

PHY-602: Statistical Physics, UMass Amherst, Problem Set #6

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Due: Wednesday, Oct 25. (Late homework receives 50% credit.)

I. DENSITY OF STATES IN 1D AND 2D

Determine the single-particle density of states for non-relativistic spinless particles of mass m in $d = 2$ and $d = 1$ dimensions.

II. DIMENSIONAL DEPENDENCE OF THE DEBYE LAW

Often, we are interested in the behavior of systems of reduced dimensionality, *e.g.* two-dimensional thin films, or one-dimensional wires. Determine how the Debye law for low-temperature scaling of the specific heat of vibrations changes with the dimension d of the system. (In this problem, we are only interested in the scaling of the specific heat with temperature T , so that you can ignore irrelevant constant prefactors).

III. GAPLESS MODES AND HEAT CAPACITY

Suppose there are some mysterious excitations in a solid with dispersion relation $\omega \sim k^z$, with some unknown exponent z . These excitations are completely decoupled from the vibrational degrees of freedom in the solid that give the familiar Debye contribution to the specific heat at low temperatures. Assuming that the additional excitations propagate in three dimensions (*i.e.*, are not confined to the surface), determine their additional contribution to the specific heat (up to some dimensionless integral that you do not have to compute), and show that the temperature-dependence can be used to extract the exponent z .

IV. ORTHO/PARA HYDROGEN (*NOT GRADED*)

Hydrogen molecules can exist in ortho and para states.

1. The nuclear spins of H_2 in para-hydrogen form a singlet (antisymmetric) state. Since the total nuclear H_2 wavefunction must be antisymmetric, para-hydrogen can only have rotational wavefunctions that are symmetric with respect to permutation of the two protons. The orbital angular momentum can thus only take even values; that is,

$$H_{\text{para}}^{\text{rotation}} = \frac{\hbar^2}{2I} \ell(\ell + 1), \quad (1)$$

where $\ell = 0, 2, 4, \dots$. Calculate the rotational partition function of para-hydrogen, and evaluate its low- and high-temperature limits.

2. In ortho-hydrogen, the nuclear spins are in a triply degenerate symmetric state (triplet), hence the rotational modes are restricted to $\ell = 1, 3, 5, \dots$. Calculate the rotational partition function of ortho-hydrogen, and evaluate its low- and high-temperature limits.
3. For an equilibrium gas of N dihydrogen molecules calculate the partition function. (*Hint. Sum over contributions from mixtures of N_p para- and $N_o = N - N_p$ ortho-hydrogen particles. Ignore vibrational degrees of freedom.*)
4. Write down the expression for the rotational contribution to the internal energy $\langle E_{\text{rot}} \rangle$, and comment on its high-temperature limit.