PHY-602: Statistical Physics, UMass Amherst, Problem Set #9

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Due: Wednesday, Nov 15. (Late homework receives 50% credit.)

I. CRITICAL POINT OF THE VAN DER WAALS GAS

Consider an interacting gas described by the van der Waals equation of state

$$p = \frac{k_B T}{v - b} - \frac{a}{v^2},$$

with $v = V/N$.

1. Compute the values (T_c, v_c, p_c) at the critical point.
2. Rewrite the equation of states in terms of the rescaled variables $\bar{T} = T/T_c, \bar{v} = v/v_c$, and $\bar{p} = p/p_c$ (this is usually referred to as the "law of corresponding states").
3. Show that near the critical point, as $T \to T_c$ from below

$$v_{\text{gas}} - v_{\text{liquid}} \sim (T_c - T)^{1/2}.$$

II. QUANTUM STATISTICS

Imagine a fictitious quantum system with n degenerate single-particle energy levels with the same energy $\epsilon = 0$. What is the partition function and the entropy of the system if it contains two distinguishable particles? two identical bosons? two identical fermions? (ignore the spin of the fermions, or more precisely, suppose that the fermions are spin-polarized with the same spin quantum state).

III. MAGNETIC DIPOLES

(based on a quals problem)
Consider a mole of magnetic dipoles with $\mathbf{J} = 1$ in the presence of a fixed magnetic field B. The position of each dipole is frozen in space but with a magnetic moment allowed to flip between values $0, \pm\mu_B$ (Bohr magneton).

1. Find the thermal average magnetic moment per dipole and describe the behavior at very low and very high temperatures.
2. What is the Helmholtz free energy of this system?
3. Compute the entropy of this system and sketch it as a function of temperature.
4. Suppose a field of $B = 1$T is applied to the system while it is in good thermal contact with its $T = 1$K environment. Subsequently, the sample is thermally isolated from its environment (i.e., no heat flow) and then the field is reduced to 0.01T. Assume no interactions between the magnetic dipoles.

   Quantitatively describe the final state of the system, in terms of its temperature and its average magnetic moment.