

PHY-602: Statistical Physics, UMass Amherst, Problem Set #11

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Due: Wednesday, Dec 6. (Late homework receives 50% credit.)

I. SPECIFIC HEAT EXPONENT

At a second order phase transition, the free energy density scales as

$$f = A |T - T_c|^{2-\alpha} + \text{regular and higher order terms},$$

with α a critical exponent.

1. How does the specific heat behave near the critical point?
2. Compute the exponent α for the Ising model within mean-field theory.

II. TRICRITICAL POINT

Consider a Landau expansion of the free energy of the form

$$F = \frac{a_0(T - T_c)}{2} m^2 + \frac{b}{4} m^4 + \frac{c}{6} m^6,$$

with $c > 0$.

1. Show that there is a line of critical transitions $T = T_c$, $b > 0$ which joins a line of first order transitions $T = T_c + \frac{3b^2}{16ca_0}$ with $b < 0$ at a point $(T = T_c, b = 0)$. This special point is called a tricritical point.
2. Compute the magnetization exponent β at the tricritical point.
3. Sketch the phase diagram of this model in the (T, b) plane.