.2.1: PMTs optimal voltage.

11.3 Energy Loss of Cosmic Rays

FIG. 11.3.1 shows the distribution of energies from the PMTs, proportional to the energy deposited in the scintillator, centered around -0.35×10^{-8} volts.

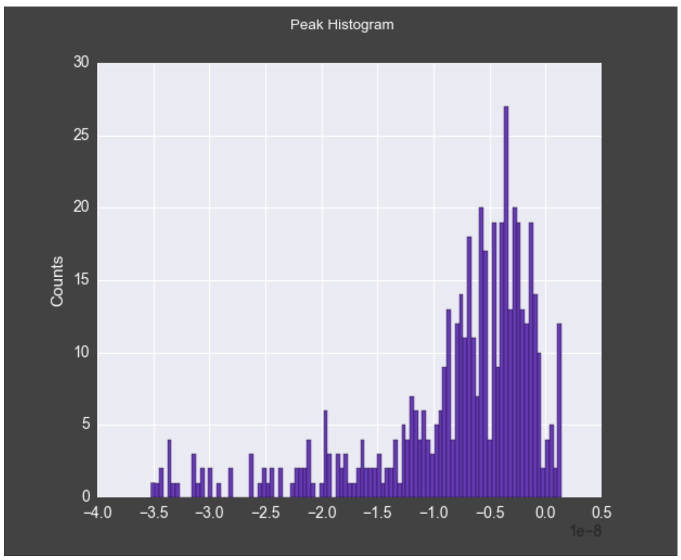

Figure 11.3.1: Tube A: energy deposited histogram.

11.4 Discussion

Ideally, both PMTs would display more plateauing, but the slowing of growth in the trigger rate is enough to indicate that the majority of cosmic ray events are being detected. The continuing steady growth after the decrease in slope in each figure can indicate two things: cosmic rays that only pass through the corners of the scintillator, and noise. For cosmic ray experiments that use the scintillating apparatus only as a trigger, often a second apparatus such as an ionizable gas chamber, or additional scintillators are placed below it. In these experiments triggering too often due to noise or particles passing through the corners is not as detrimental to the results as failing to trigger when a cosmic ray passes through both detecting apparatuses. It is for this reason we declare the optimal voltage for each PMT to be approximately 100 V from where the growth begins to taper off to ensure this does not occur.

Since we are only trying to put ourselves firmly in the plateau region, being rigorously quantitative is excessive; any value 100 V beyond the change in slope before the PMTs give way to internal noise will suffice. The bend for tube A begins at about 1290 V. The bend for tube B begins at about 1250 V. Thus the optimal values for tube A is 1390 V and 1350 V for tube B.

Tube A begins to succumb to noise at 1450 V, but tube B does not have any issue with noise even at 1500 V. This indicates that tube B is more robust than tube A.

As expected, the energy deposited in the scintillator is very near zero: -0.35×10^{-8} volts. This is exactly as it should be for minimum ionizing particles, and this read out is an excellent indication that we are in fact triggering on cosmic rays.

Notice in FIG. 11.3.1 that some values of energy deposited are positive. This is due to errors in our python code determining where the first peak in the waveform capture was. Occasionally it would select noise prior to the initial drop off, or subsequent positive reflections for the peak of interest to integrate over. The integration of subsequent positive reflections was particularly curious, because its peak detection algorithm looked for negative derivatives in slope, and should have no excuse for beginning its integration window after a negative peak on an increasing slope. These errors were infrequent however, and did little to change the overall distribution of energy deposited.

11.5 Conclusion

With the coincidence logic set up, PMTs characterized, and cosmic rays confirmed as the trigger source, a variety of particle experiments are now open for exploration. Note that PMT characterization will have to be performed for *your own* PMTs, and less time consuming methods than using an event counter without a built in timer are likely available. Introduction of more scintillators and PMTs into your setup can allow for more interesting experiments. For example, using two small scintillator paddles placed above a spark chamber, and another paddle below, you can determine that a muon decayed within the spark chamber if the top two PMTs activate their discriminators, but the bottom PMT doesn't. The methods introduced here are an effective procedure for getting a cosmic ray trigger operational for whatever application is desired.

12
MWPC Drift Time Studies

12.1 Introduction

****pre-requisite reading: triggers, PMTs, fast electronics, cosmic rays, MPWCs**

MWPCs are used to track ionizing particles. In the experiment at JLAB a large beam of ionizing particles, mostly comprised of muons and pions, will be incident on our detectors. The ability of the detectors to distinguish between separate particles will be a function of how quickly the liberated charge arrives at the wire cells inside of the detector. If it takes too long for the liberated charge to arrive, two particles passing by the same wire at approximately the same time will appear on our equipment readout as one elongated pulse rather than two separate ones. Thus, it is of utmost importance to minimize the drift time in the gas to maximize our equipments timing resolution.

Drift times are a property of gas compositions. We present here the method and results of measuring the drift times for different gas mixtures in the prototype detector.

12.1.1 Mow to Measure the Drift Time

That there exists a "drift time" implies the MWPC does not produce a signal immediately. A measurement of this drift time can be made by placing a second faster responding detector (such as a plastic scintillator paired with an PMT) in the incident particle's path. Theoretically, either putting the second detector in before or after the MWPC is fine. Once the PMT fires, one needs only to count the time it takes for the MWPC to then fire. An average of many such times gives a measurement of the drift time in the detector.

However, things are a little more complicated. Just because the MWPC or PMT fired doesn't mean that there was a particle that passed through both of them. Sometimes the PMT fires on its own without a passing particle as discussed earlier in this thesis. A logic circuit and trigger must be built to ensure that the signal in both the PMT and the MWPC came from the same particle. The details of the trigger that was set up is presented in the methods section of this lab.

12.2 Methods

Before any precise measurements of the drift time were made, the MWPC signal was observed on the scope. The minimum and max times of arrival were recorded to get a ballpark on the width of the drift time. It was estimated that the drift time was on the order of 700 nanoseconds. This was recorded for use in the cosmic ray trigger. Since the signal from the MWPC would at maximum come 700 nanoseconds later, the width on the PMT pulse was set to 1000 ns to ensure that whenever the MWPC fired, the PMT would still read as ON to produce a

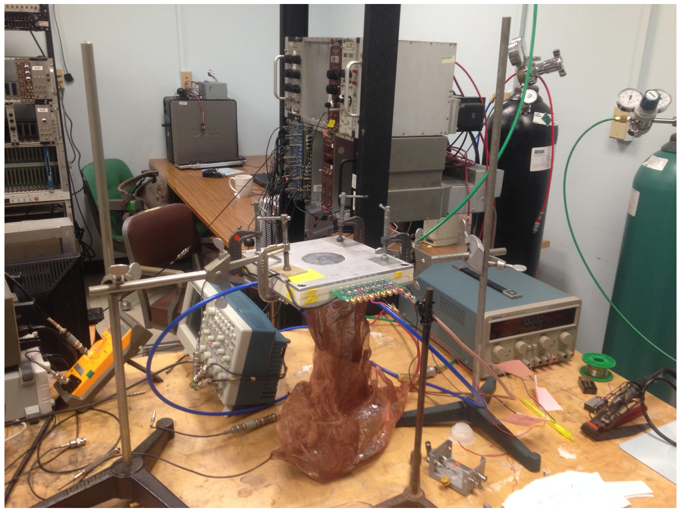

Figure 12.2.1: MWPC suspended above PMT

trigger at the coincidence box.

To make the actual precise measurement, a sodium-iodide scintillator PMT pair was placed under our prototype MWPC. Both were triggered off of cosmic rays. The signals from both devices were split and sent to either the Tektronix TDS 2042B oscilloscope, or the discriminator box. The pulse width set for the PMT on the discriminator was 1000 ns, and the pulse width for the MWPC was 50 ns. The electronics on the MWPC take longer to communicate information than the PMT, so a 700 ns delay was added to the PMT signal to ensure both signals arrived at approximately the same time. These signals were then routed into a coincidence box, which then was used as a trigger for the oscilloscope.

Whenever our scope triggered, ScopeOut (our oscilloscope waveform capture program) pulled the waveform info and measured when the trigger arrived. The difference in the earliest arriving waveform to the last arriving waveform determined drift time.

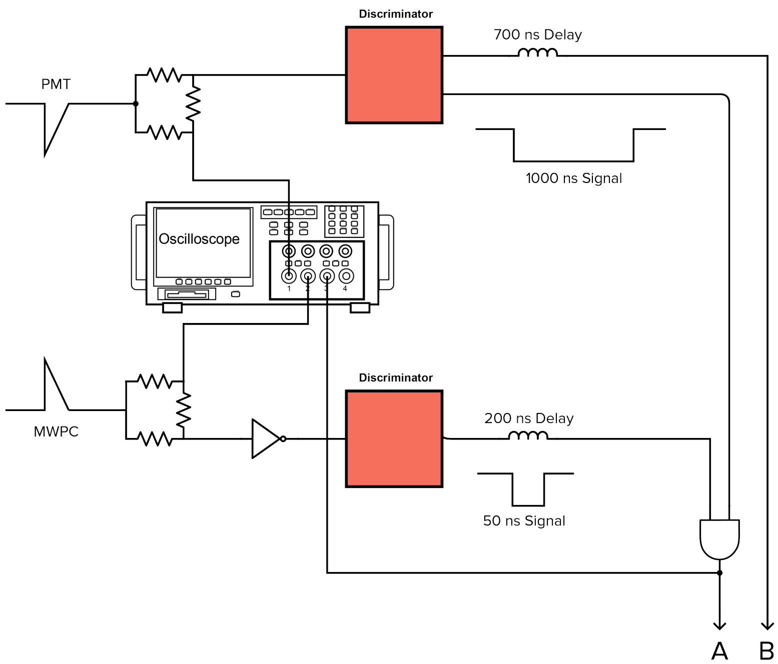

Figure 12.2.2: Trigger Logic

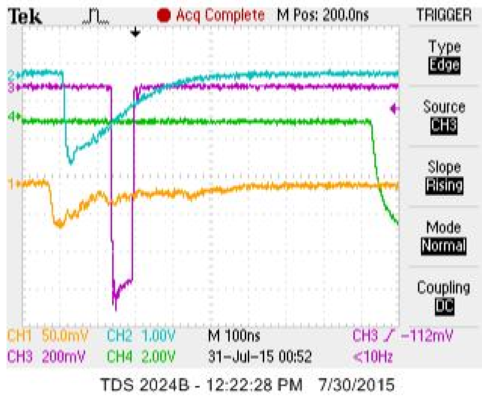

Figure 12.2.3: Scope capture of the 4 channels.

This procedure was repeated for the following gas mixtures in the MWPC:

1. Ar:CO_2 in an 80:20 ratio
2. Ar:CO_2 in a 90:10 ratio
3. Ar:CO_2:CF_4 in a 88:2:10 ratio

The voltages to achieve 10^5 gain for the gas mixtures tested; Ar:CO_2 80:20 @ 2000 V ; Ar:CO_2 90:10 @ 1800 V ; Ar:CO_2:CF_4 88:2:10 @ 2100 V.

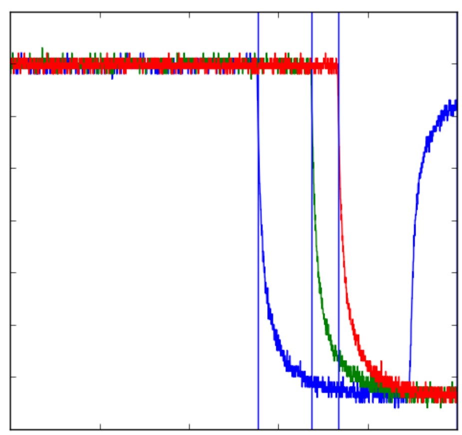

Figure 12.2.4: Three superimposed trigger signals.

12.3 Results

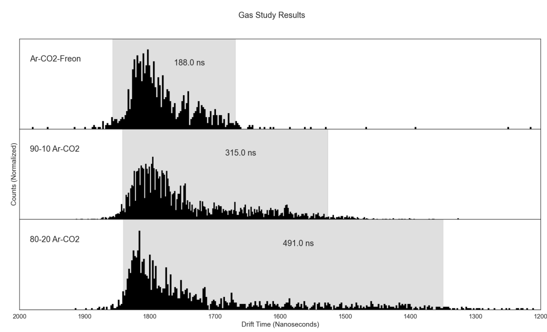

Figure 12.3.1: Drift times for the three mixtures.

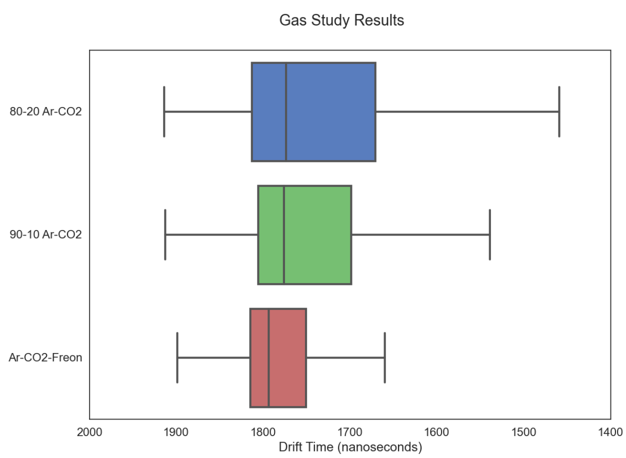

Figure 12.3.2: Box and Whisker plot

12.4 Conclusion

Clearly the drift time for the freon mixture was the fastest. This is exactly as expected, as this gas composition has much less CO_2 quencher in it. The draw back to using less quencher was discussed in the section on MWPC gas mixtures.

However, internal chamber sparking was not observed, indicating that this mixture will likely be used in the full size detectors.

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