PHY-602: Statistical Physics, UMass Amherst, Problem Set #5

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Due: Wednesday, Oct 18. (Late homework receives 50% credit.)

I. PARTICLE NUMBER DISTRIBUTION

Consider a classical ideal gas in a volume V with mean particle number $\langle N \rangle$.

1. Starting from the grand canonical distribution, show that the probability $P(N)$ to find N particles in this volume is a Poisson distribution $P(N) = \lambda^N \mathrm{e}^{-\lambda}/N!$ where you will identify the parameter λ.
2. Compute all the cumulants $\langle N^k \rangle_c$ and argue that in the limit $\langle N \rangle \gg 1$, $P(N)$ can also be approximated by a normal distribution.

II. ISOTHERMAL-ISOBARIC ENSEMBLE

Let S be a system in contact with a thermostat at temperature T and a barostat at pressure p. The system not only exchanges heat with the thermostat, it also exchange volume with the barostat. The number of particles N in S remains fixed, while the energy E and volume V of the system can fluctuate at thermal equilibrium.

1. Compute the probability of find the system S in a state with energy E_n and volume V_n at thermal equilibrium.
2. Introduce a partition function for this system, and show that it is dominated by states with volume and energy that minimize the Gibbs free energy. What conditions do the most likely volume and energy satisfy?
3. Express the fluctuations of the volume of the system in terms of the compressibility coefficient $\beta_c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$. How do the fluctuations scale with V?
4. Relate the Gibbs free energy to the chemical potential.
5. Consider a classical ideal gas in this "isothermal-isobaric ensemble". Recover the equation of state and the chemical potential of this gas using the results derived above. (You will need to introduce a "quantum of volume" ΔV to make the partition function dimensionless: what condition does ΔV have to satisfy?).

III. ROTATING IDEAL GAS

Consider a classical ideal gas at temperature T in a cylindrical container of radius R and height L rotating at constant angular velocity ω. In the rotating frame, this leads to a potential energy $-\frac{m\omega^2 r^2}{2}$ per particle using cylindrical coordinates. Compute the pressure that the gas exerts on the walls of the cylindrical container. Discuss the high and low temperature limits of your result.