PHY-602: Statistical Physics, UMass Amherst, Problem Set #4

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Due: Wednesday, Oct 11. (Late homework receives 50% credit.)

I. ATMOSPHERE THERMODYNAMICS

Let us assume the atmosphere is an ideal gas composed of molecules of mass m.

1. By considering a slab of atmosphere of height dz in (local) hydrostatic equilibrium, find a relation between the variation of pressure as a function of altitude z and the density at altitude z.
2. Assuming an isothermal atmosphere at temperature T, compute the pressure $P(z)$ at altitude z.
3. Suppose now that there is convection in an atmosphere where parcels of gas move up and down in an adiabatic way. Compute the temperature profile $T(z)$ in such an atmosphere.

II. HEAT ENGINE WITH IMPERFECT RESERVOIRS

Consider a heat engine that operates between imperfect reservoirs with variable temperature and heat capacity C. The initial temperatures of the reservoirs are given by T_H^0 and T_C^0 with $T_H^0 > T_C^0$.

1. What is the maximal work that can be extracted from this engine?
2. Define an efficiency η for this engine and give its maximum value $\eta_{\max}$.
3. Let's now assume that over an elementary cycle of the engine, the variations of the temperatures T_C, T_H of the reservoirs remain very small. Show that $\eta_{\max}$ can be expressed as some average of the Carnot efficiency over the cycles of the engine. (Your result should involve an integral over the temperature of one of the reservoirs).

III. SURFACE TENSION

Consider a droplet of liquid of radius r in equilibrium with a gas at temperature T and pressure P_0. The liquid-gas interface leads to a surface tension contribution to the energy given by $E_{\rm s} = \gamma A$ where A is the area of the droplet. By identifying the relevant thermodynamic potential, compute the pressure P_d inside the droplet.

IV. JOULE-THOMSON EXPANSION

Consider the expansion of a gas through a small orifice from a high pressure region to a low pressure region, while kept insulated so that no heat is exchanged with the environment. To analyze this process, imagine a parcel of gas pushed through the constriction so that initially, the gas parcel is entirely in the high pressure region with thermodynamic properties T_1, P_1, V_1. The final condition is the same amount of gas in the low pressure region with T_2, P_2,V_2.

1. Show that the enthalpy $H = E + PV$ is conserved in the process. What is T_2 for an ideal gas?
2. If the pressure drop dp is small, the temperature of the gas changes by an amount $dT = \mu_{\rm JT} dp$ where $\mu_{\rm JT}$ is called the Joule-Thomson coefficient. Compute $\mu_{\rm JT}$ for a non-ideal gas as a function of the specific heat c_p at constant pressure, the density n, the temperature T, and the volumetric coefficient of thermal expansion $\alpha = \frac{1}{V} \left.\frac{\partial V}{\partial T}\right|_{P,N}$.

   hint: you will need to show that $\left.\frac{\partial H}{\partial p}\right|_{T,N} = V(1 - T\alpha)$ *using a Maxwell relation.*