PHY-602: Statistical Physics, UMass Amherst, Problem Set #2

Romain Vasseur

Due: Monday, Sept 25. (Late homework receives 50% credit.)

I. PARAMAGNETIC CRYSTAL

Consider a system consisting of N spin-$\frac{1}{2}$ particles, each of which can be in one of two quantum states, $\uparrow$ (up, $S_z = \frac{1}{2}$) and $\downarrow$ (down, $S_z = -\frac{1}{2}$). In a magnetic field B, the energy of a spin in the up/down state is $\mp\mu B/2$ where μ is the magnetic moment.

1. Without any calculation, what dimensionless parameter do you expect to control the behavior of the system as a function of temperature. How do you expect the spins to behave at low and high temperature?
2. Compute the free energy, the energy and the specific heat of this system.
3. The magnetization of the system is defined as $M = \frac{\mu}{N}\sum_{i=1}^{N} S_i^z$. Compute the mean magnetization as a function of the magnetic field and of temperature, and the susceptibility $\chi = \left.\frac{\partial\langle M\rangle}{\partial B}\right|_T$. Show that the susceptibility for small fields scales as $\sim \frac{1}{T}$ (Curie's law).
4. Compute the fluctuations of the magnetization, and relate them to the susceptibility. How do they scale with the number of spins?
5. Repeat the calculation of the magnetization using the microcanonical ensemble.

II. FLUCTUATIONS IN THE GRAND CANONICAL ENSEMBLE

In the grand canonical ensemble, express the average and the fluctuations of the number of particles in terms of the grand partition function.

III. RUBBER ELASTICITY

Let's consider a one-dimensional polymer chain, made of N monomers of length a. The total length of the chain is L (in units of a), and the displacement of each monomer is either $+a$ or $-a$ with no energy difference between these two possibilities. Note that L is defined as the distance between the two end points of the chain, and that monomers can lie outside of the two end points (see figure below).

1. Compute the entropy of the polymer as a function of L and N.
2. Let us define the tension of this chain in analogy with pressure in 3D as $F = -T\frac{\partial S}{\partial L}$ (note that F is not the free energy!). Compute F and analyze its behavior for $L \ll Na$. Explain why such polymer chains are referred to as "entropic springs".

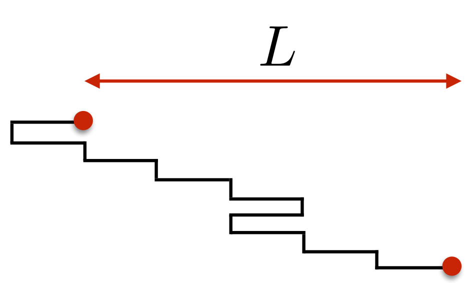

FIG. 1: Polymer chain with $L = 5a$ and $N = 9$ monomers.